International Spillovers of Conventional versus New Monetary Policy

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Abstract

We compare the international spillovers of conventional and new monetary policy shocks from a large economy to a small open economy (SOE). Building on Sims and Wu (2021, *Journal of Monetary Economics,* Vol. 118, pp. 135–160), we employ a medium-scale New Keynesian model that features the major tools of new monetary policy and conventional monetary policy in a unified framework. We extend their model to an open economy setting and use it as a measurement device to quantify the spillovers and study the economic mechanisms behind them. In our empirical application, Canada is the SOE and the US is the large economy. We find that the US expansionary monetary policy shocks that have the same positive effect on US GDP (a 0.5% increase), cause different changes to Canada's private bond yield depending on the nature of the shock. For example, if the shock is due to forward guidance, Canada's private bond yield *increases* by 0.15% but if the shock is due to quantitative easing (QE), the yield *drops* by 0.13%. We also simulate counterfactual monetary policy scenarios for the US and Canada around the Great Recession of 2008. Three main conclusions emerge from these simulations: (1) Had the Fed increased the size of its QE, the recession in the US would have been milder but Canada would have had a steeper drop in GDP; (2) had the Bank of Canada followed the Fed and engaged in QE of its own by doubling the size of its balance sheet from 3% to 6% of GDP, the drop in Canada's GDP would have been 50% smaller; and (3) had the Fed engaged in a negative interest rate policy by letting its policy rate drop to− 0.5%, instead of keeping it at the zero-lower bound, the effects on Canadian economy would be very similar.

JEL Classification Codes: E52; E58; E61; F41; F42

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I. Introduction

When the Federal Reserve used new monetary policy tools like quantitative easing (QE) and forward guidance (FG) to support the US economy after the 2008 financial crisis and after its policy rate hit the zero lower bound, it sparked great interest in both academic and policy circles about the possible international spillover effects of these policies [Bernanke (2017)]. Since then, a number of studies have investigated the international spillover effects of new monetary policy tools and compared them to those of the conventional monetary policy (see the sub-section on related literature below). We contribute to this debate by extending the model in Sims and Wu (2021) (from here on, SW) to an open economy setting. A distinguishing feature of SW is that they model the conventional and major new monetary policy tools (QE, FG and negative interest rate policy (NIRP)) in a unified framework. Another feature of their model is a separate financial sector that buys both government and private bonds (also see Gertler and Karadi (2011, 2013)), and holds reserves issued by the monetary authority. Our open economy extension retains these attractive features of their framework and allows us to compare not only the international spillovers of conventional and new monetary policies, but also the spillovers resulting from the different tools of new monetary policy. To our knowledge, this is the first paper to develop a quantitative DSGE model that combines the traditional interest rate policy and the three popular new monetary policy tools (namely QE, FG and NIRP) together in an open economy framework.

There are two economies in our model: (1) a large foreign economy that we model as a closed economy; and (2) a home small open economy (SOE) that engages in international trade of goods and financial assets with the large economy. The SOE takes as given the prices of foreign goods and financial assets, and foreign income. When a monetary policy shock hits the large foreign economy, its effects are transmitted to the SOE through changes in foreign prices (for both goods and financial assets), foreign income and exchange rate. There are no reverse spillovers from the SOE to the large foreign economy.[1]

The large foreign economy in our model is almost identical to the closed economy in SW. We call it *Sims-and-Wu economy.* The Sims-and-Wu economy is a New Keynesian (NK) economy with the usual nominal frictions in the form of price and wage rigidities, and real frictions in the form of habit formation in consumption and adjustment cost of capital. There is a fiscal authority that finances its expenditure through lumpsum taxes on consumers and by issuing long-term bonds. There is a monetary authority that conducts monetary policy by choosing nominal interest rate on its reserves according to a Taylor-type rule. In addition to the standard NK features, the economy has a number of non-standard features.

The first non-standard feature of the Sims-and-Wu economy is the existence of a private long-term bond market. The only user of capital in the economy is a wholesale firm (details below) that faces a loan-in-advance constraint (as in Carlstrom et al. (2017)) and must finance a certain fraction of its investment by issuing private long-term bonds. The private long-term bonds are held by the monetary authority and financial intermediaries. The equilibrium in the private long-term bond market determines the bond price and yield. It is through this market that QE affects the real economy. If the monetary authority increases its demand for private long-term bonds, it will increase the bond price and decrease its yield to maturity. This will reduce the cost of borrowing for the wholesale firm and increase investment.

The second non-standard feature is a financial sector (as in Gertler and Karadi (2011, 2013)) consisting of a continuum of financial intermediaries (FI's). The FI's hold private and government long-term bonds. They also hold reserves issued by the monetary authority. They finance these assets with own equity and deposits from consumers. The FI's face a leverage constraint that restricts their demand for bonds, reduces bond prices and generates excess returns from bond holdings. Negative shocks to the financial sector can tighten the leverage constraint and increase the cost of borrowing for the wholesaler. QE by the monetary authority can reverse these effects by increasing the demand for bonds.

The third non-standard feature is the balance sheet of the monetary authority. The monetary authority issues reserves (a liability) to finance its purchases of the long-term private and government bonds (assets). The reserves are held by the FI's as assets and the monetary authority pays interest on them. The interest rate on reserves is the actual policy rate in this economy. When the policy rate implied by the Taylor rule is positive, the monetary authority sets the rate on reserves equal to the Taylor-rule policy rate. When the policy rate implied by the Taylor rule is not positive, the monetary authority can either set the rate on reserves to zero (assuming a zero lower bound (ZLB)) or allow it to go negative with the Taylor-rule rate (the NIRP).

There are four monetary policy tools available to the monetary authority in the Sims-and-Wu economy: (1) The policy rate, i.e. the rate on reserves; (2) QE, the purchase (sale) of long-term private and government bonds; (3) FG, the commitment by the monetary authority to keep the policy rate lower for longer; and (4) NIRP, a negative interest rate on reserves. The first tool, the policy rate, is the conventional monetary policy. The other three are the tools of new monetary policy.

For the home SOE, we keep all elements of the Sims-and-Wu economy and make the following additions to it:[2] (1) Home SOE consumers consume both home and foreign goods; (2) There is an exporting firm that exports final home good; (3) The wholesaler issues bonds in both home and foreign currencies. The foreign-currency bonds are held by the foreign financial intermediaries. (4) The FI's hold both home and foreign-currency bonds. The foreign-currency bonds that they hold are issued by the foreign wholesaler and the foreign fiscal authority.

We simulate the large foreign economy as a closed economy under the assumption that its interactions with the home SOE are so small relative to its own size that they have no effect on it. When we simulate the home SOE, we take as exogenous the simulated large foreign economy variables that appear in the home SOE equilibrium equations. The effects of shocks to the large foreign economy spillover to the home SOE through these exogenous variables. For example, if a shock reduces the policy rate in the foreign economy and results in an increase in foreign output, other things being equal, it will increase exports of the home SOE. For another example, if a shock increases the price (in foreign currency) of a foreign-currency bond (and hence reduces its yield to maturity), other things being equal, the home SOE wholesaler would issue more foreign-currency bonds as they would generate more funds at a lower cost of borrowing. Both these examples are about partial equilibrium effects. The final net effect on a variable would depend on the overall general equilibrium solution to the model. For example, when the price of a foreign-currency bond increases in the second example above, if at the same time the home currency appreciates by a lot, it is possible that the foreign bond price in home currency may actually decrease.

We calibrate the home SOE in our model to the Canadian economy and the large foreign economy to the US. We then use our model as a measurement device to quantify the spillovers from the US to Canada caused by the Fed's use of different monetary policy tools. Our quantitative general equilibrium model also enables us to better understand the economic mechanisms behind these spillovers. To save on space and keep our discussion focused, we concentrate on the spillovers to Canadian GDP and its components that are affected by the US monetary policy (consumption of home goods, investment and exports). We also discuss the spillovers to other variables that help us understand the underlying economic mechanisms. We report the results of two broad sets of experiments in this paper.

In the first set of experiments, we build on SW's idea of comparing the relative strength of the four monetary policy tools (conventional monetary policy, QE, FG and NIRP) in generating the same change in the US GDP. For each of the four tools, they calibrate expansionary monetary policy shocks in such a way that the equilibrium effect on US GDP is roughly a 0.5% increase. We use the same shocks as they do but focus on the spillover effects to the Canadian economy. The Fed's four policy tools affect the Canadian private-bond yield differently. QE decreases the bond yield by 0.13%.[3] The other three tools increase it: the conventional monetary policy increases it by 0.09%, FG by 0.12% and NIRP by 0.13%. These effects on Canadian private-bond yield are a combination of the effects on the US private-bond yield and changes in the equilibrium nominal exchange rate. These various effects on Canadian bond yields, in turn, affect Canada's investment differently: QE first increases investment by 0.2% and then decrease it by 0.8%; conventional monetary policy decreases it by 1.3%; FG decreases it by 1.8%; and NIRP decreases it by 1.9%. The other main effect is on Canada's terms of trade. The relative price of Canadian good increases resulting in decreases in Canadian consumption of home good and Canadian exports. These effects are broadly similar across the four monetary policy tools though a bit more pronounced in the case of QE. The combined effect on Canadian GDP is a drop of slightly more than 0.4%. This effect is similar across the four tools. However, the recovery in the GDP after the initial drop is fastest in the case of QE, followed by the conventional monetary policy and the slowest in the case of FG and NIRP.

In the second set of experiments, we simulate a number of counterfactual monetary policy scenarios around the 2008 Great Recession. In the benchmark scenario, we calibrate the shocks to the US and Canadian economies to match the drops in GDP and investment after the recession. We also match the overall size of the Fed's QE in the wake of the recession. We then ask three counterfactual questions: (1) Had the Fed engaged in a more or less aggressive QE, how would the spillover effects to the Canadian economy have changed? (2) Had the Bank of Canada, which did not engage in QE after the 2008 recession, also engaged in QE, how would the outcomes for the Canadian economy have been different? (3) Had the Fed let its policy rate go negative, how would the effects of this policy have spilled over to Canada? The broad answer (see Section IV for more in-depth answers to all three questions) to the first question is that the benchmark increase of 19% of GDP in the Fed's balance sheet would cause a 6% decline in Canada's GDP. A less aggressive QE (an increase of 10% of GDP in the Fed's balance sheet) would have caused a drop of 4% in Canada's GDP. A more aggressive QE (an increase of 30% of GDP in the Fed's balance sheet) would have caused a 10% drop in Canada's GDP. The broad answer to the second question is that QE by the Bank of Canada would have mitigated the negative effect on Canada's GDP: An increase in the Bank of Canada's balance sheet from 3% to 6% of GDP would have lessened the negative effect on GDP from a 6% drop to a 3% drop. The broad answer to the third question is that if the counterfactual negative policy rate by the Fed were similar to the negative policy rates tried by a number of central banks in the last decade, say -0.5%, the effects on Canadian economy would be very similar to the benchmark scenario.

A. Related Literature

The literature that explores the international spillovers of new monetary policy tools and compares them with those of the conventional monetary policy builds on two literatures: (1) The literature on *international spillovers of conventional monetary policy* [Gali (2015, pp. 252–254) provides a brief and informative overview of this literature]; and (2) the literature on the *domestic effects of the new monetary policy* [Kuttner (2018) and Bernanke (2020) are two excellent surveys of this literature].

In the literature on international spillovers of new monetary policy, researchers have taken two broad methodological approaches. The first is to use the reduced-form empirical methods, identify the conventional and/or new monetary policy shocks originating in a large economy (mostly the US) and explore their effects on various macroeconomic variables in other economies (For examples see Curcuru et al. (2018), Gilchrist et al. (2019), Neely (2015), Rogers et al. (2018) and the literature cited in these papers.).

The second approach is to use medium-scale dynamic stochastic general equilibrium (DSGE) models to build quantitative theories of international spillovers of conventional and new monetary policy tools. These models allow researchers to think explicitly about the transmission mechanisms of the policies and run counterfactual experiments to examine the effects of alternative policy paths. This is the approach that we take in this paper. The papers closest to ours in terms of the general approach are Alpanda and Kabaca (2020), and Kolasa and Wesolowski (2020). But there are important differences.

Alpanda and Kabaca (2020), similar to our paper, evaluate the international spillovers of new monetary policy using a DSGE model. But they focus on QE and study two symmetric economies. We, following SW, model all major tools of new monetary policy in a unified framework and focus on spillovers from a large economy to an SOE. Our model is more suitable to study the international spillovers of new and conventional monetary policies from the US, a large economy, to Canada, an SOE.

Kolasa and Wesolowski (2020) develop a two-country model with asset market segmentation to investigate the effects of QE by a large country central bank on a small open economy. Similar to us, they also restrict international trade in short-term bonds. This assumption not only reflects the experiences of SOEs but is also crucial in explaining the difference between spillover effects of QE and conventional monetary policy on domestic long term yields. Our paper is different from theirs because we model QE differently and the QE transmission mechanism is also different. We also model spillovers due to FG and NIRP. Another difference is in empirical application: They study spillovers from the European Union, the US and the UK to Poland.

