MATHEMATICS 213 WINTER 2010
QUIZ 3 SOLUTION

1. Given that $A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}$. Find $adjA$. Make sure that you show all the work.

Solution:

$$\det A^{-1} = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{vmatrix} \underset{R_3 - R_1}{=} \begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -1, \text{ so } detA = \frac{1}{\det A^{-1}} = -1$$

and

$$adjA = (\det A)\, A^{-1} = \begin{bmatrix} -1 & -2 & 0 \\ 0 & -2 & -3 \\ -1 & -3 & -1 \end{bmatrix}$$

2. Let $A = \begin{bmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{bmatrix}$. Find all values of x for which A is not invertible.

Solution:

$$\det A = \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \underset{R_3 - xR_1}{\overset{R_2 - xR_1}{=}} \begin{vmatrix} 1 & x & x \\ 0 & 1-x^2 & x-x^2 \\ 0 & x-x^2 & 1-x^2 \end{vmatrix} = (1-x)^2 \begin{vmatrix} 1 & x & x \\ 0 & 1+x & x \\ 0 & x & 1+x \end{vmatrix} =$$

$$(1-x)^2 \begin{vmatrix} 1+x & x \\ x & 1+x \end{vmatrix} = (1-x)^2 (2x+1).$$

Thus, A is not invertible when $detA = 0$, that is, when $x = 1$ or $x = -\frac{1}{2}$.

3. Show that no 3×3 matrix A exists such that $A^2 + I = 0$, but there exists a 2×2 matrix A such that $A^2 + I = 0$.

Solution:

We prove that no 3×3 matrix A exists such that $A^2 + I = 0$ by contradiction. Suppose that there exists a 3×3 A exists such that $A^2 + I = 0$. Then $A^2 = -I$ and so $(\det A)^2 = (-1)^3 \det I = -1$ which is impossible. Thus, no 3×3 matrix A exists such that $A^2 + I = 0$.

Now we prove that there exists a 2×2 matrix A such that $A^2 + I = 0$. Consider $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then $A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$. Thus, $A^2 + I = 0$.