MATHEMATICS 271 L60 SUMMER 2003
ASSIGNMENT 2

This assignment is to be handed to your lab instructor at 9:30 a.m. on July 24, 2003. Late assignments will not be accepted and are given a mark of zero. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his/her own words. Students should attempt all problems. However, only two problems will be marked for credit. Make sure that your papers are stapled together.

1. Let $\mathcal{P}$ be the statement: "*If* $A \subseteq B \cup C$ *and* $B \subseteq C \cup A$ *then* $A \triangle B = C$."
and let $\mathcal{Q}$ be the statement: "*For all sets* A, B *and* C, *if* $A \triangle B = A \triangle C$ *then* $B \subseteq C$."

(a) Is $\mathcal{P}$ true for all sets A, B and C? Prove your answer.

(b) Is $\mathcal{Q}$ true? Prove your answer.

(c) Write the converse of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true for all sets A, B and C? Prove your answer.

2. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $\mathcal{R}$ be a relation on 2^A defined by:

$$\textit{For all } X, Y \in 2^A, \qquad X\mathcal{R}Y \textit{ if and only if } X \cup (A - Y) = A.$$

(a) Is $\mathcal{R}$ reflexive? irreflexive? symmetric? antisymmetric? transitive? Prove your answers.

(b) Is $\mathcal{R}$ an equivalence relation on 2^A? Explain.

(c) Let $S = \{1, 3, 5, 7, 9\}$. How many $T \in 2^A$ are there so that $S\mathcal{R}T$? Explain.

3. Let $A = \{1, 2, 3, 4\}$. Let $\mathcal{R}$ and $\mathcal{S}$ be relations on A. Prove or disprove each of the following statements:

(a) If $\mathcal{R}$ and $\mathcal{S}$ are symmetric then $\mathcal{R} \cup \mathcal{S}$ is symmetric.

(b) If $\mathcal{R} \cup \mathcal{S}$ is symmetric then $\mathcal{R}$ and $\mathcal{S}$ are symmetric.

(c) There exists a relation $\mathcal{T}$ on A so that $\mathcal{T}$ is not reflexive but $\mathcal{I}$ is both symmetric and antisymmetric.

(d) There exists a relation $\mathcal{V}$ on A so that $\mathcal{V}$ is not transitive but $\mathcal{V}$ is both symmetric and antisymmetric.

4. Let $S = \{1000, 1001, 1002, ..., 9999\}$.

(a) How many number in S have at least one digit is a 2 or a 5?

(b) How many number in S have at least one digit that is a 2 and at least one digit that is a 5?

(c) How many number in S have the property that the sum of its digits is even?

(d) How many number in S have the property that the digits appear in increasing order (that is, the first digit is smaller than the second digit, the second digit is smaller than the third digit, and the third digit is smaller than the fourth digit)?