Another difference between our paper and the two papers above is that we, following SW, incorporate the idea of an endogenous QE policy. This is an important assumption to match the Fed's QE policy (especially its second and third rounds of QE) as it unfolded in the wake of the 2008 financial crisis.

II. Model

We build on the models in Gali and Monacelli (2016), and Sims and Wu (2021) (SW from here on). Gali and Monacelli (2016) build an SOE model with staggered prices and wages to study gains from wage flexibility in a currency union. SW, building on Gertler and Karadi (2011, 2013) and Carlstrom et al. (2017), develop a closed-economy New Keynesian model that features a financial sector and a central bank that employs both conventional and new monetary policy tools.[4]

There are two countries in our model: home (Canada) and foreign (the United States). The foreign country is not affected by economic events in the home country and we model it as a closed economy. This assumption is justified by the relatively large size of the US economy and the small size of its trade with Canada as a fraction of its GDP.[5] The foreign economy in our model is almost identical to the US economy in SW.[6] The focus of our study is the home economy that we model as a small open economy (SOE). The SOE takes all foreign variables (import prices, bond prices and foreign income) as given. We model the SOE by making the following additions to the closed economy in SW: (1) We allow the representative consumer to consume foreign goods (imports) in addition to home goods; (2) we introduce an exporter who buys the final good and sells it to the foreign country; (3) we allow the wholesale firm to issue both home- and foreign-currency bonds; and (4) we allow financial intermediaries to hold both home- and foreign-currency bonds (both private and government). The first two additions represent the real side of the SOE and the last two the financial side. We now provide some details of our model.

A. Household

The first part of the household's problem in our model is identical to that in SW except that we have changed some of their notation, changed the utility function from log to CRRA and introduced a shock to utility. The problem of the representative household is

$$\max_{\{C_{t+\tau}, L_{1,t+\tau}, D_{t+\tau}\}} \mathbb{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} Z_{t+\tau} \left(\frac{(C_{t+\tau} - hC_{t+\tau-1})^{1-\sigma}}{1-\sigma} - \omega \frac{L_{1,t+\tau}^{1+\varphi}}{1+\varphi} \right) \right) \tag{1}$$

subject to the following *nominal* period budget constraint:

$$P_{C,t}C_t + D_t - D_{t-1} \leq W_{1,t}L_{1,t} + \Pi_t - P_{C,t}T_t - P_{C,t}\chi + (R^D_{t-1} - 1)D_{t-1}. \quad (2)$$

C_t is a composite consumption good which is a CES (constant elasticity of substitution) aggregate of consumption on home and foreign goods, $L_{1,t}$ is the labor supply (work hours) of the household and Z_t is a preference shock. We assume that $\log Z_t$ follows an AR(1) process. $P_{C,t}$ is the consumer price index, which is a CES aggregate of home and foreign prices. $W_{1,t}$ is the nominal wage a household receives from the labor unions. Π_t is the net nominal dividend from all financial and non-financial firms. T_t is the real lump sum tax paid to the fiscal authority. χ is the real transfer from the household to the new financial intermediaries. D_t is the household's deposits with financial intermediaries. These deposits pay a nominal gross interest rate of R^D_t. The household chooses sequences of consumption (C_t), work hours (L_t) and deposits (D_t) to maximize the expected value of utility over the infinite horizon.

As in Gali and Monacelli (2016), the household combines home-produced goods, $C_{H,t}$, and foreign-produced goods (imports), $C_{F,t}$ to produce the composite consumption good, C_t according to the following CES production function:

$$C_t = \left((1-\nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1 - 1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} C_{F,t}^{\frac{\eta_1 - 1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1 - 1}}. \quad (3)$$

The problem of the household is to choose $C_{H,t}$ and $C_{F,t}$ that minimize the cost of composite consumption C_t subject to the CES production function above.

The solution to the household's problem satisfies the nine equations (from eqn. 01/72 to eqn. 09/72) in the appendix.[7]

B. Labor Market

The labor market in our model is identical to that in SW. The following flow chart illustrates the flow of labor in our model economy:

$$\text{Household} \longrightarrow \text{Labor unions} \longrightarrow \text{Labor packer} \longrightarrow \text{Wholesale firm}$$

The household supplies labor ($L_{1,t}$) to labor unions at a wage ($W_{1,t}$) that is equal to the marginal rate of substitution of leisure for consumption.

There is a unit mass of labor unions indexed by $i \in [0, 1]$. Each labor union takes labor services from the household and repackages them into *specialized labor* $L_{2,t}(i)$, which is specific to union i, and sells it to a labor packer at wage $W_{2,t}(i)$. The labor unions face Calvo-type wage rigidity. Each period, θ_w fraction of unions do not choose optimal wage. Instead, they update their last-period wage by using an indexation formula. The remaining fraction $1 - \theta_w$ of unions reoptimize and choose a new optimal wage $W^*_{2,t}(i)$.

The labor packer combines specialized union labor $L_{2,t}(i)$ into a final labor bundle $L_{2,t}$ according to a CES aggregator. The packer sells $L_{2,t}$ to the wholesale firm—the only user of labor for production—at the economy-wide wage of $W_{2,t}$.

The labor-market equilibrium conditions are in eqns. 10/72 to 15/72 in the appendix.

C. Non-Financial Firms

There are five types of non-financial firms in our model: (1) A representative wholesale firm produces output using its own capital, accumulated through purchases of new capital from the capital-good firm, and labor hired from the labor packer. (2) A continuum of retail firms repackage wholesale output for resale to the final-good firm. Retail firms behave as monopolistic competitors and are subject to Calvo-type price stickiness. (3) A competitive final-good producer aggregates retail output into a final good that is meant for consumption (by both the household and government), investment and export. (4) A representative capital-good firm purchases final output and transforms it into new physical capital subject to an adjustment cost. (5) A representative exporter buys the final good and sells it to the foreign country.

The firms of the first four types behave in exactly the same way as in SW except that we allow the wholesale firm to issue foreign-currency bonds in addition to home-currency bonds. This change is motivated by the observation that close to half (in terms of value) of all outstanding bonds issued by Canadian non-financial corporations were issued in foreign currencies, mostly in the US dollars.[8] We add an export firm to the model along the lines of Gali and Monacelli (2016).[9] We now describe the problem of each type.

Wholesale Firm.—A representative wholesale firm produces according to a Cobb-Douglas technology:

$$\text{(eqn. 16/72)} \qquad Y_{2,t} = A_t(u_tK_t)^{\alpha}L_{2,t}^{1-\alpha}.$$

$Y_{2,t}$ is the flow of output and $L_{2,t}$ is the labor input. Parameter $\alpha \in (0, 1)$ is the share of capital in production. A_t is total-factor productivity that follows an exogenous stochastic process. u_t is capital utilization. K_t is the stock of physical capital owned by the firm. A higher utilization of capital leads to faster depreciation. Function $\delta(u_t)$ maps utilization into depreciation. Physical capital evolves according to:

$$\text{(eqn. 18/72)} \qquad K_{t+1} = \hat{I}_t + (1 - \delta(u_t))K_t,$$

where $\hat{I}_t$ is new gross investment. The wholesale firm is constrained to finance a constant fraction $\psi \in [0, 1]$ of investment by issuing private debt. This constraint, which SW and Carlstrom et al. (2017) call the "loan-in-advance constraint," is

$$\text{(4)} \qquad \psi P_{k,t}\widehat{I}_t \leq \bar{Q}_{H,t}^{Pvt}(\bar{B}_{H,t}^{Pvt} - \kappa\bar{B}_{H,t-1}^{Pvt}).$$

$P_{k,t}$ is the price at which the wholesale firm purchases new physical capital. The left-hand side of the constraint is a fraction ψ of the wholesale firm's investment expenditure $P_{k,t}\widehat{I}_t$. The right-hand side is the net addition to the wholesale firm's debt (i.e. outstanding bonds).[10] The firm has $\bar{B}_{H,t-1}^{Pvt}$ units of bonds outstanding from the previous period that are equal to $\kappa\bar{B}_{H,t-1}^{Pvt}$ units of bonds this period because of drop in return from one to κ. $\bar{B}_{H,t}^{Pvt}$ is the firm's new bond issue. The difference $\bar{B}_{H,t}^{Pvt} - \kappa\bar{B}_{H,t-1}^{Pvt}$ is the net addition to the wholesale firm's debt. This debt is priced at $\bar{Q}_{H,t}^{Pvt}$. The loan-in-advance constraint forces the firm to issue new debt to finance at least ψ fraction of its current-period investment expenditure.

Although the loan-in-advance constraint in (4) looks very similar to the one in SW (see their equation 2.21), it is not the same. To be consistent with the borrowing pattern of Canadian corporations, we model the wholesale firm to issue bonds in home and foreign currencies. The values of the two types of bonds aggregate according to

$$\bar{Q}_{H,t}^{Pvt}\bar{B}_{H,t}^{Pvt} = Q_{H,t}^{Pvt}B_{H,t}^{Pvt} + Q_{H,FC,t}^{Pvt}B_{H,FC,t}^{Pvt},$$

where $B_{H,t}^{Pvt}$ is the total quantity of outstanding bonds issued in *home currency* by the wholesale firm, $B_{H,FC,t}^{Pvt}$ is the total quantity of outstanding bonds issued in *foreign currency* and

$$\bar{B}_{H,t}^{Pvt} = \left[\left(\frac{1}{1-\upsilon_3}\right)^{1/\eta_3}\left(B_{H,t}^{Pvt}\right)^{1+1/\eta_3} + \left(\frac{1}{\upsilon_3}\right)^{1/\eta_3}\left(B_{H,FC,t}^{Pvt}\right)^{1+1/\eta_3}\right]^{\frac{\eta_3}{\eta_3+1}} \tag{5}$$

is a CES-type aggregate. The firm takes the bond prices $Q_{H,t}^{Pvt}$ and $Q_{H,FC,t}^{Pvt}$ as given ($\bar{Q}_{H,t}^{Pvt}$, which we define below, is a CES-type aggregate of $Q_{H,t}^{Pvt}$ and $Q_{H,FC,t}^{Pvt}$) and chooses $B_{H,t}^{Pvt}$ and $B_{H,FC,t}^{Pvt}$ to maximize its bond-sale proceeds

$$Q_{H,t}^{Pvt}B_{H,t}^{Pvt} + Q_{H,FC,t}^{Pvt}B_{H,FC,t}^{Pvt}$$

subject to (5). The solution to this problem gives the wholesale firm's bond supply functions that are in eqns. 26/72 and 27/72 in the appendix. The shadow price of $\bar{B}_{H,t}^{Pvt}$ is $\bar{Q}_{H,t}^{Pvt}$ which is in eqn. 28/72 in the appendix. To see the intuition behind the bond supply functions, take the ratio of eqns. 26/72 and 27/72:

$$\frac{B_{H,FC,t}^{Pvt}}{B_{H,t}^{Pvt}} = \frac{\upsilon_3}{1-\upsilon_3}\left(\frac{Q_{F,t}^{Pvt}}{Q_{H,t}^{Pvt}}\right)^{\eta_3}.$$

The relative supply of foreign-currency bonds depends on their relative price. If a unit of foreign bond can bring in more proceeds (in home currency), i.e. $Q_{F,t}^{Pvt}/Q_{H,t}^{Pvt}$ increases, the wholesale firm would like to issue more foreign bonds, i.e. $B_{H,FC,t}^{Pvt}/B_{H,t}^{Pvt}$ will increase. The elasticity of relative supply is given by $\eta_3 > 0$. Parameter υ_3 determines the openness of the wholesale firm to foreign borrowing. For example, if the home- and foreign-bond prices are equal, i.e. $Q_{F,t}^{Pvt} = Q_{H,t}^{Pvt}$, the ratio of foreign to home bonds will be $\upsilon_3/(1-\upsilon_3)$. If $\upsilon_3 = 0$, the wholesale firm will not issue any foreign-currency bonds.

As in SW, bonds issued in home currency are held by the home financial intermediaries (indexed by $j \in [0,1]$) and the home monetary authority:

$$B_{H,t}^{Pvt} = B_{H,t}^{Pvt}\left(fi\right) + B_{H,t}^{Pvt}\left(ma\right),$$

where $B_{H,t}^{Pvt}\left(fi\right) \equiv \int B_{H,t}^{Pvt}\left(j\right) dj$ is the sum of private-bond holdings of the financial intermediaries and $B_{H,t}^{Pvt}\left(ma\right)$ are the private-bond holdings of the monetary authority. We assume for simplicity that the foreign-currency bonds issued by the wholesale firm are held only by the foreign financial intermediaries. Because of the SOE assumption, the wholesale firm takes the price of foreign-private bonds, $Q_{H,FC,t}^{Pvt}$, as given.

