Math 323

Lab #1

Login with your ID and pass word. If you do not have this, you need to go the 7th floor of the Math Science building and get it. You will have to wait at least 45 minutes until you can use your password after getting it.

1. In an automobile plant, two tasks are performed by robots. The first entails welding two joints; the second, tightening three bolts. Let Y_1 denote the number of defective welds and Y_2 the number of improperly tightened bolts produced per car. Past data indicates that the joint density for (Y_1, Y_2) is as follows:

	Y_1		
Y_2	0	1	2
0	.84	.06	.01
1	.03	.01	.005
2	.02	.008	.004
3	.01	.002	.001

   (a) Find the probability that there will be no errors made by the robots. (.84)
   (b) Find the probability that there will be exactly one error made. (.09)
   (c) Find the probability that there will be no improperly tightened bolts. (.91)
   (d) Find the marginal distributions of Y_1 and Y_2.

2. A fair coin is tossed four times in succession. Let Y_1 = the number of heads, and Y_2 = number of tails before a head appears (e.g. for THHT, $Y_1 = 2$, $Y_2 = 1$). Produce the joint distribution.

3. In a healthy individual age 20 to 29 years, the calcium level in the blood, Y_1, is usually between 8.5 and 10.5 mg/dl and the cholesterol level, Y_2, is usually between 120 and 240 mg/dl. Assume that for a healthy individual in the age group the random variable (Y_1, Y_2) is uniformly distributed. That is, assume that the joint density for (Y_1, Y_2) is

$$f(y_1, y_2) = \begin{cases} c & 8.5 \le y_1 \le 10.5, \quad 120 \le y_2 \le 240 \\ 0 & elsewhere \end{cases}$$

   (a) Plot the density
   (b) Find c. (1/240)
   (c) Find the probability that an individual's calcium level will lie between 9 and 10 mg/dl while the cholesterol level is between 125 and 140 mg/dl. (15/240)
   (d) Find the marginal densities for Y_1 and Y_2. (1/2, 1/120)

4. In studying the behavior of air support roofs, the random variables Y_1, the inside barometric pressure (in inches of mercury) and Y_2, the outside pressure, are considered. Assume that the joint density of (Y_1, Y_2)is given by

$$f(y_1, y_2) = \begin{cases} c / y_1 & 27 \le y_2 \le y_1 \le 33 \\ 0 & elsewhere \end{cases}$$

   c= 1/(6-27ln33/27) ~1.72
   (a) Find the marginal densities for Y_1 and Y_2. [c(1-27/y1), c(ln33-lny2)]
   (b) Find the probability that the inside pressure is at most 30 and the outside pressure is at most 28. [c(ln30 -28ln28+27ln27+1) ~ c(.09) = 1.72(.09) = .1548]

5. Are the random variables Y_1 and Y_2 independent in questions 1,2, 3, and 4? (no, no, yes,no)

6. In an automobile plant, two tasks are performed by robots. The first entails welding two joints; the second, tightening three bolts. Let Y_1 denote the number of defective welds and Y_2 the number of improperly tightened bolts produced per car. Past data indicates that the joint density for (Y_1, Y_2) is as follows:

	Y_1		
Y_2	0	1	2
0	.84	.06	.01
1	.03	.01	.005
2	.02	.008	.004
3	.01	.002	.001

(a) Find $E[Y_1]$, $E[Y_2]$, $E[2Y_1+5Y_2]$, $E[Y_1 Y_2]$(.12, .148, .98, .064)
(b) Are Y_1 and Y_2 independent? Show by using $E[Y_1 Y_2]$ (.no)
(c) Find the covariance. (.0462)
(d) Find the $V[Y_1]$, and $V[Y_2]$ (.1456, .2681)
(e) Find the correlations coefficient and interpret. (.2338)
(f) Find $V[2Y_1+5Y_2]$ (8.2089)

7. In a healthy individual age 20 to 29 years, the calcium level in the blood, Y_1, is usually between 8.5 and 10.5 mg/dl and the cholesterol level, Y_2, is usually between 120 and 240 mg/dl. Assume that for a healthy individual in the age group the random variable (Y_1, Y_2) is uniformly distributed. That is, assume that the joint density for (Y_1, Y_2) is

$$f(y_1, y_2) = \begin{cases} c & 8.5 \le y_1 \le 10.5, \quad 120 \le y_2 \le 240 \\ 0 & elsewhere \end{cases}$$

(a) Find $E[Y_1]$, $E[Y_2]$, $E[.5Y_1+Y_2]$, $E[Y_1 Y_2]$. (9.5mg/dl, 180mg/dl, 184.75mg/dl, 1710)
(b) Are Y_1 and Y_2`` independent? Show by using $E[Y_1 Y_2]$ (yes)
(c) Find the covariance. (0)
(d) Find the correlation coefficient. (0)
(e) Find the $V[Y_1]$, and $V[Y_2]$. (.3333, 1200)
(f) Find the $V[.5Y_1+Y_2]$ (1200.0833)

8. It is noted that 1% of the items coming off a production line are defective and nonsalvageable, 5% are defective but salvageable, and the rest are nondefective.
(a) Find the probability that if 10 items are selected at random that 4 are defective and nonsalvageable, 3 are defective but salvageable, and the rest are nondefective.
4.3606×10^{-9}
(b) Find the probability that if 10 items are selected at random that at least 1 is nondefective. (~1)
(c) Find the probability that if 5 items are selected that at least 1 is defective and nonsalvageable. (.049)
(d) If 100 items are randomly selected (assume that the number of items produces is sufficiently large) find
    i. The expected number of defective and nonsalvageable items along with its variance. (1, .99)
    ii. The expected number of defective and salvageable items along with its variance. (5, 4.75)
    iii. The expected number of nondefective items along with its variance. (94, 5.64)
    iv. Find Cov(Y_s Y_t). (-.05, -.94, -4.7)
    v. Find the $V[Y_3-Y_2]$ if Y_3 represents the number of nondefective and Y_2 repsresents the number of defective but salvageable. (19.79)

9. Do as many questions as possible in the text from section 5.1-5.9