

THE UNIVERSITY OF CALGARY
MATHEMATICS 251
FINAL EXAMINATION, FALL 2001
TIME: 2 HOURS

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Total (max. 80)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [80]. THIS EXAM HAS 9 PAGES INCLUDING THIS ONE.

[5] 1. Find and simplify $\frac{d}{dx} \left(\sqrt{x \tan^2 x - 4} \right)$.

[5] 2. Find and simplify $\frac{d}{dx} \left(\frac{\sin(6 - x^{3/2})}{e^{x^5} - 2} \right)$.

[6] 3. Find and simplify the equation of the tangent line to the curve $y = 2x - \cos 3x$ at the point where $x = 0$.

[6] 4. Find and simplify $\frac{dy}{dx}$ where $y \ln x - x \ln y = 8$.

[6] 5. USE THE DEFINITION OF DERIVATIVE to find $\frac{d}{dx} \left(\frac{1}{2x-1} \right)$.

[6] 6. Use the derivative of the natural logarithm function, and implicit differentiation, to prove the formula for $\frac{d}{dx} e^x$.

[8] 7. A spotlight is on the ground pointing towards a high wall 20 metres away. A child 1 metre tall is standing near the wall so that her shadow is cast onto the wall as in the picture. The child begins to run at 2 metres per second towards the spotlight. How fast is the length of her shadow changing at the instant that she is halfway between the wall and the spotlight?

[10] 8. For the function $f(x) = x - 3x^{2/3}$, find the following, if any: the x and y intercepts, horizontal and vertical asymptotes, intervals of increase and decrease, relative maxima and minima, intervals of concave up and concave down, and inflection points. Then draw the graph of the function.

[8] 9. Find the point(s) on the curve $x^2 - 2y^2 = 1$ which are closest to the point $(0, 6)$.

[5] 10. Find and simplify $\int \sec 2x \tan 2x \, dx$.

[5] 11. Find and simplify $\int_0^1 \frac{x^3}{(x^4 + 3)^{3/2}} \, dx$.

[5] 12. Find constants a and b so that the function $f(x) = \begin{cases} ax^2 + b, & x \leq 2 \\ x^3, & x > 2 \end{cases}$ is differentiable at $x = 2$.

[5] 13. Find and simplify $\lim_{x \rightarrow 0} (e^{3x} - 1) \cot 4x$.