$Q_{H,FC,t}^{Pvt}$ is the price of foreign private bonds in home currency. The corresponding price in foreign currency is $\text{USD}Q_{H,FC,t}^{Pvt} = Q_{H,FC,t}^{Pvt}/E_t$, where USD represents foreign currency and E_t is the nominal exchange rate defined as the price of one unit of foreign currency in terms of home currency. The realized return on the foreign-private bonds in foreign currency is

$$\text{USD}R_{H,FC,t}^{Pvt} = \frac{\text{USD}1 + \kappa \text{USD}Q_{H,FC,t}^{Pvt}}{\text{USD}Q_{H,FC,t-1}^{Pvt}},$$

which we can convert to a return in home currency of

$$R_{H,FC,t}^{Pvt} = \text{USD}R_{H,FC,t}^{Pvt}\frac{E_t}{E_{t-1}}.$$

Apart from the choice of the wholesale firm between home and foreign bonds, the rest of the problem of the wholesale firm in our model is identical to that in SW. The objective of the wholesale firm is to choose labor, capital, capital utilization and bonds to maximize its real profit

$$\frac{\Pi_{2,t}}{P_{C,t}} = p_{2,t}Y_{2,t} - w_{2,t}L_{2,t} - p_{K,t}\widehat{I}_t + \bar{Q}_{H,t}^{Pvt}(\bar{b}_{H,t}^{Pvt} - \kappa \bar{b}_{H,t-1}^{Pvt}\pi_{C,t}^{-1}) - \bar{b}_{H,t-1}^{Pvt}\pi_{C,t}^{-1},$$

subject to eqn. 16/72, eqn. 18/72 and (4) [see page 10 above], where $p_{2,t} \equiv P_{2,t}/P_{C,t}$ is the relative price of wholesale firm's output and $p_{k,t} \equiv P_{K,t}/P_{C,t}$ is the relative price of new capital. The first term on the right-hand side is the firm's revenue from sale of output. The second term is its labor cost. The third term is the cost of new investment. The fourth term is the real value of new bond issue ($\bar{b}_{H,t}^{Pvt} \equiv \bar{B}_{H,t}^{Pvt}/P_{C,t}$ and $\pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$) and the last term is the real coupon payment on outstanding bonds. The first-order conditions for this problem, together with other equilibrium conditions, are in eqns. 16/72 to 28/72 in the appendix.

Retailers.—Retail firms are indexed by $f \in [0,1]$. They repackage wholesale output $Y_t(f) = Y_{2,t}(f)$ and sell it to a competitive final-good firm. The final output, Y_t, is a CES aggregate of retail outputs with elasticity of substitution $\epsilon_p > 1$. Hence, retailer f faces the demand curve:

$$Y_t(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\epsilon_p} Y_t, \tag{6}$$

where $P_{H,t}(f)$ is the price of retail output of firm f. The price of the final good, $P_{H,t}$, is given by

$$P_{H,t}^{1-\epsilon_p} = \int_0^1 P_{H,t}(f)^{1-\epsilon_p} df. \tag{7}$$

The nominal profit of a retail firm is

$$\Pi_{3,t}(f) = P_{H,t}(f)Y_t(f) - P_{2,t}Y_{2,t}(f). \tag{8}$$

By using $Y_{2,t}(f) = Y_t(f)$ and the demand function, we obtain:

$$\Pi_{3,t}(f) = P_{H,t}(f)^{1-\epsilon_p} P_{H,t}^{\epsilon_p} Y_t - P_{2,t} P_{H,t}(f)^{-\epsilon_p} P_{H,t}^{\epsilon_p} Y_t. \tag{9}$$

The retail firms set prices in a Calvo fashion. In each period, a retailer faces a constant probability $1 - \theta_p$ of being able to adjust its price, with $\theta_p \in [0,1]$. Non-updated prices are indexed to lagged CPI inflation ($\pi_{C,t-1}$).

The relevant equilibrium conditions from retailers' problem are in eqns. 29/72 to 31/72 in the appendix.

Final-Good Producer.—The final-good producer buys $Y_t(f)$ at price $P_{H,t}(f)$ from retailers and combines them into a composite final good Y_t. The equilibrium conditions from the final-good producer's problem are in eqns. 32/72 to 34/72 in the appendix.

Capital Producers.—A representative capital producer generates new physical capital according to

$$\widehat{I}_t = \left(1 - O\left(\frac{I_t}{I_{t-1}}\right)\right) I_t. \tag{10}$$

I_t is the final output allocated to investment. $O(\cdot)$ is an adjustment-cost function. The capital producer chooses I_t to maximize

$$P_{k,t}\left[1 - O\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_{H,t} I_t. \tag{11}$$

We assume the following functional form for $O(\cdot)$:

$$O\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

The equilibrium conditions for the capital producer's problem are in eqns. 35/72 and 36/72 in the appendix.

Exporters.—We follow Gali and Monacelli (2016) to model exports. The exporting firm is a price taker and the demand for exports from the foreign country is

$$X_t = \nu_2 \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_2} Y_{F,t} \tag{12}$$

where $P_{F,t}$ is the home-currency price of the foreign good and $Y_{F,t}$ is the real GDP of the foreign country. We note that

$$P_{F,t} \equiv \mathrm{USD}P_{F,t} E_t, \tag{13}$$

where $\mathrm{USD}P_{F,t}$ is the price of foreign good in foreign currency and E_t is the nominal exchange rate. Let $p_{F,t} \equiv P_{F,t}/P_{C,t}$ and $p_{H,t} \equiv P_{H,t}/P_{C,t}$. The equilibrium condition for the exporter's problem is in eqn. 37/72 in the appendix.

D. Financial Intermediaries

There is a unit mass of financial intermediaries (FI's) indexed by $j \in [0,1]$. Each period, $1-\vartheta$ fraction of financial intermediaries exit and are replaced by an equal mass of new entrants such that the total mass remains the same. The exiting FI's return their networth to the household. The new entrants receive an initial endowment of networth from the household. Each FI holds private bonds, $B_t^{Pvt}(j)$, government bonds, $B_t^{Gov}(j)$, and interest bearing reserves, $S_t(j)$, with the central bank. These assets are matched by the FI's liabilities in the form of household deposits, $D_t(j)$ and net worth, $N_t(j)$. The balance sheet of financial intermediary j is:

$$Q_t^{Pvt} B_t^{Pvt}(j) + Q_t^{Gov} B_t^{Gov}(j) + S_t(j) = D_t(j) + N_t(j).$$

The value of financial intermediary j at t is:

$$V_t(j) = (1-\vartheta)\,\mathbb{E}_t \Lambda_{t,t+1} n_{t+1}(j) + \vartheta \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}(j).$$

The two constraints facing the FI are:

$$V_t(j) \geq \theta_t \left(Q_t^{Pvt} b_t^{Pvt}(j) + \Delta Q_t^{Gov} b_t^{Gov}(j) \right),$$

where $b_t^{Pvt}(j) \equiv B_t^{Pvt}(j)/P_{C,t}$ and $b_t^{Gov}(j) \equiv B_t^{Gov}(j)/P_{C,t}$, and

$$s_t(j) \geq \varsigma_t d_t(j),$$

where $s_t(j) \equiv S_t(j)/P_{C,t}$ and $d_t(j) \equiv D_t(j)/P_{C,t}$. The first constraint above is central to the problem of the FI. If this constraint is not binding, the FI's will buy unrestricted quantities of bonds, bid up the bond prices and drive down bond returns to the level of return on deposits. In that case, there would be no excess returns from holding bonds and the FI would be indifferent among the three assets that it can hold. In our model, this constraint is always binding:

$$V_t(j) = \theta_t \left(Q_t^{Pvt} b_t^{Pvt}(j) + \Delta Q_t^{Gov} b_t^{Gov}(j) \right).$$

The second constraint is only binding when the monetary authority allows the rate on reserves to be negative. So far, the problem of the FI above is identical to that in SW and the relevant equilibrium conditions are in appendix eqns. 38/72 to 46/72.

What is new in our model is that we allow the FI's to also hold foreign-private and foreign-government bonds, both issued in foreign currency. This modeling choice is also motivated by data. For example, in 2006, Canada's biggest five banks held government bonds worth \$155.7 billion. Out of these, \$67.6 billion (43.4%) were foreign government (mostly the US-government) bonds. To do so, we decompose the first two terms on the left-hand side of the balance-sheet equation as:

$$Q_t^{Pvt} B_t^{Pvt}(j) = Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt} B_{F,t}^{Pvt}(j)$$

and

$$Q_t^{Gov} B_t^{Gov}(j) = Q_{H,t}^{Gov} B_{H,t}^{Gov}(j) + Q_{F,t}^{Gov} B_{F,t}^{Gov}(j).$$

This formulation provides two more channels through which the effects of foreign monetary policy spillover to the home SOE. For example, when the interest rate on foreign-private bonds increases, the FI's will increase their holdings of these bonds.

The FI's take the prices of foreign-private $\left(Q_{F,t}^{Pvt}\right)$ and foreign-government $\left(Q_{F,t}^{Gov}\right)$ bonds as given. We assume the foreign private bond market to be frictionless and hence

$$Q_{F,t}^{Pvt} = Q_{H,FC,t}^{Pvt},$$

i.e. the price at which the home FI's buy foreign-private bonds is the same as the price at which home wholesale firms sell foreign-private bonds. And because of the SOE assumption, home firms, both financial and non-financial, take this price as exogenously given.

For the private bonds, the FI's problem is to choose $B_{H,t}^{Pvt}(j)$ and $B_{F,t}^{Pvt}(j)$ to minimize the total cost of private bonds

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt} B_{F,t}^{Pvt}(j)$$

subject to

$$B_t^{Pvt}(j) = \left[\left(\frac{1}{1-\nu_4} \right)^{-1/\eta_4} \left[B_{H,t}^{Pvt}(j) \right]^{\frac{\eta_4-1}{\eta_4}} + \left(\frac{1}{\nu_4} \right)^{-1/\eta_4} \left[B_{F,t}^{Pvt}(j) \right]^{\frac{\eta_4-1}{\eta_4}} \right]^{\frac{\eta_4}{\eta_4-1}}.$$

The solution to this problem gives the demand function in eqns. 47/72 and 48/72 in the appendix. A similar problem for government bonds gives the demand function in eqns. 49/72 and 50/72 in the appendix. The appendix eqns. 51/72 to 60/72 contain other equilibrium conditions that mainly consist of the definitions of some bond prices and returns.

E. Monetary Authority

We model the monetary authority in the same way as do SW. The monetary authority conducts monetary policy in two ways: (1) by adjusting the interest rate on reserves and (2) by adjusting its holdings of home-private and home-government bonds. The choice of the interest rate on reserves is guided by the following Taylor-type rule:

$$\begin{aligned} \ln R_t^{Pol} &= (1-\rho_r) \ln \left(R^{Pol} \right)^{SS} + \rho_r \ln R_{t-1}^{Pol} \\ &\quad + (1-\rho_r)[\phi_\pi(\ln \pi_t - \ln \pi^{SS}) + \phi_y(\ln Y_t - \ln Y_{t-1})] + s_r \varepsilon_{r,t} \end{aligned}$$

where $\left(R^{Pol} \right)^{SS}$ and π^{SS} are steady state values of the policy rate and the CPI inflation target, $0 < \rho_r < 1$, and ϕ_π and ϕ_y are non-negative parameters. There is no restriction on R_t^{Pol} and its realized value depends on inflation and output gaps, and on the policy-rate shock ($\varepsilon_{r,t}$). The realized value of R_t^{Pol} leads to one of the following three scenarios regarding the monetary authority's choice of the interest rate on reserves.

Scenario 1: $R_t^{Pol} > 1$.—When $R_t^{Pol} > 1$, the monetary authority sets the interest rate on reserves equals to the policy rate implied by the Taylor rule: $R_t^S = R_t^{Pol}$.

Table 1: Three scenarios

Scenario	Taylor-rule rate $\left(R_t^{Pol}\right)$	Rate on reserves $\left(R_t^S\right)$	Rate on deposits $\left(R_t^D = \max\left\{1, R_t^S\right\}\right)$
1	$R_t^{Pol} > 1$	$R_t^S = R_t^{Pol}$	$R_t^D = R_t^S$
2	$R_t^{Pol} \leq 1$	$R_t^S = \max\left\{1, R_t^{Pol}\right\}$ (which implies $R_t^S = 1$)	$R_t^D = 1$
3	$R_t^{Pol} \leq 1$	$R_t^S = \max\left\{\underline{R}, R_t^{Pol}\right\}$ (which implies $R_t^S \in [\underline{R}, 1]$)	$R_t^D = 1$

Scenario 2: $R_t^{Pol} \leq 1$ ***and the monetary authority does not want negative interest rate on reserves.***—When $R_t^{Pol} \leq 1$, the monetary authority may decide that it will not reduce the rate on reserves below zero. In this scenario, $R_t^S = \max\left\{1, R_t^{Pol}\right\}$, which implies $R_t^S = 1$ because $R_t^{Pol} \leq 1$.

