MATHEMATICS 271 L60 SUMMER 2004
ASSIGNMENT 1

This assignment is to be handed to your lab instructor at 9:30 a.m. on July 15, 2004. Late assignments will not be accepted and are given a mark of zero. Students should attempt all problems. However, only one problem will be marked for credit.

1. In this question, a, b and c are integers.Prove or disprove each of the following:

(a) If $a \mid b$ and $a \mid c$ then $a \mid ma + nb$ for all integers m and n.

(b) If $a \mid b$ and $b \mid c$ then $a \mid c$

(c) If $a \mid bc$ then $a \mid b$ or $a \mid c$

(d) If $a \mid b + 2c$ and $a \mid 2b + c$ then $a \mid b$ and $a \mid c$.

2. Prove or disprove each of the following:

(a) For all real numbers x and y, if x is rational and y is irrational then $x + y$ is irrational.

(b) For all real numbers x and y, if x is rational and y is irrational then xy is irrational.

(c) For all real numbers x and y, if $x+y$ is irrational then x is irrational or y is irrational.

(d) For all real numbers x and y, if xy is irrational then x is irrational or y is irrational.

3. Prove or disprove each of the following:

(a) For all integers m, $m^2 + 3m + 2$ is even.

(b) For all integers m and n, if $m^2 + 3m = n^2 + 3n$ then $m = n$.

(c) For all integers m and n, if $3m^2 + m = 3n^2 + 3n$ then $m = n$.

(d) For all integers $n \geq 0$, $n^2 + n + 41$ is a prime.

4. Prove or disprove each of the following:

(a) For all real numbers x, $x^2 - x + 1 > 0$.

(b) For all real numbers x and y, if $x^2 - x + 1 = y^2 - y + 1$ then $x = y$.

(c) For all integers y, there exists an integer x so that $x^3 + x = y$.

(d) For all irrational number x, there is an irrational number y so that xy is an integer.