## STATISTICS 211

## Assignment \#5

## Review assignment 4 on confidence intervals, normal approximation to Binomial and the $t$-distribution.

1. (a) $5.622 \pm 2.35 \frac{.068}{\sqrt{41}} \quad 5.622 \pm .025 \quad[5.597 \mathrm{~g}, 5.647 \mathrm{~g}]$
(b)Yes because it falls in the $\mathbf{9 8 . 1 2 \%} \mathbf{C I}$
2. $\quad 124 \pm 2.26 \frac{18}{\sqrt{10}} \quad 124 \pm 12.8641$ [111.1359, 136.8641]
3. $\quad$ Sample $\%=80 \% \quad \mathrm{SE} \%=\frac{\sqrt{(.8)(.2)}}{\sqrt{2250}} \times 100 \%=.8433 \%$ $\mathbf{8 0 \%} \pm 1.95$ (.8433\%) $\mathbf{8 0 \%} \pm \mathbf{1 . 6 4 4 4 \%} \quad$ [78.36\%, 81.64\%] Results are not valid because the sample is self-selected (not a random sample)
4. Sample $\%=\frac{150}{850} \times 100 \%=17.65 \% \quad \mathrm{SE} \% \sim \frac{\sqrt{(.1765)(.8235)}}{\sqrt{850}} \times 100 \%=1.3077 \%$ $17.65 \% \pm 2.35(1.3077 \%) \quad 17.65 \% \pm 3.0731 \% \quad[14.5769 \%$, 20.7231\%]
5. Sample $\%=\frac{90}{400} \times 100 \%=22.5 \% \quad$ SE $\% \sim \frac{\sqrt{(.225)(.775)}}{\sqrt{400}} \times 100 \%=2.0879 \%$ $22.5 \% \pm 1.95(2.0879 \%) \quad 22.5 \% \pm 4.0714 \% \quad[18.4286 \%, 26.5714 \%]$
6. 

Ho: avg job length is $\mathbf{2 7}$ days Ha: avg job length is less than 27 days
Expected avg $=27$ SEavg $=\frac{S D}{\sqrt{\# \text { of draws }}}=\frac{2.1}{\sqrt{50}}=.297$
$z=\frac{25.3-27}{.297}=-5.724 \mathbf{P}(\mathbf{Z}<-5.724) \sim 0 \quad \sim 0 \%$ chance
There is $\sim \mathbf{0 \%}$ chance that the sample average for time spent on a job would take less than $\mathbf{2 5 . 3}$ days on avg if the true avg is $\mathbf{2 7}$ days. Most likely it takes less than 27 days on avg to complete the job since the observed sample avg was 25.3 days
7.

Ho:corporate extortion costs companies \$3.35 million on avg
Ha: corporate extortion costs companies more than $\$ 3.35$ million on avg
Expected avg $=3.35$ million SEavg $=\frac{S D}{\sqrt{\# \text { of draws }}}=\frac{1.21}{\sqrt{65}}=.15$
$z=\frac{3.71-3.35}{.15}=2.4 \quad \mathbf{P}(\mathbf{Z}>-2.4)=.0082 \quad \sim 0.82 \%$ chance
There is $\boldsymbol{\sim} \mathbf{0 . 8 2 \%}$ chance that the sample avg cost of extortion is 3.71 million or higher if the true avg is 3.35 million. Since we observed a sample average of 3.71 million, most likely the average extortion costs more than 3.35 milliion since the observed avg was 3.71 million.
8.

Ho: daily average is $\$ 5000$
Ha: daily average is greater than $\$ 5000$
Expected avg $=5000$ SEavg $=\frac{S D+}{\sqrt{\# \text { of draws }}}=\frac{507}{\sqrt{20}}=\mathbf{1 1 3 . 3 6 8 6}$
$t=\frac{5200-500}{113.3686}=\mathbf{1 . 7 6 4} \quad \mathbf{d f}=\mathbf{2 0 - 1}=\mathbf{1 9} \quad \mathbf{P}(\mathbf{t}>\mathbf{1 . 7 6 4}) \sim \mathbf{P}(\mathbf{t}>\mathbf{1 . 7 2})=.05 \quad$ less than a $5 \%$ chance
There is less than a $5 \%$ chance that the sample avg of daily revenue is $\$ 5200$ or more if the true avg is $\$ 5000$. Since we observed a sample avg of $\$ 5200$, most likely the average daily revenue is more than $\$ 5000$.
9. Ho: average fill $=\mathbf{1 9 . 2} \mathbf{~ o z}$