Scenario 3: $R_t^{Pol} \leq 1$ ***and the monetary authority is open to negative interest rate on reserves.***—When $R_t^{Pol} \leq 1$, the monetary authority may decide to allow negative interest rate on reserves. Suppose $\underline{R} < 1$ is the lowest interest rate on reserves that the monetary authority is willing to allow, then $R_t^S = \max\left\{\underline{R}, R_t^{Pol}\right\}$. In this scenario, because $R_t^{Pol} \leq 1$, $R_t^S \in [\underline{R}, 1]$.

Under Scenario 1, the FI's set $R_t^D = R_t^S$. Under Scenarios 2 and 3, the FI's set $R_t^D = 1$. So the FI's choice of the interest rate on deposits can be summarized as

$$R_t^D = \max\left\{1, R_t^S\right\}.$$

We summarize the choices of R_t^S and R_t^D in the three scenarios in Table 1.

In addition to the choice of R_t^S, the monetary authority can also buy or sell home-private and home-government bonds. This is what is commonly known as quantitative easing (QE). Note that the monetary authority can potentially engage in QE in any of the above three scenarios. However, in practice, monetary authorities have resorted to QE only when $R_t^{Pol} \leq 1$ (i.e. Scenarios 2 and 3). To see how QE works in this model, note that the balance sheet of the monetary authority is

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(ma) + Q_{H,t}^{Gov} B_{H,t}^{Gov}(ma) = S_t,$$

where the assets, which consist of bond holdings, are on the left-hand side and the liabilities, which consist of reserves, are on the right-hand side. We can express this balance sheet in real terms as

(eqn. 61/72) $$Q_{H,t}^{Pvt} b_{H,t}^{Pvt}(ma) + Q_{H,t}^{Gov} b_{H,t}^{Gov}(ma) = s_t.$$

The monetary authority's choices of $b_{H,t}^{Pvt}(ma)$ and $b_{H,t}^{Gov}(ma)$ follow the following AR(1) processes:

(eqn. 66/72, Exog.) $$b_{H,t}^{Pvt}(ma) = (1-\rho_1)\left(b_H^{Pvt}(ma)\right)^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) + s_1\varepsilon_{1,t}$$

and

(eqn. 67/72, Exog.) $$b_{H,t}^{Gov}(ma) = (1-\rho_2)\left(b_H^{Gov}(ma)\right)^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) + s_2\varepsilon_{2,t},$$

We will call $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ the QE shocks. We also consider endogenous QE policies under which the monetary authority's choices of $b_{H,t}^{Pvt}(ma)$ and $b_{H,t}^{Gov}(ma)$ could alternatively follow Taylor-rule-type reaction functions:

(eqn. 66/72, Endo.)

$$\begin{aligned} b_{H,t}^{Pvt}(ma) &= (1-\rho_1)\left(b_H^{Pvt}(ma)\right)^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) \\ &\quad + (1-\rho_1)\Psi_1\left[\phi_\pi\left(\ln\pi_{C,t} - \ln\pi_C^{SS}\right) + \phi_y\left(\ln Y_t - \ln Y_{t-1}\right)\right] + s_1\varepsilon_{1,t} \end{aligned}$$

and

(eqn. 67/72, Endo.)

$$\begin{aligned} b_{H,t}^{Gov}(ma) &= (1-\rho_2)\left(b_H^{Gov}(ma)\right)^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) \\ &\quad + (1-\rho_2)\Psi_2\left[\phi_\pi\left(\ln\pi_{C,t} - \ln\pi_C^{SS}\right) + \phi_y\left(\ln Y_t - \ln Y_{t-1}\right)\right] + s_2\varepsilon_{2,t}. \end{aligned}$$

The appendix eqns. 61/72 to 67/72 contain the equilibrium conditions related to the monetary authority.

F. *Fiscal Authority*

We also model the fiscal authority in the same way as do SW. The nominal period budget constraint of the fiscal authority is:

$$P_{C,t}G_t + B^{Gov}_{H,t-1} = P_{C,t}T_t + \Pi_{ma,t} + Q^{Gov}_{H,t}B^{Gov}_{H,t} - \kappa Q^{Gov}_{H,t}B^{Gov}_{H,t-1}.$$

The fiscal authority consumes an exogenous and stochastic amount of final good, G_t. The money to pay for these expenses comes from lump-sum taxes on the household (T_t), profit of the monetary authority $(\Pi_{ma,t})$ and borrowing $\left(B^{Gov}_{H,t}\right)$. The real outstanding government debt is constant and equal to

$$\frac{B^{Gov}_{H,t}}{P_{C,t}} = \frac{B^{Gov}_{H,t-1}}{P_{C,t-1}} = \bar{b}^{Gov}$$

and the lump-sum taxes adjust every period to satisfy the government's budget constraint. The market-clearing condition for home-government bonds is

$$B^{Gov}_{H,t} = B^{Gov}_{H,t}(fi) + B^{Gov}_{H,t}(ma),$$

where $B^{Gov}_{H,t}(fi) \equiv \int B^{Gov}_{H,t}(j)\,dj$. The equilibrium conditions related to the fiscal authority are in eqns. 68/72 to 70/72.

G. *Foreign Exchange Market*

There are two important equations related to the foreign-exchange market. The first is the evolution of the real exchange rate (RER). By definition, the RER is

$$\text{RER}_t = \frac{\text{USD}P_{F,t}E_t}{P_{C,t}} = \frac{P_{F,t}}{P_{C,t}} = p_{F,t}.$$

Similarly,

$$p_{F,t-1} \equiv \frac{P_{F,t-1}}{P_{C,t-1}} = \frac{E_{t-1}\text{USD}P_{F,t-1}}{P_{C,t-1}}.$$

By taking the ratio of the two we get

$$\frac{p_{F,t}}{p_{F,t-1}} = \frac{E_t}{E_{t-1}} \frac{\text{USD}\pi_{F,t}}{\pi_{C,t}},$$

which we can rearrange to get eq. 71/72 in the appendix.

The second equation is the balance-of-payment equilibrium condition. The total demand for home currency (in home currency units) in the foreign-exchange market in period t is

$$\underbrace{P_{H,t}X_t}_{\text{Exports}} + \underbrace{\left(B^{Pvt}_{F,t-1} + B^{Gov}_{F,t-1}\right) E_t}_{\text{Interest income of FI's}} + \underbrace{Q^{Pvt}_{F,t}\left(B^{Pvt}_{H,FC,t} - \kappa B^{Pvt}_{H,FC,t-1}\right)}_{\text{Net new debt of wholesale firm}}.$$

The total supply of home currency in units of home currency in period t is

$$\underbrace{P_{F,t}C_{F,t}}_{\text{Imports}} + \underbrace{E_t B^{Pvt}_{H,FC,t-1}}_{\text{Interest paid by wholesale firm}} + \underbrace{Q^{Pvt}_{F,t}(B^{Pvt}_{F,t} - \kappa B^{Pvt}_{F,t-1}) + Q^{Gov}_{F,t}(B^{Gov}_{F,t} - \kappa B^{Gov}_{F,t-1})}_{\text{Net addition to foreign-bond holdings of FI's}}.$$

Equating the demand and supply of home currency, and dividing both sides by $P_{C,t}$ gives the balance-of-payment equilibrium condition in eqn. 72/72 of the appendix.[11]

This completes our description of the SOE model. There are 72 equilibrium equations and 72 endogenous variables. We provide a complete list of endogenous variables in Appendix A. The list also includes a short description of each variable. The full set of equilibrium equations is in Appendix B. In an online appendix that accompanies this paper, we solve the model for its non-stochastic steady state, provide other details about the model and derive the equilibrium conditions that are new to our model. We refer the reader to SW for derivations of the equilibrium conditions that are common between SW and this paper.

Our model is an SOE version of the model in SW. On the real side, we add imports and exports by closely following Gali and Monacelli (2016). On the financial side, we allow the wholesale firm to issue bonds in both home and foreign currencies. We also allow home financial intermediaries to hold both home and foreign bonds. We summarize the flow of bonds in our model in Figure 1.

The bonds that the wholesaler issues are $\bar{B}^{Pvt}_{H,t}$, which is a CES aggregate of $B^{Pvt}_{H,t}$ and

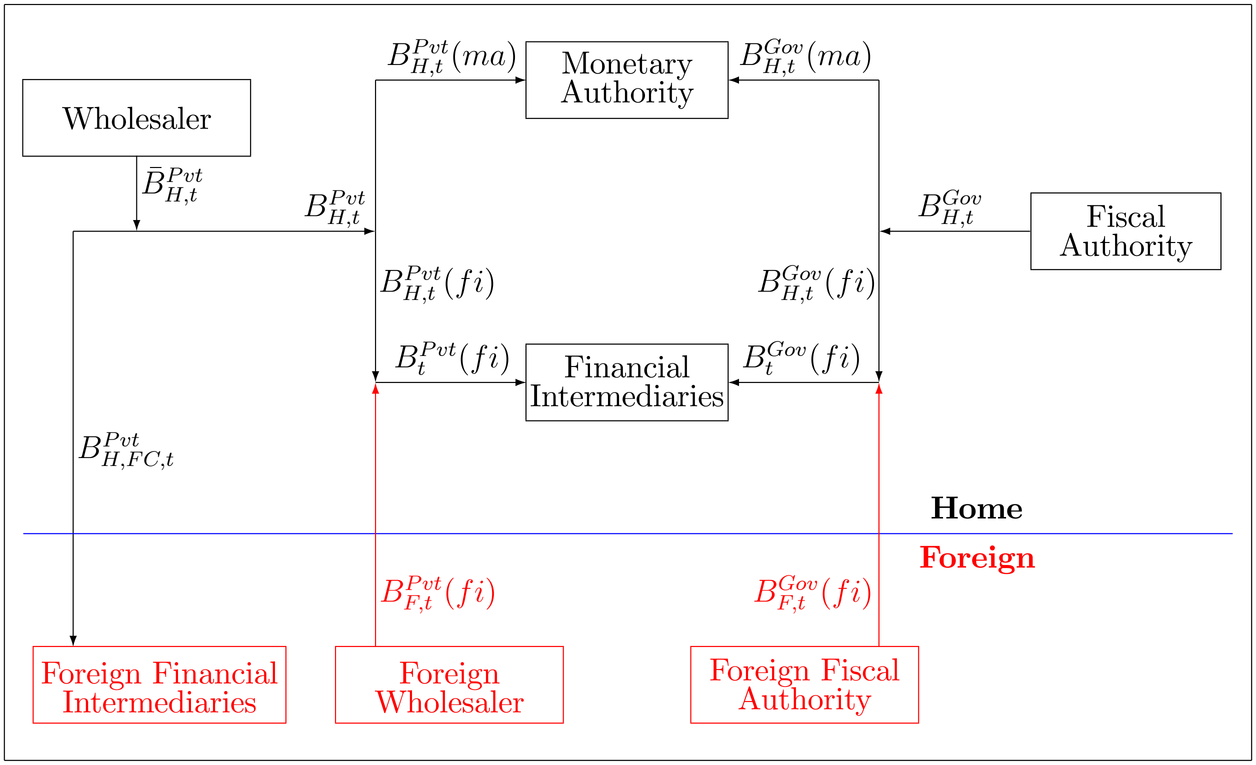


Figure 1: Bond flow chart

$B^{Pvt}_{H,FC,t}$, and it also satisfies

$$\bar{B}^{Pvt}_{H,t} = \frac{Q^{Pvt}_{H,t}}{\bar{Q}^{Pvt}_{H,t}} B^{Pvt}_{H,t} + \frac{Q^{Pvt}_{F,t}}{\bar{Q}^{Pvt}_{H,t}} B^{Pvt}_{H,FC,t}.$$

The wholesale firm issues $B^{Pvt}_{H,t}$ bonds in home currency. These bonds are held by the monetary authority, $B^{Pvt}_{H,t}(ma)$, and the financial intermediaries, $B^{Pvt}_{H,t}(fi)$:

$$B^{Pvt}_{H,t} = B^{Pvt}_{H,t}(ma) + B^{Pvt}_{H,t}(fi).$$

The wholesale firm also issues $B^{Pvt}_{H,FC,t}$ bonds in foreign currency. The choice between $B^{Pvt}_{H,t}$ and $B^{Pvt}_{H,FC,t}$ depends on the home bias of the wholesale firm, the relative price of the two types of bonds and an elasticity parameter.

