THE UNIVERSITY OF CALGARY
MATHEMATICS 249 L07/L08
FINAL EXAMINATION, FALL 2007
TIME: 2 HOURS

NAME____________________________________ID_____________Section______

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12	
Total (max. 75)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\lim_{x\to\infty}\left(\dfrac{x^2}{8-3x^2}\right)$.

[5] 2. Use l'Hôpital's rule to find $\lim_{x\to 0}\left(\dfrac{5^x-1}{\ln(5x+1)}\right)$.

[5] 3. Find $\dfrac{d}{dx}\left(\dfrac{\sqrt{4-3x}}{\sin(4e^x)}\right)$.

[5] 4. Find $\dfrac{d}{dx}\Big(\ln(2x)\cos(x^2)\Big)$.

[6] 5. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(3x - x^2\right)$.

[6] 6. Use implicit differentiation to find y' where $2\sqrt{xy} = x^2 + 2y^2$.

[15] 7. For the function $f(x) = 3x^{5/2} - 5x^{3/2}$, $(x \geq 0)$, you are given that

$$f'(x) = \frac{15}{2}x^{1/2}(x-1) \quad \text{and} \quad f''(x) = \frac{15(3x-1)}{4x^{1/2}}.$$

(a) Find the critical points.

(b) Find the intervals of increase and decrease of $f(x)$. Use them to determine whether each critical point in part (a) is a local maximum, local minimum, or neither.

(c) Find the absolute maximum and absolute minimum of $f(x)$ for x in the interval $[0, 4]$.

(d) Find the intervals where $f(x)$ is concave up and where it is concave down, and find any inflection points.

(e) Find a number $a > 0$ so that the tangent line to the curve at the point $(a, f(a))$ passes through the origin $(0, 0)$.

[6] 8. Find constants a and b so that the function $f(x) = \begin{cases} x+2 & \text{if } x \leq a \\ 2x+3 & \text{if } a < x < b \\ 3x+1 & \text{if } x \geq b \end{cases}$ is continuous at both $x = a$ and $x = b$.

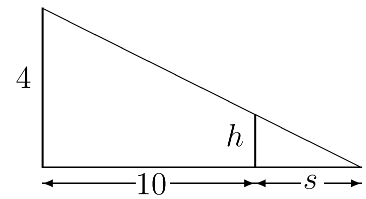
[6] 9. Prove that $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$. You may use the formulas for the derivative of $\sin x$ or $\cos x$ (or both).

[5] 10. Find and simplify $\int_1^4 \left(6\sqrt{x} - \frac{32}{x^3}\right)\, dx$.

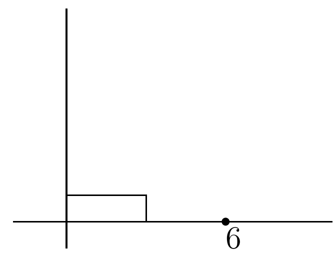
[5] 11. Find and simplify $\int e^{2x} \sec^2(e^{2x})\, dx$.

[6] 12. Do **ONE** of the following two problems:

(a) A magic beanstalk starts growing at a spot 10 metres from a 4 metre lamppost. The beanstalk grows at 1 metre per hour. Find the rate at which the length s of the beanstalk's shadow is increasing at the instant when the height h of the beanstalk is 3 metres.



(b) A rectangle has two of its sides on the x and y axes, and its upper right corner on the left half of the parabola $y = (x-6)^2$ as shown. Find the maximum possible area of the rectangle.



[75]