Ha: average fill is not equal to $19.2 \mathbf{~ o z}$.
Expected avg $=$ 19.2 SEavg $=\frac{S D+}{\sqrt{\# \text { of draws }}}=\frac{.67}{\sqrt{60}}=.0865$
$Z=\frac{19.2-19.0}{.0865}=2.31 \quad \mathbf{P}(\mathbf{z}>2.31) \sim .0107 \quad \mathbf{1 . 0 7 \%}$ chance.
There is approximately a $1.07 \%$ chance that the sample avg fill for $\mathbf{6 0}$ boxes is $\mathbf{1 9 . 2} \mathbf{~ o z ~ o r ~ m o r e ~ i f ~ t h e ~ t r u e ~ a v g ~ i s ~} \mathbf{1 9} \mathbf{~ o z}$. The difference between 19.2 and 19 oz is considered to be significant. Since we observed a sample avg of 19.2 oz , most likely the true average fill is more than 19.0 oz .
(b) Since this is a non directional test, the $\mathbf{p}$-value $=\mathbf{2 ( 1 . 0 7 \% )}=\mathbf{2 . 1 4 \%}$
10. Ho: at least .25 pounds of beef on average Ha: less than .25 pounds of beef on average
Expected avg $=.25 \quad$ SEavg $=\frac{S D+}{\sqrt{\# \text { of draws }}}=\frac{.06}{\sqrt{100}}=. .006$
$z=\frac{.237-.25}{.006}=-2.17 \quad P(\mathbf{z}<-2.17) \sim .015 \quad 1.5 \%$ chance.
There is approximately a $1.5 \%$ chance that the sample avg fill for 100 burgers is .237 pounds or less if the true avg is .25 pounds or more. Since we observed a sample avg of .237 , most likely the true average pounds of beef on average is less than 0.25 . The difference between 237 and .25 pounds is considered to be significant. Based on the sample data, Big Burger is guilty of false advertising.
11.

Ho: the machine is operating properly (\% is at most $5 \%$ for bottles that are not full)
Ha: the machine is not operation properly ( $\%$ is more than $5 \%$ for bottles that are not full)

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\begin{aligned}
& \text { Sample } \%=\frac{7}{100} \times 100 \%=7 \% \quad \text { Assuming that the machine is operating properly, than the } \\
& \mathrm{SE} \%=\frac{\sqrt{(.05)(.95)}}{\sqrt{100}} \times 100 \%=2.1794 \% \\
& \mathrm{Z}=\frac{7-5}{2.1794}=.9177 \quad \mathrm{P}(\mathbf{z}>.9177) \sim .17 .88 \quad 17.88 \% \text { chance. }
\end{aligned}
$$

There is approximately a $\mathbf{1 7 . 8 8 \%}$ chance of observing a sample $\%$ of $7 \%$ or higher (for a sample of 100 bottles) of bottles that are not full when the true \% is at most $5 \%$. This observed difference between $7 \%$ and the expected $5 \%$ is most likely due to random sampling or chance error.
12. Ho: the \% of people against the war in Iraq is the same as last year (65\%) Ha: the \% of people against the war in Iraq is higher than last year (greater than 65\%)

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\begin{aligned}
& \text { Sample } \%=\frac{701}{1002} \times 100 \%=69.96 \% \quad \text { Assuming that the } \% \text { is the same than the } \\
& \text { SE } \%==\frac{\sqrt{(.65)(.35)}}{\sqrt{1002}} \times 100 \%=1.5068 \% \\
& Z=\frac{69.96 \%-65 \%}{1.5068}=3.29 \quad \text { P(z> 3.29) } \sim .0 \quad \sim 0 \% \text { chance. }
\end{aligned}
$$

There is approximately a $0 \%$ chance of observing a sample \% of $\mathbf{6 9 . 9 6 \%}$ or higher (for a sample of 1002 people) of people who are against the war when the true \% is $65 \%$. This observed difference between $\mathbf{6 9 . 9 6 \%}$ and the expected $65 \%$ is considered to be a significant difference.
13.

