Math 323
Lab #3

Minitab instructions
Click on **Start** in the lower left-hand corner of the screen. Click **MathStats Packages** and then **Minitab 14 for Windows**. Finally, click on **Minitab.**
You are now in the program. Our main interest will be in the DATA window, which serves as the worksheet for entering data, and the MENU bar that allows the selection of commands to perform various tasks. The SESSION window permits entry of typed commands that duplicate the aforementioned procedures ... and so will not be used initially. However, results will be presented in this window.

Binomial Instruction.
1. The probability that a person who undergoes a kidney operation will recover is 0.6. Find the probability that of 5 patients who undergo similar operations,
   (a) none will recover (b) not more than one will recover (c) at least 3 will recover

From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>BINOMIAL...... a dialog box appears. There are various ways in which you can find the required answers
ii.   If a single probability is needed as in part a) of the example:
                Select PROBABILITY from the options listed (clicking on the circle, enters a dot.)
                In the box by NUMBER OF TRIALS, type the number **5**.
                In PROBABILITY OF SUCCESS, insert **0.6**.
                As INPUT CONSTANT, specify the "y" value from the question (**0** for part a)). Click on OK
                The probability then comes up on the SEESION screen.   [0.0102]
iii.   When a sum of probabilities is involved, as in part b), begin in the same way {CALC>PROB DISTR>BINOM}
                Select CUMULATIVE PROBABILITY from the options.
                Enter NUMBER OF TRIALS and PROBABILITY OF SUCCESS (5 and 0.6) as before.
                As INPUT CONSTANT, specify the "y" from the question (**1** for part b)). Click on OK
                The number appearing on the SESSION screen will be the $P(Y \le y)$. {$P(Y \le 1)$ for b)}
                                                            [0.08704]
     This method could also be used for part c), but the required probabilities would have to be expressed in terms of the cumulative probability and the complement........$P(Y \ge 3) = 1 - P(Y \le 2)$ [0.68256]

iv.   When several different questions are asked about the same distribution, it would be helpful to have all of the probabilities for each individual "y" calculated at once.
                Enter the values 0,1,2,3,4,5 into c1, the first column of the worksheet.
                Then proceed as in the previous examples:
                From the menu bar, select CALC>PROBABILITY DISTRIBUTIONS>BINOMIAL; select PROBABILITY from options; enter NUMBER OF TRIALS and PROBABILITY OF SUCCESS (5 and 0.6).
                In the box associated with INPUT COMUMN indicate C1 (where you have listed the possible values of Y) and request OPTIONAL STORAGE in C2. CLICK on OK>
                On the worksheet, the probabilities for each value of Y appear in column C2.
        Repeating the procedure with CUMULATIVE PROBABILITY and choosing OPTIONAL STORAGE in C3 will complete a table of x values, individual probabilities, and cumulative probabilities in the WORKSHEET.

Minitab instructions for the Poisson Distribution.
The Poisson instructions are similar to the binomial.

2. The number of mistakes in one page of a solutions manual to a statistics textbook follows a Poisson distribution with a rate of 2.2 mistakes per page.

(a) Find the probability that a randomly chosen page contains exactly 3 mistakes.
(b) Find the probability that a randomly chosen page contains at most 4 mistakes.
(c) Find the probability that a randomly chosen page contains at least 7 mistakes.

From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>POISSON...... a dialog box appears. There are various ways in which you can find the required answers

ii. If a single probability is needed as in part a) of the example:

   Select PROBABILITY from the options listed (clicking on the circle, enters a dot.)
   In the box by MEAN, type 2.2
   As INPUT CONSTANT, specify the "y" value from the question (**3** for part a)). Click on OK
   The probability then comes up on the SEESION screen.   [0.1966]

iii. When a sum of probabilities is involved, as in part b), begin in the same way {CALC>PROB DISTR>POISSON}
   Select CUMULATIVE PROBABILITY from the options.
   Enter MEAN 2.2.
   As INPUT CONSTANT, specify the "y" from the question (**4** for part b)). Click on OK
   The number appearing on the SESSION screen will be the $P(Y \le y)$. {$P(Y \le 4)$ for b)}
                                                                        [0.9275]

   This method could also be used for part c), but the required probabilities would have to be expressed in terms of the cumulative probability and the complement........$P(Y \ge 7)= 1-P(Y \le 6)$ [0.0075]

Play around with MINITAB to see other distributions and options.