The home financial intermediaries hold both private, $B^{Pvt}_{t}(fi)$, and government, $B^{Gov}_{t}(fi)$, bonds. Within each caltegory, they hold both home and foreign bonds that satisfy

$$B_t^{Pvt}(fi) = \frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}} B_{H,t}^{Pvt}(fi) + \frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}} B_{F,t}^{Pvt}(fi)$$

and

$$B_t^{Gov}(fi) = \frac{Q_{H,t}^{Gov}}{Q_t^{Gov}} B_{H,t}^{Gov}(fi) + \frac{Q_{F,t}^{Gov}}{Q_t^{Gov}} B_{F,t}^{Gov}(fi).$$

The financial intermediaries' choices between home and foreign bonds of each type depend on the degree of their home bias, the relative price of the two types of bonds and the elasticity parameters.

The fiscal authority issues $B_{H,t}^{Gov}$ bonds, all in home currency. Of these, $B_{H,t}^{Gov}(ma)$ are held by the monetary authority and $B_{H,t}^{Gov}(fi)$ are held by the finanacial intermediaries:

$$B_{H,t}^{Gov} = B_{H,t}^{Gov}(ma) + B_{H,t}^{Gov}(fi).$$

The monetary authority, holds home-currency bonds issued by the wholesale firm, $B_{H,t}^{Pvt}(ma)$, and by the fiscal authority, $B_{H,t}^{Gov}(ma)$.

III. Calibration and Computation of Solution

We model the foreign country as a closed economy, which is almost identical to the Sims-and-Wu economy. The only changes that we make to Sims-and-Wu economy are to generalize log utility to CRRA and add a preference shock. These changes add three parameters to their model. Our modified model of Sims-and-Wu economy has 45 parameters in total. In Tables 1 and 2 in the online appendix, we list these parameters and their values or targets for the home SOE. The online-appendix Table 1 has 14 parameters, all of which are related to the dispersion or persistence of shocks in the model. The remaining 31 parameters are in the online-appendix Table 2.

We take most of these parameter values and targets from SW. The only differences are the following: (1) The ratio of the value of the MA's bond holdings to GDP is 6% in SW. We keep that target for the foreign economy but change it to 3.34% for the home economy to match the Bank of Canada's government bond holdings in 2006. (2) The steady-state government expenditure to GDP ratio is 22.13% in the home country. For the foreign country, we keep it at 20% as in SW. (3) The steady-state government debt (as a percentage of GDP) is 40.54% for the home country and 41% for the foreign country. (4) In the home country, the credit-to-GDP ratio is 1.59, which gives $\psi = 75.82\%$. For the foreign country, as in SW, the credit-to-GDP ratio is 1.65, which gives $\psi = 81\%$. (5) We need to adjust parameters Ψ_1 and Ψ_2 to achieve specific QE targets when QE is endogenous (see discussion in SW). We set $\Psi_1 = \Psi_2 = -2$ for the home economy and $\Psi_1 = \Psi_2 = -56$ for the foreign economy. Sims and Wu had $\Psi_1 = \Psi_2 = -7$. (3) The target for L_1^{SS} is 1 for home country and 7.44 for foreign country.[12]

In addition to the 45 parameters that we list in the online-appendix Tables 1 and 2, the home economy has 10 SOE-specific parameters. There are five elasticity parameters: η_1, η_2, η_3, η_4 and η_5, and five home/foreign bias (or home/foreign share) parameters: ν_1, ν_2, ν_3, ν_4 and ν_5. We follow Gali and Monacelli (2016) and set all elasticity parameters equal to one. Later, we perform sensitivity analysis on these parameters. We calibrate the five home/foreign share parameters to match certain targets in the Canadian data. We list these parameters and their targets in Table 2.

Table 2: SOE-specific Parameters

Parameter	Description	Target
ν_1	Foreign bias in consumption	Import to GDP ratio = 33.72%
ν_2	Degree of openness	Export to GDP ratio = 36.28%
ν_3	Share of foreign bonds in wholesale firm's total bond issue	$\frac{\left(b_{H,FC}^{Pvt}\right)^{SS}}{\left(b_H^{Pvt}\right)^{SS}+\left(b_{H,FC}^{Pvt}\right)^{SS}} = 48.17\%$
ν_4	Share of foreign private bonds in FI's private bond holdings	Balance of payment equilibrium
ν_5	Share of foreign government bonds in FI's government bond holdings	$\frac{\left(b_F^{Gov}(fi)\right)^{SS}}{\left(b_F^{Gov}(fi)\right)^{SS}+\left(b_H^{Gov}(fi)\right)^{SS}} = 43.42\%$

Parameter ν_1 represents the steady-state share of imports in total consumption in the model. We pick ν_1 such that the steady-state import-to-GDP ratio in the model is 33.72%, which is the same as Canada's import-to-GDP ratio in 2006, the year before the 2007 crisis. Similarly, we pick ν_2 such that the steady-state export-to-GDP ratio is 36.28%, which is the same as Canada's export-to-GDP ratio in 2006. We pick ν_3 to match the share of foreign-currency bonds (48.17%) in the total outstanding bonds issued by Canadian non-financial corporations. We pick ν_5 to match the share of foreign government bonds in Canada's five largest banks' government bond holdings.[13] This share was 43.42% in 2006. We pick ν_4 such that the balance of payment condition is satisfied in the steady state.[14]

The solution to our model is based on 51 non-linear equilibrium equations for the foreign economy and 72 non-linear equilibrium equations for the home economy.[15] We list the 72 equations for the home SOE in Appendix B. The equations for the foreign economy are a 51-equation subset of the 72 equations. In our description of the solution and in the online appendix accompanying this paper, we focus on the 72-equation equilibrium system that represents the home SOE. In the online appendix, we derive the equilibrium conditions that are new to our paper and provide the full non-stochastic steady-state solution to the model. We solve the model by using a linear approximation around the non-stochastic steady state. In the non-stochastic steady state, the zero-lower-bound (ZLB) constraint does not bind. When the ZLB constraint binds, we follow Guerrieri and Iacoviello (2015) to solve a piecewise linear version of the model. We use Dynare [Adjemian et al. (2011)] to solve and simulate the model.[16]

IV. International Spillovers of Conventional versus New Monetary Policy

In this section, we use our model as a measurement device to quantify and compare the international spillovers of conventional monetary policy, quantitative easing (QE), forward guidance (FG) and negative interest rate policy (NIRP). All monetary policy shocks originate in the US and spillover to Canada through goods and asset markets. We use the same monetary policy shocks as SW did for Figure 1 in their paper. Their goal was to quantify the conventional and new monetary policy steps by the Fed that would generate a similar effect on the US GDP. In order to do so, they came up with the following monetary policy interventions for the US: (1) They hit the economy with a -1% shock to its annualized policy rate. This is the conventional monetary-policy stimulus. (2) For QE, they allow the central bank to increase its balance sheet by about 4% of GDP. (3) For FG, they shock the economy with a -2.2% change in the annualized policy rate. (4) For NIRP, they hit the economy with a -2.4% shock to its annualized policy rate. All monetary-policy shocks hit the economy in period 7. They generate a binding zero lower bound (ZLB) in the US economy with a sequence of liquidity shocks (shocks to θ_t) of 1.5 standard deviations for periods 1 to 6. They simulate their model twice: once with the liquidity shocks in periods 1 to 6 only and then with the liquidity shocks in periods 1 to 6 and the monetary-policy shock in period 7. They plot the differences between the impulse response functions (IRFs) generated by the two simulations in Figure 1 of their paper.

In our Figure 2 below, the US simulations are identical to those in Figure 1 in SW. To save on space, we refer the reader to SW for discussion on the US IRFs. What is new in this paper are the simulations for the Canadian economy. It is important to note that we do not hit the Canadian economy with any exogenous shock. The shocks to the US economy spillover to the Canadian economy through both real and financial channels. The Canadian monetary authority (The Bank of Canada, BoC), continues to follow the Taylor rule (see eqn. 63/72 in Appendix B) to conduct monetary policy.

An important goal of monetary policy is to affect real economic activity in the short run. In our SOE model, Canada's real GDP is the sum of home consumption, investment, government spending and exports. We plot Canada's real GDP and its four components in

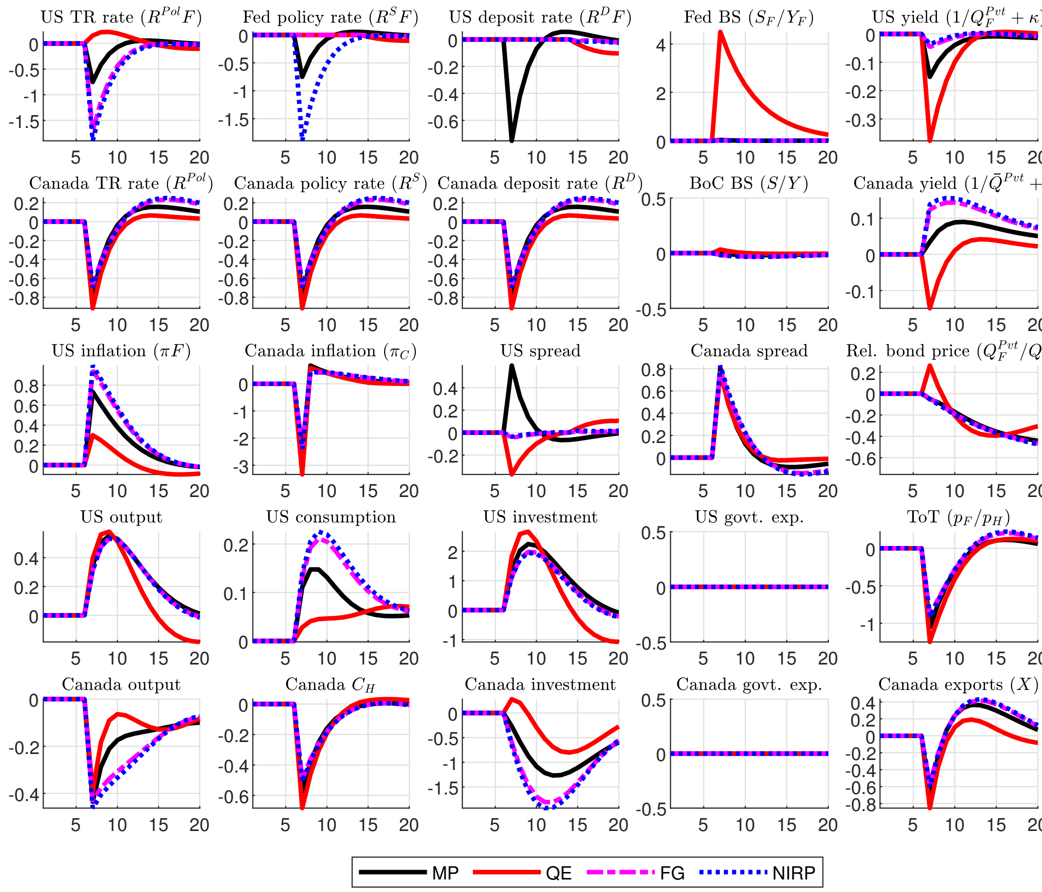


Figure 2: Exogenous monetary policy shocks to US economy only

row 5 of Figure 2. In the following discussion, we mostly focus on spillover effects of the US monetary policies on these real variables.

We start with the conventional monetary policy (solid black lines). When the US policy rate drops, the US output (Figure 2, panel [4,1]) increases. This has a positive effect on Canada's exports. However, Canada's terms of trade (panel [4,5]) worsen mainly due to an appreciation of Canadian currency. This has a negative effect on Canada's exports. Overall, the terms-of-trade effect dominates and Canada's exports decline (panel [5,5]). The net effect of monetary policy on government spending is almost zero (panel [5,4]). In our model, as in SW, monetary policy affects investment through the price of private bonds that the wholesale firm issues to finance investment due to the loan-in-advance constraint. The drop in the US policy rate increases the US private bond prices and decreases their yield to maturity (panel [1,5]). This reduction in the cost of borrowing encourages US investment (panel [4,3]). The spillover effects on Canada's investment are more nuanced. The increase in the US private bond price is more than offset by the appreciation of Canadian currency. So much so that the composite price index, in terms of home currency, of home and foreign private bonds $\left(\bar{Q}^{Pvt}\right)$ decreases. This implies an increase in the private bond yield (panel [2,5]). This increase in the cost of borrowing leads to a decrease in Canada's investment (panel [5,3]). Canada's consumption of home good (panel [5,2]) also declines due to the worsening terms of trade. The sum of these negative effects is an overall decrease in Canada's GDP (panel [5,1]). This is consistent with the findings in Rey (2016) and Blanchard et al. (2016), who also find a similar contractionary effect of foreign expansionary monetary policy on home GDP.

