MATHEMATICS 271 L01 FALL 2003
ASSIGNMENT 1

Due at NOON on Friday, September 26. Your assignment must be handed in at the beginning of the lab on September 26. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.
Marked assignments will be returned during the lab on Friday, October 3.

1. In this question, prove your answers using **only** the definitions of rational and irrational numbers and the fact that $\sqrt{2}$ is irrational. Let $\mathcal{P}$ be the statement : "If $x + y$ is rational and $x - y$ is irrational then x is irrational and y is irrational."

(a) Is $\mathcal{P}$ true for all real numbers x and y? Prove your answer.

(b) State the *converse* of $\mathcal{P}$. Is the *converse* of $\mathcal{P}$ true for all real numbers x and y? Prove your answer.

(c) State the the *contrapositive* of $\mathcal{P}$. Is the *contrapositive* of $\mathcal{P}$ true for all real numbers x and y? Explain.

(d) State the the *negation* of $\mathcal{P}$. Is the *negation* of $\mathcal{P}$ true for some real numbers x and y? Explain.

2. In this question, a, b and c are integers. Let $\mathcal{P}$ be the statement : "if $a \mid b$ and $a \mid c$ then $a \mid 2b + c$ and $a \mid b + 2c$." and let $\mathcal{Q}$ be the statement : "if $a \mid b$ and $a \mid c$ then $a \mid 2b + c$ and $a \mid 3b + 2c$.".

(a) Is $\mathcal{P}$ true? Prove your answer.

(b) State the *converse* of $\mathcal{P}$. Is the *converse* of $\mathcal{P}$ true? Prove your answer.

(c) Is $\mathcal{Q}$ true? Prove your answer.

(d) State the *converse* of $\mathcal{Q}$. Is the *converse* of $\mathcal{Q}$ true? Prove your answer.

3. For each of the following statements, determine whether the statement is true or false and **prove your answer**.

(a) For all integers y, there is an integer x so that $x^3 + x = y$.

(b) For all integers x and y, if $2x^2 + x = 2y^2 + y$ then $x = y$..

(c) For all integers m and n, if $m \mid n$ and $n \mid m$ then $n = m$.

(d) For all natural numbers m and n, if $m \mid n$ and $n \mid m$ then $n = m$.