1. Let Y be a random variable with density

   $$f(y) = 2y \qquad 0 < y < 1 \qquad \text{and let U=3Y+6}$$

   (a) Find f(u) using the method of distribution function. $f(u) = \dfrac{2}{9}(u-6) \quad 6 < u < 9$

   (b) Verify (a) by using the method of transformation.
   (c) Find E[U] and V[U] by using f(u). (8,.5)
   (d) Verify (c) by using E[Y] and V[Y].

2. Let Y be uniformly distributed over (0,4) and let $U = (Y-3)^2$. Find f(u).

   $$f(u) = \frac{1}{8\sqrt{u}} \quad 1 \le u < 9$$

3. Let Y be a random variable with density $f(y) = \dfrac{1}{4}y \quad 0 \le y \le \sqrt{8}$ and let U=Y+3.

   (a) Find f(u). $f(u) = \dfrac{2}{8}(u-3) \quad 3 \le u < \sqrt{8}+3$

   (b) Find E[U] and V[U] using f(u). (4.8856,0.4445)
   (c) Verify (b) by using E[Y] and V[Y].

4. Let Y be a random variable with density $f(y) = \dfrac{1}{4}ye^{-y/2} \quad y \ge 0$. Find f(u).

   $$f(u) = (2-u)e^{-(2-u)} \quad u \le 2$$

5. Vehicles arrive at a highway toll booth at random instances from both the south and north. Assume that they arrive at average rates of 5 and 3 per five minute period respectively. Let Y_1 denote the number arriving from the south during a 5 minute period and let Y_2 denote the number arriving from the north during this same time. Assume that Y_1 and Y_2 are independent. Let U equal the total # of cars that arrive from the North and South in a certain period.

(a) Find the moment generating function for U if U is the total number of cars that arrive in a 5 minute period. $m_U(t) = e^{8(e^t - 1)}$

(b) Find f(u). $f(u) = \dfrac{e^{-8}8^k}{k!}$ $k = 0,1,2,...$

(c) Find the probability that a total of 4 vehicles arrives during a five minute period. (0.0573).

(d) Find the probability that given a total of 10 cars arrive in 5 minutes, 8 came from the south. (.1473).

(e) What density function does this probability resemble and what are its parameters?
[binomial(10, 5/8)]

6. Look at other questions in the text using the moment generating method.

7. Assume that Y_1 and Y_2 are independent and uniformly distribution over (0,1) and (0,2) respectively. Find the joint density for (U_1, U_2) where $U_1 = 2Y_1 + Y_2$ and $U_2 = Y_1 + 3Y_2$.
$f(u_1, u_2) = 1/10$ $0 < u_1 < 4$, $0 < u_2 < 7$

8. Let Y_1 and Y_2 be independent and uniformly distribution random variables over the intervals (0,2) and (0,3) respectively.

(a) Find f(u) if $U = Y_1 Y_2$. $f(u) = 1/6(\ln 3 - \ln u/2)$ $0 < u < 6$

(b) Find f(u) if $U = Y_1/Y_2$. $f(u) = \begin{cases} 3/4 & 0 < u < 2/3 \\ 1/3u^2 & 2/3 \le u < \infty \end{cases}$

9. Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4 from a distribution $f(y) = 2y$ $0 < y < 1$,

(a) Find the density function of $g_3(y_3)$. $g_3(y_3) = 24(y_3^5 - y_3^7)$ $0 < y_3 < 1$

(b) Find P(1/2<Y3). 243/256

10. Let $Y_1, Y_2, ... Y_n$ be independent random variables, each with a beta distribution, with $\alpha = \beta = 2$.

(a) Find the probability distribution function of $Y_{(n)} = \max(Y_1, Y_2, ... Y_n)$.

$$F_{Y_{(n)}}(y) = \begin{cases} 0 & y < 0 \\ (3y^2 - 2y^3)^n & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

(b) Find the density function of $Y_{(n)}$. $f_{Y_{(n)}}(y) = 6ny(1-y)(3y^2 - 2y^3)^{n-1}$ $0 \le y \le 1$

(c) Find $E[Y_{(n)}]$. (.6286)

11. Suppose that the length of time Y it takes a worker to complete a certain task has the probability density function given by $f(y) = e^{-(y-\theta)}$ $y > \theta$ where θ is a positive constant that represents the minimum time until task completion. Let $Y_1, Y_2, ... Y_n$ denote a random sample of completion times from this distribution.

(a) Find the density function for $Y_{(1)} = \min(Y_1, Y_2, ... Y_n)$. $f_{Y_{(1)}}(y) = ne^{-n(y-\theta)}$ $y \ge \theta$

(b) Find $E[Y_{(1)}]$. $(1/n + \theta)$

12. Do as many questions as possible from the text in chapter 6.