The spillover effect of the US QE (solid red lines) on the Canadian bond yield is different from that of the US conventional monetary policy. In the case of QE, the US private bond yield declines by so much (panel [1,5]) that the net effect on the Canadian private bond yield is negative (panel [2,5]). This reduction in the cost of borrowing has a small positive effect on Canada's investment (panel [5,3]), though it reverses within a year when the bond yield increases. The spillover effects of the US QE on Canada's exports and consumption of home good (panels [5,5] and [5,2]) are qualitatively very similar to those of conventional monetary policy, though quantitatively they are a bit more pronounced. The net negative effect on output (panel [5,1]) is also qualitatively similar but quantitatively milder in the case of QE. Also, after the initial drop, the Canadian GDP recovers faster in the case of QE.

FG (purple dashed lines) and NIRP (blue dotted lines) increase Canada's private bond yield (panel [2,5]) by even more than does the conventional monetary policy. As a result, the drop in Canada's investment (panel [5,3]) is bigger and the recovery in the GDP (panel [5,1]), after the initial fall, is slower.

To sum up, the key mechanisms in our model through which the US monetary policy shocks, both conventional and new, spillover to Canada's real economy are the terms of trade and private bond yield faced by the Canadian wholesale firm. The effect on terms of trade is similar across the four types of monetary policies that we consider (panel [4,5]). It is the effects on Canada's private bond yield that differ significantly by the type of policy tool used by the Fed (panel [2,5]). The conventional monetary policy, FG and NIRP all increase Canada's private bond yield. FG and NIRP increase it by more than does the conventional monetary policy. The QE, on the other hand, decreases Canada's private bond yield.

These different effects of various US monetary policy measures on Canada's private bond yield are the sum of two opposing effects: (1) the drop in the US private bond yields; and (2) the appreciation of the Canadian currency. The magnitude of the appreciation of Canadian currency is very similar across different US monetary policy measures. We do not plot the exchange rate IRFs separately in Figure 2 because they look almost identical to the IRFs of terms of trade (panel [4,5]). The drop in the US private bond yield is smallest in the case of FG and NIRP, intermediate in the case of conventional monetary policy and largest in the case of QE (panel [1,5]). The net effect on Canada's private bond yield of the drop in US private bond yield and the appreciation of Canadian currency (which lowers US bond prices in Canadian currency and hence increases the yield that the wholesale firm has to pay on foreign bonds) is positive in the cases of FG, NIRP and conventional monetary policy and negative in the case of QE. If we rank the IRFs of private bond yields in the US and Canada in response to various monetary policy measures (panels [1,5] and [2,5]), FG and NIRP are at the top, conventional monetary policy is in the middle and QE is at the bottom.

In other words, the net effect of various US monetary-policy measures on Canada's private bond yield is directly related to their effect on the US private bond yield. Because different monetary policy measures affect the US private bond yield differently, once we net out the effects of appreciation of Canadian currency, the Canadian private bond yield increases in the case of FG, NIRP and conventional monetary policy and decreases in the case of QE. If we ignore the exchange-rate effect, this finding is consistent with the main result in Gilchrist et al. (2019) "that yields on dollar-denominated sovereign debt are highly responsive to unanticipated changes in the stance of US monetary policy during both the conventional and unconventional policy regimes."

A key assumption behind the simulations in Figure 2 is that there is no monetary policy shock directly hitting the Canadian economy. However, highly integrated economies like those of the US and Canada are often hit by similar exogenous shocks. To explore the implications of this scenario, we modify the experiments in Figure 2 and allow, in addition to the same monetary policy shocks to the US economy, a -1% shock to Canada's policy rate. The results of this experiment are in the online-appendix Figure 1. The main difference between the two sets of results is that when Canada also engages in expansionary monetary policy synchronously to the US, Canada's currency depreciates. This depreciation reverses the direction of spillovers to Canada's GDP and its components. Canada's net exports increase because of an improvement in its terms of trade. Canada's consumption of home good also increases for the same reason. Canada's investment increases because the two effects (the drop in the US private bond yields and the appreciation of Canadian currency) that worked opposite to each other in the experiments in Figure 2 above, now work in the same direction to lower Canada's private bond yield.

In general, the differences between the results of our two sets of experiments are in line with the conventional wisdom. For example, Blanchard et al. (2016, p. 565) write: "Standard models, along Mundell-Fleming or more recent incarnations, predict that, for a given monetary policy interest rate, capital inflows lead to an appreciation of the currency, and thus to a contraction in net exports and in output. Only with a decrease in the policy rate can capital flows be expansionary." Our contribution is to show that within these general trends, the spillover effects of foreign monetary policy on home country differ quantitatively (and in some cases, even qualitatively) by the type of monetary policy. The QE in the foreign country has the most favorable real effects on home economy whereas FG and NIRP have the least favorable effects.

To isolate the spillover effects operating through trade in goods and services from those due to trade in assets, we run another set of experiments in which we do not allow the Canadian wholesale firm and financial intermediaries to trade US bonds but keep the trade in goods and services open (see the online-appendix Figure 2 for results). In this case, the effects of US monetary policy on Canada's terms of trade and net exports are smaller compared to the effects in Figure 2 above. Also, because there is no trade in assets, Canada's private bond yield moves in the same direction as the Bank of Canada policy rate and Canada's investment increases.

V. Counterfactual Experiments: Global Financial Crisis 2008

In the previous section, we used the same set of hypothetical monetary policy and leverage shocks as did SW to quantify and compare the international spillovers of various monetary policy tools. In this section, first we calibrate the monetary policy and leverage shocks to match certain features of the US and Canadian economies around the 2008 financial crisis. We then run a number of counterfactual monetary policy experiments to explore their effects on spillovers to the Canadian economy.

Before the 2008 financial crisis, the size of the Fed's balance sheet was around 6% of the US GDP. After the third round of QE in 2014, the size of the Fed's balance sheet had increased to 25% of the GDP. Had the Fed engaged in a more or less aggressive QE, how would the spillover effects to the Canadian economy have changed? The Bank of Canada, on the other hand, did not engage in any QE around the 2008 crisis. Had the Bank of Canada also engaged in QE, how would the outcomes for the Canadian economy have been different? The Fed never tried a negative interest rate policy (NIRP). Had the Fed let its policy rate go negative, how would the effects of this policy have spilled over to Canada? In this section, we run some counterfactual experiments to answer these questions. We do so in four steps. First, we construct a benchmark scenario in which we calibrate credit shocks, i.e. shocks to θ, in such a way that, when combined with the actual QE policies of the Fed and the Bank of Canada in terms of the sizes of their balance sheets, they produce drops in output and investment that are similar to what happened in the US and Canada from 2008 onwards. Second, we counterfactually change the magnitude of QE done by the Fed to see how it would have changed the outcomes for the Canadian economy. Third, we run a counterfactual experiment in which we allow the Bank of Canada to engage in QE in the wake of the 2008 crisis. Fourth, we conduct a few counterfactual experiments in which the Fed lets its policy rate go negative, with or without doing any QE. For ease of exposition, we divide the four steps into four separate subsections, namely A, B, C and D.

A. Benchmark

There were steep drops in output and investment in both the US and Canada between 2008 and 2010. We use these drops as our targets and calibrate the exogenous liquidity shocks (shocks to θ_t) to the two economies accordingly.[17] In Figure 3, we plot actual versus simulated time series for selected variables. A few comments are in order.

The recovery in output and investment for the US is much slower in the model (Figure 3, panels [1,1] and [1,3]). The model does a much better job of matching the recovery in output and investment for Canada (panels [2,1] and [2,3]). The time that the US economy spends at the ZLB is shorter in the model (panel [3,1]). The model fails to replicate the initial drop in US consumption but matches the subsequent drop (from 2012 onwards) better (panel [3,2]).

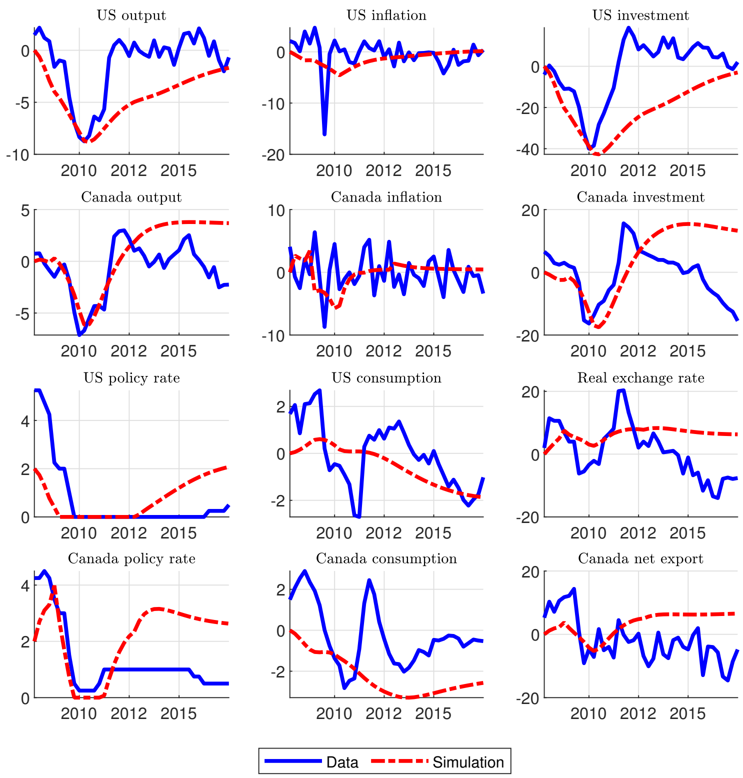
The model qualitatively captures the initial contours of the real exchange rate (panel [3,3]) and net exports (panel [4,3]), though quantitatively the model fluctuations are much smaller. One reason for this poor fit is that the 2008 crisis caused permanent shocks to Canada's real exchange rate and net exports. Canada used to have current-account surpluses with the US before the crisis but started to have regular current-account deficits after the crisis. Our model cannot capture the effects of the permanent shocks because all variables must eventually return to their starting steady-state values.

The primary purpose of Figure 3 is not to match the data, which would be difficult because we introduce only one shock to both economies. Instead, the purpose of the figure is to give the reader a perspective on our benchmark simulation and set the stage for the subsequent counterfactual experiments.

B. Counterfactual Changes in QE by the Fed

In the aftermath of the 2008 crisis, the Fed increased its balance sheet from 6% to 25% of the US GDP. In this set of counterfactual experiments, we change the magnitude of the Fed's QE to see how the spillovers to the Canadian economy change.

In Figure 4, we compare the benchmark (a 19 percentage-point increase in the Fed's balance sheet from 6% to 25% of the US GDP) with three counterfactual scenarios. In



Notes: (1) The data source for the US is FRED, Federal Reserve Bank, St. Loius. The data sources for Canada are the Bank of Canada and Statistics Canada. (2) We use the filter proposed by Hamilton (2018) to detrend output, investment and consumption series for both US and Canada. We use 8 lags in estimating the trend.

Figure 3: Benchmark, 2008 crisis and its aftermath (model vs. data)

Counterfactual 1, there is no change is the Fed's balance sheet and it stays at 6% of the GDP. In Counterfactual 2, there is a 10 percentage-point increase in the Fed's balance sheet from 6% to 16% of the GDP. In Counterfactual 3, there is a 30 percentage-point increase in the Fed's balance sheet from 6% to 36% of the GDP.

