

FACULTY OF SCIENCE Department of Mathematics and Statistics

Applied Mathematics 581 / Mathematics 681

Stochastic Calculus for Finance

(see Course Descriptions for the applicable academic year: http://www.ucalgary.ca/pubs/calendar/)

Syllabus

<u>Topics</u>	<u>Time</u>
Introduction: basics of probability, stochastic processes and math finance	1
Conditional expectation, martingales in discrete and continuous times, examples	2
Discrete-time (B,S)-security markets: capital, portfolio, arbitrage, completeness, self-financing, risk-neutral valuation and measure, options, Cox-Ross-Rubinstein option pricing formula	3
Brownian motion: definition and properties, quadratic variation, Markov property, reflection principle and application to first passage time distribution	6
Stochastic calculus: Ito integral, Ito processes, Ito formula, integration by parts formula, multivariable stochastic calculus, stochastic differential	8
equations, examples Continuous-time (B,S)-security markets: equivalent probability measures and the Girsanov Theorem; financial capital, self-financing portfolios, arbitrage, market completeness; risk-neutral valuation and measure, first and second fundamental theorems of asset pricing, applications to option pricing, Black-Scholes-Merton formulas	8
Stopping times, American options	1
Stochastic interest rates and their derivatives	2
Stochastic models in energy and commodity markets, derivatives	2
Value-At-Risk and risk management	2
Poisson processes, jump diffusions and applications to finance	1
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Course Outcomes for AMAT 581

Students successfully completing this course will be able to:

- 1. Define Brownian motion and analyze its behavior using basic probability and stochastic process theory
- 2. Construct Ito's integral and stochastic differential equations (SDE) based on Brownian motion
- 3. Develop Ito's formula, and use it as a tool to analyze the stochastic dynamics of functions of Brownian motion
- 4. Formulate and analyze the most important elements of a financial model using Ito calculus
- 5. State the fundamental theorems of asset pricing and recreate the key steps in their derivations; use these theorems to distinguish important features of a financial model
- 6. Derive the risk neutral evaluation formula and use it to calculate the prices of contingent claims on financial assets modeled as Ito processes
- 7. Survey a list of advanced topics (American options, stochastic interest rate models, commodity markets, risk management, Poisson processes, jump diffusion and Levy processes) and summarize the key points.

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