MATH 271 ASSIGNMENT 2

Due 4:00 PM Friday, February 26, 2010. Put your assignments in the appropriate wooden box (corresponding to your lecture section and last name) outside MS569. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. (a) Show algebraically that $a^{n+1} - b^{n+1} = a(a^n - b^n) + b^n(a - b)$.
   (b) Use part (a) to prove **using mathematical induction** (or well ordering) that $(a - b) \mid (a^n - b^n)$ for all integers a, b, n with $n \geq 1$.
   (c) Use part (b) to prove that $11 | (7^{271} + 4^{271})$.
   (d) Prove part (b) again by proving that $a^n - b^n = (a - b) \sum_{i=0}^{n-1} a^{n-1-i} b^i$ for all integers $n \geq 1$, using telescoping. (See Example 4.1.10, page 205.)

2. The sequence $a_1, a_2, a_3, \ldots$ is defined by: $a_1 = 0$ and $a_{n+1} = a_n + 2n + 1$ for all integers $n \geq 1$.
   (a) Calculate a_2, a_3 and a_4.
   (b) Use part (a) (and more data if you need it) to guess a simple formula for a_n for all positive integers n.
   (c) **Use mathematical induction** (or well ordering) to prove that your guess in part (b) is correct.
   (d) Prove that a_n is composite for all integers $n \geq 3$.

3. You are given the following "while" loop:

   [*Pre-condition*: m is a nonnegative integer, $a = 1$, $b = 1$, $i = 0$.]

   while $(i \neq m)$
      1. $a := a + 2b$
      2. $b := b - 2a$
      3. $i := i + 1$

   end while

   [*Post-condition*: $a = (-1)^m(1 - 4m)$.]

   Loop invariant $I(n)$ is: $i = n, \quad a = (-1)^n(1 - 4n), \quad b = (-1)^n(1 + 4n)$.

   (a) Prove the correctness of this loop with respect to the pre- and post-conditions.
   (b) Suppose the "while" loop is as above, with the same pre-condition, except that statements 1 and 2 are switched (so the new statements 1 and 2 are: 1. $b := b - 2a$, 2. $a := a + 2b$). Run through this new loop a few times to get data. Then find a post-condition that gives the final value of a, and an appropriate loop invariant, and prove the correctness of this new loop.