UNIVERSITY OF CALGARY

FACULTY OF SCIENCE
Department of Mathematics and Statistics

Mathematics 273 Numbers and Proofs

(see Course Descriptions for the applicable academic year: http://www.ucalgary.ca/pubs/calendar/)

Syllabus

Topics	**Number of Hours**
(1) Sets and Functions: - functions, domain, codomain - The graph of a function - Composition of functions, injections, surjection	3
(2) Integers: - Division algorithm, Euclidean algorithm - Prime numbers, prime factorization - Equivalence relations, modular arithmetic - Induction, recursion - The Binomial Theorem	9
(3) Rational Numbers: - Defined via equivalence relations	2
(4) Real Numbers: - Limits - Sequences - Real numbers defined as equivalence classes of Cauchy sequences of real numbers - Completeness of the real numbers, upper bounds, Bolzano-Weierstrass theorem - Basic topology of the real line - Base-p expansions (time permitting)	12
(5) Complex Numbers: - Quadratic equations - Addition, multiplication and division of complex number - The complex plane - Properties of complex numbers - Polar representation and De Moivre's theorem - Roots of complex numbers - The fundamental theorem of algebra	10
TOTAL HOURS	**36**

Course Outcomes

Proof reading and writing. By the end of this course, students will be fluent in proof reading and writing, which is the logical language used to communicate mathematics. Specifically, by the end of this course, students should be able to

1. list the different proof methods (including direct proofs, inductive proofs, proof by contrapositive and proof by contradiction) and explain the difference between them.
2. read and recreate proofs of mathematical statements about the topics covered in this course, such as sets, functions and the number systems (natural, integer, rational, real and complex numbers).
3. construct mathematical proofs using a variety of methods, including direct proofs, inductive proofs, proof by contrapositive and proof by contradiction.

Subject specific knowledge. By the end of this course, students will be able to construct the integers, rational numbers, real numbers and complex numbers using fundamental operations on sets and functions. Specifically, by the end of this course students should be able to

4. restate all of the technical definitions and named theorems covered in the course and use these definitions and theorems from memory to construct solutions to problems and/or proofs.
5. verify that an abstract mathematical object satisfies a given definition, or is a counterexample.
6. apply standard problem-solving techniques (such as proof by induction, the Chinese remainder theorem, the Euclidean algorithm, and other formulaic theorems) to par- ticular problems or situations.
7. generate original solutions to a variety of mathematical problems related to the topics covered in this course, such as sets, functions and the number systems (natural, integer, rational, real and complex numbers).

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