MATH 271 ASSIGNMENT 1

Due 4:00 PM Friday, February 5, 2010. Put your assignments in the appropriate wooden box (corresponding to your lecture section and last name) outside MS569. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

Notes: Statements of converses, contrapositives, and negations should always be as simple as possible. $\mathbb{Q}$ denotes the set of all rational numbers, and $\mathbb{Z}$ denotes the set of all integers.

1. For each true statement below, give a proof. For each false statement below, write out its negation and prove that. [*Hint*: you may use Exercise 16, page 147.]
   (a) $\forall q \in \mathbb{Q}\ \exists r \in \mathbb{Q}$ so that $q + r \in \mathbb{Z}$.
   (b) $\forall q \in \mathbb{Q}\ \exists r \in \mathbb{Q}$ so that $q + r \notin \mathbb{Z}$.
   (c) $\forall q \in \mathbb{Q}\ \exists r \in \mathbb{Q}$ so that $qr \in \mathbb{Z}$.
   (d) $\forall q \in \mathbb{Q}\ \exists r \in \mathbb{Q}$ so that $qr \notin \mathbb{Z}$.
   (e) $\forall q \in \mathbb{Q}\ \exists r \in \mathbb{Q}$ so that $q + r \notin \mathbb{Z}$ and $qr \in \mathbb{Z}$.

2. In this question you may assume without proof that every integer is either even or odd (but not both) and also that consecutive integers have opposite parity, but otherwise use only the definitions of even and odd integers.
   (a) Prove using contradiction or the contrapositive: $\forall a \in \mathbb{Z}$, if a is odd then $a/2 \notin \mathbb{Z}$.
   (b) Write out the *converse* of the statement in (a). Is it true? Explain.
   (c) Let $\mathcal{S}$ be the statement: $\forall a \in \mathbb{Z}$, if a is odd then $\lfloor a/2 \rfloor$ is odd. If $\mathcal{S}$ is true, prove it. If $\mathcal{S}$ is false, write out its negation and prove that.
   (d) Prove or disprove: $\forall a \in \mathbb{Z}$, if a is odd then $\lfloor a/2 \rfloor$ is odd or $\lceil a/2 \rceil$ is odd.
   (e) Write out the *contrapositive* of the statement in (d). Is it true? Explain.

3. (a) Let N be your U of C ID number. Use the Euclidean algorithm to calculate $d = \gcd(N, 271)$. Then use your calculations to find integers x and y so that $Nx+271y = d$.
   (b) Suppose a certain student's ID number M satisfies

   $$\gcd(M, 2010) > \gcd(M, 271) > 1.$$

   Find all possible values for $\gcd(M, 2010)$. Be sure to explain your reasoning. [*Note*: both 271 and 67 are prime.]
   (c) Suppose that the ID number M from part (b) lies between 10020000 and 10030000. Find M. Be sure to explain your reasoning.