When the size of QE is smaller relative to the benchmark (Counterfactuals 1 and 2), the US private bond yield (Figure 4, panel [1,5]) increases by more, and the US output (panel [4,1]) and investment (panel [4,3]) decrease by more. Although, Canada's terms of trade (panel [4,5]) improve by more, the larger drop in the US output (panel [4,1]) leads to an overall drop in Canada's exports (panel [5,5]). The larger increases in the US yield (panel [1,5]), result in smaller increases in Canada's yield (panel [2,5]). These result in smaller drops

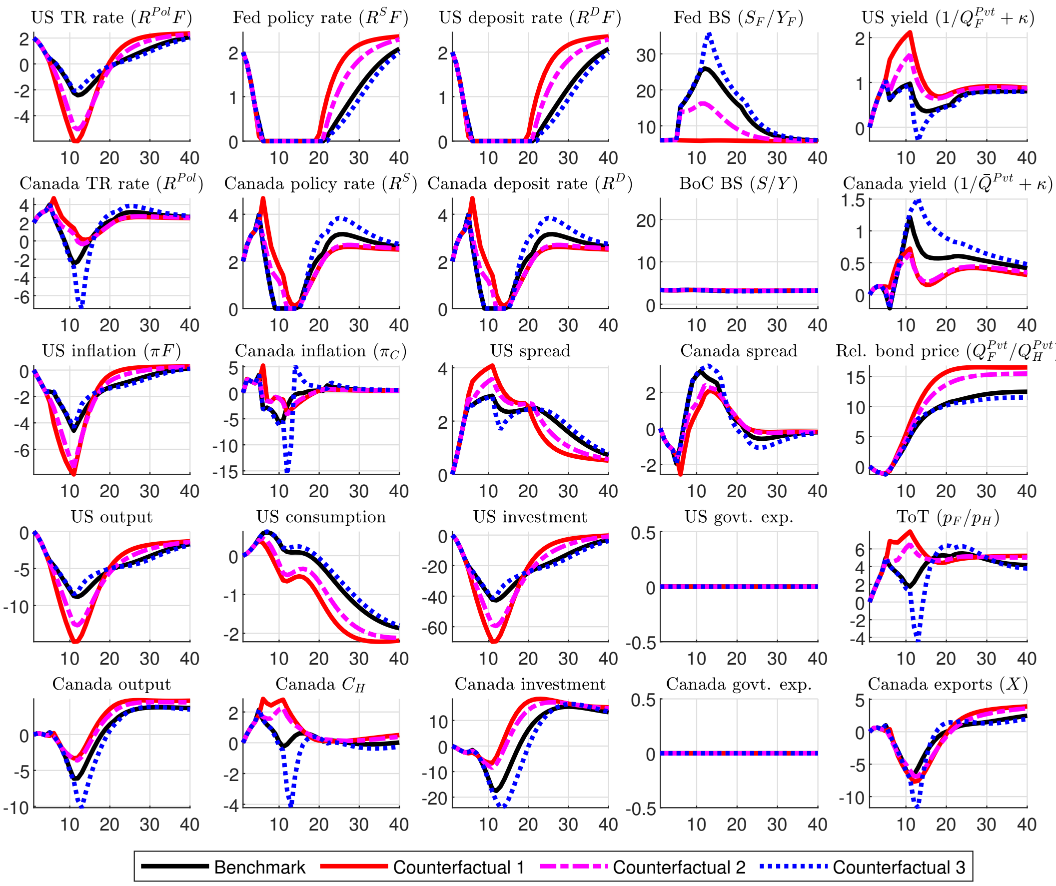


Figure 4: Benchmark vs. Counterfactuals 1, 2 and 3

in Canada's investment (panel [5,3]) and output (panel [5,1]). The main message of Figure 4 is that more QE is better for the US economy because it mitigates the negative effects of liquidity shocks on the US output (panel [4,1]) and investment (panel [4,3]). However, more QE in the US increases Canada's private bond yield (panel [2,5]) by more and hence causes steeper drops in Canada's output (panel [5,1]) and investment (panel [4,3]).

The effects on the Canadian economy in these experiments are driven entirely by the spillovers from the US. There is no difference in shocks to the Canadian economy between the benchmark and the three counterfactual scenarios in Figure 4. Similarly, there is no change in the Bank of Canada's monetary policy beyond the endogenous response of the Taylor-rule policy rate.

These experiments also show that it is possible that a change in policy in a large country may have bigger effects on other countries, though it may not affect the large country itself by much. To see this, compare the benchmark with Counterfactual 3 in Figure 4. The effects on the US output (panel [4,1]) and investment (panel [4,3]) are quite similar for these two experiments. However, the drops in Canada's output (panel [5,1]) and investment (panel [5,3]) are greater in the case of Counterfactual 3 compared to the benchmark. In other words, a larger than the benchmark increase in QE in the US has very little additional effect on the real variables in the US but it has discernibly larger effects on real variables in Canada. The primary reason for these differences is the change in Canada's terms of trade (panel [4,5]), which affects both exports (panel [5,5]) and consumption of home goods (panel [5,2]). This is an example of unintended consequences of monetary-policy changes in a large country like the US on smaller countries that have close trade and financial ties to it.

Another interesting pattern that emerges from these simulations is that the relationship between the size of QE and its effect on bond yields is not linear. For example, in panel [2,5] the changes in Canadian yield are similar for Counterfactuals 1 and 2 but much larger for the benchmark. That means the increase in the Fed's balance sheet from 6% to 16% has a much smaller effect than the increase from 6% to 25%.

C. Counterfactual QE by the Bank of Canada

When the Fed implemented its QE policies between 2008 and 2014, the Bank of Canada did not follow. It did slash its policy rate from more than 4% to almost zero within a few quarters (see panel [4,1] of Figure 3) but it did not engage in QE and kept its balance sheet small. In our next counterfactual experiment, we allow the Bank of Canada to engage in QE. The results of this experiment are in the online-appendix Figure 3.

The first thing to note in the online-appendix Figure 3 is that all the US variables remain unaffected when the Bank of Canada engages in QE. This is because of our assumption that Canada's is a small open economy and has no effect on the US economy.

The effects of this counterfactual experiment on the Canadian economy differ predictably from the benchmark. The Canadian bond yield does not initially increase by as much as in the benchmark scenario. And, once the QE in Canada really picks up (panel [2,4]), the Canadian bond yield drops below zero (panel [2,5]). The ultimate effects on real variables are generally positive. Output (panel [5,1]), investment (panel [5,3]) and net exports (panel

[5,5]) drop by much less and recover faster. The drop is consumption (panel [5,2]) is milder and slower.

This counterfactual experiment suggests that had the Bank of Canada followed the Fed and engaged in QE, the real economic outcomes would have been better for Canada. There are a couple of obvious reasons why the Bank of Canada decided against the QE. First, the US experiment with QE was new and it was not clear how it would pan out. As Bernanke (2020) discusses in detail, there was a lot of skepticism even among the Fed officials about the effectiveness of QE. Second, the Canadian financial sector was in a much better shape than its US counterpart, so the Bank of Canada felt confident that it could get the Canadian economy back on track without doing any QE. However, had the magnitude of negative shocks hitting the Canadian economy in 2008 been bigger, the Bank might have taken the QE route. For example, in the wake of the current new coronavirus pandemic, the Bank of Canada did not hesitate to top up its near zero policy rate with a healthy doze of QE.

D. Counterfactual Negative Interest Rate Policy by the Fed

The negative interest rate policy (NIRP) is perhaps the newest of new monetary policy tools. A number of central banks have tried NIRP since 2012 (See Figure 1 in Ulate (2021)) but the Fed and the BoC have not tried it so far. We have already compared the international monetary policy spillovers due to NIRP with those due to QE, FG and the conventional monetary policy (see Figure 2 above). Now we conduct another two sets of counterfactual experiments.

In the first set of experiments, the Fed allows its policy rate to go negative but only up to a certain point. We try the following three lower limits: -0.5%, -1% and -2%. This NIRP is in addition to the QE that the Fed does in the benchmark scenario. In this sense, this experiment is different from the NIRP experiment that we report in Figure 2 above. To save on space, we report the results of this experiment in the online-appendix Figure 4. In the presence of QE, the NIRP does not have any dicernible effect on the US economy. The spillover effects on the Canadian economy are also negligible when the lower limits are -0.5% and -1%. When the lower limit is -2%, we see some additional spillover effects to Canadian variables. The Canadian currency appreciates a bit more, Canada's terms of trade (panel [4,5]) decreases a bit more and Canada's exports (panel [5,5]) also decline a bit more. Canadian bond yield (panel [2,5]) increases slightly more and the drop in Canada's investment (panel [5,3]) is slightly deeper. Because of the currency appreciation, Canadians shift consumption to the foreign good and the consumption of home good declines more (panel [5,2]). The overall effect on Canada's GDP is that instead of falling by around 6% as in the benchmark, it drops by around 7% when the Fed lets its policy rate go to -2%.

In the experiments reported in the last paragraph, the negative policy rate is bounded below. In the next counterfactual experiment, we do not impose any lower limit on the negative policy rate and also do not let the Fed do any QE when it is engaging in NIRP.[18] Once again, to save on space, we report the results of this experiment in the online-appendix Figure 5. Panels [1,2] and [1,4] show the policy rate and the size of the Fed's balance sheet under the benchmark and the counterfactual NIRP. The increase in the US yield is higher (panel [1,5]) and the US output (panel [4,1]), consumption (panel [4,2]) and investment (panel [4,3]) drop by more. So, in the face of negative liquidity shocks, the NIRP is unable to provide the same level of support to the US economy that the QE does in the benchmark scenario. The NIRP proves better for the Canadian economy. The Canadian bond yield (panel [2,5]) increases by less, so the drop in investment is smaller (panel [5,3]). Because foreign price increases by more (panel [4,5]), Canadians consume more of the home good (panel [5,2]). Despite the bigger increase in foreign price (panel [4,5]), Canada's exports decrease by a little more (panel [5,5]) because of the much bigger drop in foreign GDP (panel [4,1]). The overall effect on Canadian GDP is that it drops by just around 3% compared to a 6% drop in the benchmark scenario.

The first broad message of the counterfactuals in this sub-section is that NIRP does not have any noticeable spillovers when it is added to the QE of the benchmark magnitude. The second broad message is that, if NIRP is followed without a limit, it would produce worse outcomes for the US and better outcomes for Canada.

VI. Concluding Remarks

Our methodological contribution is that we take a state-of-the-art dynamic New Keynesian model that combines both conventional and new monetary policy tools in a unified framework, and modify it to an open-economy setting. This modification allows us to compare, not only the international spillovers of conventional and new monetary policies as others have done before but also, the international spillovers caused by the different tools of new monetary policy, something that is new to the literature.

Our empirical contribution is to quantify the nature and size of monetary policy spillovers from a large economy (the US) to an SOE (Canada) that has close trade and financial ties to the large economy.

The open-economy framework that we have developed in this paper is rich enough that it can be used in a number of other applications. For example, instead of thinking about an SOE and its interactions with a large economy, the framework can be modified to think about the monetary-policy interactions between two large economies like the US and the European Union. The importance of the study of such interactions has further increased in recent years when both the Fed and the European Central Bank have significantly expanded their use of new monetary policy tools.

We have assumed producer currency pricing in this paper. Future work could use this model to explore the new monetary policy spillovers under the dominant-currency pricing paradigm [Gopinath et al. (2020)], which is empirically more relevant for some SOE's.

The framework in this paper also provides a rich environment to quantitatively assess Rey's hypothesis [Rey (2016)] about the lack of monetary policy independence in small open economies.

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Notes

[1]It is a reasonable assumption given that in our empirical application the US is the large foreign economy and Canada is the SOE. The kind of reverse spillovers that Obstfeld (2019) talks about are more relevant when we think about the US versus the rest of the world.

[2]We discuss the justifications for these additions in Section II.

[3]We report the maximum positive or negative effect here. See Section IV for detailed impulse responses.

[4]Following Bernanke (2020), we use the label 'new monetary policy' to refer to the monetary policy tools that are often called 'unconventional monetary policy' in the literature.

[5]The US GDP in 2006 was 10.3 times larger than Canada's. Canada's exports to and imports from the US as fractions of Canada's GDP were 21.4% and 15.7%. So the economic events in the US are likely to have significant effects on Canada's economy. On the other hand, the US exports to and imports from Canada as fractions of US GDP were just 1.5% and 2.0%. So the economic events in Canada are likely to have negligible effects on the US economy. [The data sources for the numbers reported in this note are FRED St. Louis and Statistics Canada.]

[6]We have changed the utility function from log to constant relative risk aversion and added a shock to utility. We have also changed some of their notation to accomodate the addition of open-economy variables.

[7]We provide a brief description of the model in the main text and collect all equilibrium conditions in the appendix. In total, there are 72 equilibrium conditions for the SOE. We number them as eqn. 01/72, eqn. 02/72, ... , eqn. 72/72.

[8]On 1st April 2007, the value of the outstanding bonds issued by Canadian non-financial corporations in Canadian dollars was $124.5 billion (Statistics Canada, CANSIM V31185504). The value (in Canadian dollars) of the bonds issued in other currencies was $115.7 billion (V31185537), out of which $110.5 billion (95%) were in US dollars (V31185570).

[9]The export-to-GDP ratio in Canada was 36.28% in 2006. [Source: Statistics Canada (2007). "National Income and Expenditure Accounts: Quarterly Estimates, First Quarter 2007." Catalogue No. 13-001_XIB, Table 2, p. 24.]

[10]All private and government bonds in this model are perpetuities that cost Q and pay one unit of the currency, in which they are issued, the next period. This coupon payment declines at a depreciation rate of κ. So paying Q_t dollars for a bond in period t entitles the holder to receive one dollar in period $t+1$, κ dollars in period $t+2$, κ^2 dollars in period $t+3$ and so on. The gross yield to maturity of this bond is $1/Q+\kappa$. The duration of the bond is $1/(1-\kappa)$.

[11]We do not make additional assumptions to induce stationarity. Schmitt-Grohe and Uribe (2003) show that an SOE model without stationarity-inducing features produces dynamics at business-cycle frequencies that are almost identical to those generated by the models that add an assumption to induce stationarity.

[12]At the end of 2006, the number of employed people in the US was 119.1 million and in Canada it was 16.0 million. The ratio of the two is 7.44. Given the SOE structure of our model, we could normalize $L_1^{SS}=1$ in the foreign country as well and recalibrate parameter ω accordingly.

