MATH 271 ASSIGNMENT 4

Due 4:00 PM Friday, April 9, 2010. Put your assignments in the appropriate wooden box (corresponding to your lecture section and last name) outside MS569. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. If $f : \mathbb{R} \to \mathbb{R}$ is a function (where $\mathbb{R}$ is the set of all real numbers), we define the function $f^{(2)}$ to be the composition $f \circ f$, and for any integer $n \geq 2$, define $f^{(n+1)} = f \circ f^{(n)}$. So $f^{(2)}(x) = (f \circ f)(x) = f(f(x))$, $f^{(3)}(x) = (f \circ f^{(2)})(x) = f(f(f(x)))$, and so on. We also define $f^{(1)}$ to be f.

   (a) Use Theorem 3.5.1 on page 167 to prove that $\lfloor x - \lfloor x \rfloor \rfloor = 0$ for every real number x.

   (b) Suppose that $f(x) = x - \lfloor x \rfloor$ for all $x \in \mathbb{R}$. Calculate and simplify $f^{(2)}(x)$ and $f^{(3)}(x)$. [*Hint*: part (a).] Then guess a simple formula for $f^{(n)}(x)$ for all integers $n \geq 1$. Use induction (or well ordering) to prove your guess.

   (c) Suppose that $g(x) = x + \lfloor x \rfloor$ for all $x \in \mathbb{R}$. Calculate and simplify $g^{(2)}(x)$ and $g^{(3)}(x)$ (and more if you need them). [*Hint*: Theorem 3.5.1 on page 167.] Then guess a simple formula for $g^{(n)}(x)$ for all integers $n \geq 1$. Use induction (or well ordering) to prove your guess.

2. Let $[n] = \{1, 2, 3, \ldots, n\}$, where $n \geq 3$ is an integer.

   (a) Define the relation $\mathscr{R}$ on the power set $\mathscr{P}([n])$ by: for all sets $A, B \in \mathscr{P}([n])$, $A\mathscr{R}B$ if and only if $A - B = \{1, 2\}$. Is $\mathscr{R}$ reflexive? Symmetric? Transitive? Give reasons.

   (b) Find the number of sets $B \in \mathscr{P}([n])$ so that $\{1, 2, 3\}\mathscr{R}B$.

   (c) Define the relation $\mathscr{S}$ on the power set $\mathscr{P}([n])$ by: for all sets $A, B \in \mathscr{P}([n])$, $A\mathscr{S}B$ if and only if $A - B \subseteq \{1, 2\}$. Is $\mathscr{S}$ reflexive? Symmetric? Transitive? Give reasons.

3. For sets A and B, define a relation R on $A \cup B$ by: for all $x, y \in A \cup B$, xRy if and only if $(x, y) \in A \times B$. Prove or disprove each of the following statements.

   (a) For all sets A and B, if R is reflexive then $A = B$.

   (b) For all sets A and B, if R is symmetric then $A = B$.

   (c) For all nonempty sets A and B, if R is symmetric then $A = B$.

   (d) For all nonempty sets A and B, if R is transitive then $A = B$.

   (e) For all sets A and B, if $A = B$ then R is an equivalence relation.