THE UNIVERSITY OF CALGARY
MATHEMATICS 249
FINAL EXAMINATION, FALL 2005
TIME: 2 HOURS

NAME_____ID_____

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total (max. 70)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [70]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\lim_{x \to 4} \left(\dfrac{7 - \sqrt{3x + 37}}{x - 4} \right)$.

[5] 2. Find $\lim_{x \to -\infty} \left(\dfrac{5 - 2x^2}{5 - 3x^2} \right)$.

[5] 3. Find $\dfrac{d}{dx}\left(\dfrac{e^{4x}-x^{-3}}{\cos 4x}\right)$.

[5] 4. Find $\dfrac{d}{dx}\Big(\sin(x^{3/2}\ln x)\Big)$.

[6] 5. USE THE DEFINITION OF DERIVATIVE to find $\frac{d}{dx}\left(2x^2 - 9x\right)$.

[6] 6. Use implicit differentiation to find $\frac{dy}{dx}$ where $y^3x + \ln(x^3 - y) = 5$.

[12] 7. For the function $f(x) = 2x^3 - 15x^2 + 4$:
(a) Find $f'(x)$ and $f''(x)$.

(b) Find the intervals of increase and decrease; and all local maximum and minimum values of $f(x)$.

(c) Find the intervals where $f(x)$ is concave up and where it is concave down.

(d) Use (b) and (c) to sketch the graph of $f(x)$.

[5] 8. Prove the formula for $\frac{d}{dx}(\tan x)$. You may use the formulas for $\frac{d}{dx}(\sin x)$ and $\frac{d}{dx}(\cos x)$.

[5] 9. A line passing through the point $(10, 0)$ is tangent to the curve $y = x^3$ at some point P. (P is not the origin $(0, 0)$.) Find the x-coordinate of P.

[5] 10. Find and simplify $\int_{-1}^{1} (x + 25)\, dx$.

[5] 11. Find and simplify $\int x \sec(1 - 2x^2) \tan(1 - 2x^2)\, dx$.

[6] 12. Find nonnegative numbers x and y so that $x + y = 13$ and so that $4\sqrt{x} + 6\sqrt{y}$ is as large as possible.