MATHEMATICS 271 L01 FALL 2010
Quiz 1 Solutions

1. Write the *negation* (in good English) of each of the following statements. The answer " It is not the case that ..." is **not** acceptable.
(a) For all real numbers x and y, if x is rational and y is irrational then $x+y$ is irrational and xy is irrational.
Solution: There exist real numbers x and y so that x is rational and y is irrational, but $x+y$ is rational or xy is rational.

(b) There exists a positive integer n such that for all rational numbers x, nx is an integer.
Solution: For all positive integers n, there exists a rational number x such that nx is not an integer.

(c) For all real numbers x and y, if $\lfloor x \rfloor \leq \lfloor y \rfloor$ then $x \leq y$.
Solution: There exist real numbers x and y so that $\lfloor x \rfloor \leq \lfloor y \rfloor$ but $x > y$.

2. Prove that the statement: "For all integers a and b, if $a^2 + a = b^2 + b$ then $a = b$." is false by writing down the negation of the statement and then prove that.
Solution: The negation of the above statement is "There exist integers a and b so that $a^2 + a = b^2 + b$ but $a \neq b$."

For example, consider the case $a = 0$ and $b = -1$. We see that $a^2 + a = 0^2 + 0 = 0 = 1 - 1 = b^2 + b$, but $a = 0 \neq -1 = b$.

3. Let $\mathcal{P}$ be the statement: "For all integers a, b and c, if $a \mid b$ and $a \mid c$ then $a \mid (2b - 3c)$."
(a) Prove that $\mathcal{P}$ is true.
Solution: Let a, b and c be integers and suppose that $a \mid b$ and $a \mid c$. Then $b = am$ and $c = an$ for some integers m and n. Thus, $2b - 3c = 2am - 3an = a(2m - 3n)$ where $2m - 3n$ is an ineger, which implies that $a \mid (2b - 3c)$.

(b) Write the converse of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true? Prove your answer.
Solution: The converse of $\mathcal{P}$ is "For all integers a, b and c, if $a \mid (2b - 3c)$ then $a \mid b$ and $a \mid c$."

The converse of $\mathcal{P}$ is false. For example, when $a = 5, b = 3$ and $c = 2$, we $2b - 3c = 0 = a \times 0$, so $a \mid (2b - 3c)$ but $a \nmid b$.