Ho: on average school $A$ and $B$ are the same
Ha: on average school A does better than school $B$.
Observed difference $=37.75-33.74=4.01$ expected difference $=0$
SE difference $=\sqrt{S E_{A}^{2}+S E_{B}^{2}}=\sqrt{\left(\frac{6.67}{\sqrt{110}}\right)^{2}+\left(\frac{6.68}{\sqrt{130}}\right)^{2}}=\sqrt{(.6357)^{2}+(.5859)^{2}}=.8645$
$z=\frac{4.01-0}{.8645}=4.6483 \sim 0 \%$ chance
There is approximately a $0 \%$ chance of observing a difference of 4.01 when the true difference is 0 . Since we observed a difference of 4.01, we can conclude that on average School A does better than school B based on the sample data.
14.

Ho: on average procedure 1 and 2 are the same
Ha: on average procedure 1 is better than procedure 2.
Observed difference $=154-150=4$ expected difference $=0$
SE difference $=\sqrt{S E_{1}{ }^{2}+S E_{2}^{2}}=\sqrt{\left(\frac{12}{\sqrt{150}}\right)^{2}+\left(\frac{15}{\sqrt{260}}\right)^{2}}=\sqrt{(.9798)^{2}+(.9303)^{2}}=\mathbf{1 . 3 5 1 1}$
$z=\frac{4-0}{1.3511}=2.9606 \quad \sim .16 \%$ chance
There is approximately a $.16 \%$ chance of observing a difference of 4 when the true difference is 0 . Since we observed a difference of 4 , we can conclude that on average procedure 1 is better (faster time) than procedure 2
15.

Ho:the percentage of adults is the same for Quebec and Ontario
Ha: the percentage of adults is not the same for Quebec and Ontario

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\text { Quebec sample } \%=\frac{80}{200} \times 100 \%=40 \% \text { Ontario sample } \%=\frac{150}{300} \times 100 \%=50 \%
$$

Observed difference $50 \%-40 \%=10 \%$
SE difference

$$
\begin{aligned}
& =\sqrt{S E_{1}^{2}+S E_{2}^{2}}=\sqrt{\left(\frac{\sqrt{(.4)(.6)}}{\sqrt{200}} \times 100 \%\right)^{2}+\left(\frac{\sqrt{(.5)(.5)}}{\sqrt{300}} \times 100 \%\right)^{2}}= \\
& \sqrt{(3.4641 \%)^{2}+(2.8868)^{2}}=4.5093
\end{aligned}
$$

$z=\frac{10-0}{4.5093}=2.2176 \quad \sim 1.32 \%$ chance

There is approximately a $1.32 \%$ chance of observing a difference of $10 \%$ or more when the true difference is $0 \%$. Since we observed a difference of $10 \%$ we can conclude that the $\%$ of people who own their business in Ontario is significantly higher than people in Quebec.
16.

Ho:the percentage unemployed in August is the same as March
Ha: the percentage unemployed in August is less than that in March
March sample $\%=\frac{75}{1000} \times 100 \%=7.5 \%$ Ontario sample $\%=\frac{65}{1000} \times 100 \%=6.5 \%$

Observed difference 7.5\%-6.5\% = 1.0\%
SE difference
$=\sqrt{S E_{1}^{2}+S E_{2}^{2}}=\sqrt{\left(\frac{\sqrt{(.075)(.925)}}{\sqrt{1000}} \times 100 \%\right)^{2}+\left(\frac{\sqrt{(.065)(.935)}}{\sqrt{1000}} \times 100 \%\right)^{2}}=$
$\sqrt{(.8329 \%)^{2}+(0.7796)^{2}}=1.1408 \%$
$z=\frac{1.0 \%-0}{1.1408 \%}=.9766 \quad \sim 18.99 \%$ chance
There is approximately a $9.68 \%$ chance of observing a difference of $1.5 \%$ or more when the true difference is $0 \%$. We can conclude that the program has not been effective ( $\%$ in March that are unemployed is the same as the \% in August) because the difference is not considered significant.
24. Review chapters from previous tutorial and chapters 26 , and 27 for quiz 4.