[13]The five banks are: (1) Royal Bank of Canada; (2) Bank of Montreal Financial Group; (3) Toronto Dominion Bank Group; (4) Scotiabank; and (5) Canadian Imperial Bank of Commerce.

[14]Just like we do for the five elasticity parameters, we perform sensitivity analysis on the five share parameters. To save on space, we have moved the entire sensitivity section to the online appendix.

[15]Our changes to the Sims-and-Wu economy add one more equation for the evolution of preference shock to SW's system of 50 equations.

[16]The Dynare codes to replicate our results are available upon request.

[17]We introduce the following exogenous shocks to θ_t. A 1.5 standard deviation negative shock hits the US economy each period from periods 2 to 6. A one standard deviation negative shock hits both the US and Canadian economies from periods 7 to 11.

[18]If we let the Fed engage in QE, the results are very similiar, but more pronounced, to those of the experiments in the previous paragraph.

A Notation

In this section, we collect in one place all variables and parameters used in the paper. We follow the following order in both lists: the uppercase English symbols appear first followed by the lowercase English, uppercase Greek and lowercase Greek symbols. Within each category, the variables and parameters are listed in lexicographical order.

A. Variables

The consumer price index, $P_{C,t}$, which is a CES aggregate of home price, $P_{H,t}$, and foreign price, $P_{F,t}$, is the numeraire for the home economy so we do not list it as a separate variable. The abbreviation USD before a variable implies that the variable is in terms of foreign currency.

1. A_t : Technology/Total factor productivity
2. C_t : Total consumption, CES aggregate of $C_{H,t}$ and $C_{F,t}$
3. $C_{F,t}$: Consumption of foreign-produced goods (imports)
4. $C_{H,t}$: Consumption of home-produced goods
5. E_t : Nominal exchange rate, units of home currency needed to buy one unit of foreign currency
6. G_t : Government expenditure
7. I_t : Investment before adjustment cost
8. $\hat{I}_t$: Investment net of adjustment cost, gross addition to capital stock
9. K_t : Capital stock
10. $L_{1,t}$: Aggregate employment (labor supplied by the household to labor unions)
11. $L_{2,t}$: Final labor bundle sold by the labor packer to the wholesale firm
12. $M_{1,t}$: An auxiliary variable, a function of the Lagrange multiplier in the wholesale firm's optimization problem
13. $M_{2,t}$: An auxiliary variable, a function of the Lagrange multiplier, in the wholesale firm's optimization problem
14. $\mathrm{MU}_{C,t}$: Marginal utility of consumption
15. Q_t^{Gov} : CES aggregate of $Q_{F,t}^{Gov}$ and $Q_{H,t}^{Gov}$
16. $Q_{F,t}^{Gov} \equiv E_t \mathrm{USD} Q_{F,t}^{Gov}$: Price of a foreign government bond in home currency
17. $Q_{H,t}^{Gov}$: Price of a home government bond
18. Q_t^{Pvt} : CES aggregate (based on parameters η_4 and ν_4) of $Q_{H,t}^{Pvt}$ (the price of home private bond) and $Q_{F,t}^{Pvt}$ (the price of foreign private bond in home currency)
19. $Q_{F,t}^{Pvt} \equiv E_t \mathrm{USD} Q_{F,t}^{Pvt}$: Price of a foreign private bond in home currency
20. $Q_{H,t}^{Pvt}$: Price of a home private bond
21. $\bar{Q}_{H,t}^{Pvt}$: CES aggregate (based on parameters η_3 and ν_3) of $Q_{H,t}^{Pvt}$ (the price of home private bond) and $Q_{F,t}^{Pvt}$ (the price of foreign private bond in home currency)
22. R_t^D : Gross nominal interest rate on deposits
23. R_t^{Gov} : CES aggregate of $R_{H,t}^{Gov}$ (the gross nominal return on home government bonds) and $R_{F,t}^{Gov}$ (the gross nominal return on foreign government bonds)

24. $R_{F,t}^{Gov} \equiv E_t \text{USD} R_{F,t}^{Gov} / E_{t-1}$: Gross nominal return (in home currency) on a foreign government bond
25. $R_{H,t}^{Gov}$: Gross nominal return on a home government bond
26. R_t^{Pol} : Policy rate implied by the Taylor rule
27. R_t^{Pvt} : CES aggregate of $R_{H,t}^{Pvt}$ (the gross nominal return on home private bonds) and $R_{F,t}^{Pvt}$ (the gross nominal return, in home currency, on foreign private bonds)
28. $R_{F,t}^{Pvt} \equiv E_t \text{USD} R_{F,t}^{Pvt} / E_{t-1}$: Gross nominal return (in home currency) on a foreign private bond
29. $R_{H,t}^{Pvt}$: Gross nominal return on a home private bond
30. R_t^S : Gross interest rate on reserves
31. T_t : Lump sum tax paid by the household to the fiscal authority
32. X_t : Exports
33. Y_t : Final-good output
34. $Y_{2,t}$: Output of the wholesale firm
35. Z_t : Preference shock
36. $b_t^{Gov}(fi)$: CES aggregate of $b_{H,t}^{Gov}(fi)$ (home government bonds) and $b_{F,t}^{Gov}(fi)$ (foreign government bonds) held by the financial intermediaries
37. $b_{F,t}^{Gov}(fi) \equiv B_{F,t}^{Gov}(fi)/P_{C,t}$: Real holdings of $B_{F,t}^{Gov} \equiv \int B_{F,t}^{Gov}(j)\, dj$ (foreign government bonds) by the financial intermediaries
38. $b_{H,t}^{Gov}(fi) \equiv B_{H,t}^{Gov}(fi)/P_{C,t}$: Real holdings of $B_{H,t}^{Gov}(fi) \equiv \int B_{H,t}^{Gov}(j)\, dj$ (home government bonds) by the financial intermediaries
39. $b_{H,t}^{Gov}(ma)$: Monetary authority's real holdings of home government bonds
40. $b_t^{Pvt}(fi)$: CES aggregate of $b_{H,t}^{Pvt}(fi)$ (home government bonds) and $b_{F,t}^{Pvt}(fi)$ (foreign government bonds) held by the financial intermediaries
41. $b_{F,t}^{Pvt}(fi)$: Real holdings of $B_{F,t}^{Pvt}(fi) \equiv \int B_{F,t}^{Pvt}(j)\, dj$ (foreign private bonds) by the financial intermediaries
42. $b_{H,t}^{Pvt} = b_{H,t}^{Pvt}(fi) + b_{H,t}^{Pvt}(ma)$: Real outstanding private bonds issued by the wholesale firm in home currency
43. $b_{H,t}^{Pvt}(fi)$: Real holdings of $B_{H,t}^{Pvt}(fi) \equiv \int B_{H,t}^{Pvt}(j)\, dj$ (home private bonds) by the financial intermediaries
44. $b_{H,t}^{Pvt}(ma)$: Monetary authority's real holdings of home private bonds
45. $b_{H,FC,t}^{Pvt} \equiv B_{H,FC,t}^{Gov}/P_{C,t}$: Real outstanding bonds issued by the home wholesale firms in foreign currency
46. $\bar{b}_{H,t}^{Pvt}$: CES aggregate of $b_{H,t}^{Pvt}$ (real outstanding bonds issued by the wholesale firm in home currency) and $b_{H,FC,t}^{Pvt}$ (real outstanding bonds issued by the wholesale firm in foreign currency)
47. $d_t \equiv D_t/P_{C,t}$: Real household deposits at financial intermediaries
48. $f_{1,t}$: An auxiliary variable in the optimal real wage $(w_{2,t}^*)$ equation
49. $f_{2,t}$: An auxiliary variable in the optimal real wage $(w_{2,t}^*)$ equation
50. $n_t \equiv N_t/P_{C,t}$: Real networth of a financial intermediary
51. $p_{2,t} \equiv P_{2,t}/P_{C,t}$: Relative price of wholesale firm's output

52. $p_{F,t} \equiv P_{F,t}/P_{C,t} = E_t \text{USD}P_{F,t}/P_{C,t}$: Real exchange rate
53. $p_{H,t} \equiv P_{H,t}/P_{C,t}$: Relative home price
54. $p^*_{H,t} \equiv P^*_{H,t}/P_{C,t}$: Optimal relative home price chosen by a retail firm
55. $p_{K,t} \equiv P_{K,t}/P_{C,t}$: Relative price of capital
56. $s_t \equiv S_t/P_{C,t}$: Real monetary authority reserves (held by financial intermediaries)
57. u_t : Capital utilization
58. $w_{1,t} \equiv W_{1,t}/P_{C,t}$: The real wage that the household receives from labor unions
59. $w_{2,t} \equiv W_{2,t}/P_{C,t}$: The real wage that the wholesale firm pays to the labor packer
60. $w^*_{2,t} \equiv W^*_{2,t}/P_{C,t}$: Optimal real wage chosen by a trade union
61. $x_{1,t}$: An auxiliary variable in the optimal relative home price ($p^*_{H,t}$) equation
62. $x_{2,t}$: An auxiliary variable in the optimal relative home price ($p^*_{H,t}$) equation
63. $\Lambda_{t,t+1} = \beta \text{MU}_{C,t+1}/\text{MU}_{C,t}$: Stochastic discount factor
64. $\Pi^{\text{real}}_{ma,t}$: Profit of the monetary authority
65. Ω_t : An auxiliary variable in the first-order conditions of a financial intermediary
66. θ_t : Liquidity shock
67. $\lambda_{1,t}$: The Lagrange multiplier on the costly enforcement constraint in the problem of a financial intermediary
68. $\lambda_{2,t}$: The Lagrange multiplier on the reserve requirement constraint in the problem of a financial intermediary
69. $\pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$: CPI inflation
70. v^p_t : Price dispersion
71. v^w_t : Wage dispersion
72. ϕ_t : An auxiliary variable in the first-order conditions of a financial intermediary (endogenous leverage)

B. Shocks

There are seven shocks in the model: (1) $\varepsilon_{1,t}$ in eqn. 66/72 for the monetary authority's government bond holdings; (2) $\varepsilon_{2,t}$ in eqn. 67/72 for the monetary authority's private bond holdings; (3) $\varepsilon_{A,t}$ in eqn. 17/72 for A_t; (4) $\varepsilon_{G,t}$ in eqn. 69/72 for G_t; (5) $\varepsilon_{R,t}$ in eqn. 63/72 for Taylor-rule policy rate R^{Pol}_t; (6) $\varepsilon_{Z,t}$ in eqn. 02/72 for Z_t; and (7) $\varepsilon_{\theta,t}$ in eqn. 46/72 for θ_t.

C. Foreign Variables

These are the foreign variables that appear in the home-economy equations. The abbreviation 'USD' before a variable means that the variable is in terms of foreign currency.

1. $\text{USD}Q^{Gov}_{F,t}$: Price of a foreign government bond
2. $\text{USD}Q^{Pvt}_{F,t}$: Price of a foreign private bond
3. $\text{USD}R^{Gov}_{F,t} = \left(\text{USD}1{+}\kappa\text{USD}Q^{Gov}_{F,t}\right)/\text{USD}Q^{Gov}_{F,t-1}$: Gross nominal return on a foreign government bond
4. $\text{USD}R^{Pvt}_{F,t} = \left(\text{USD}1{+}\kappa\text{USD}Q^{Pvt}_{F,t}\right)/\text{USD}Q^{Pvt}_{F,t-1}$: Gross nominal return on a foreign private bond
5. $Y_{F,t}$: Foreign real income
6. $\text{USD}\pi_{F,t} \equiv \text{USD}P_{F,t}/\text{USD}P_{F,t-1}$: Inflation in foreign country in foreign currency ($\text{USD}P_{F,t}$ is the numeraire for foreign economy)

B Non-Linear Equilibrium Conditions

In our model of the small open economy, there are 72 non-linear equilibrium equations in 72 endogenous variables. In this section, we list these equations. We have already listed the 72 endogenous variables in Appendix A.A. We have divided these equations into 7 blocks: (A) Household; (B) Labor market; (C) Non-financial firms; (D) Financial intermediaries; (E) Monetary authority; (F) Fiscal authority; and (G) Foreign exchange market. We have assigned each equation a unique serial number for easy reference. The numbers range from eqn. 01/72 (i.e. equation 1 of 72) to eqn. 72/72 (i.e. equation 72 of 72).

A. Household

(eqn. 01/72)
$$\mathrm{MU}_{C,t} \equiv Z_t[C_t - hC_{t-1}]^{-\sigma} - \beta h \mathbb{E}_t Z_{t+1}[C_{t+1} - hC_t]^{-\sigma}$$

(eqn. 02/72)
$$\ln Z_t = \rho_Z \ln Z_{t-1} + s_Z \varepsilon_{Z,t}$$

(eqn. 03/72)
$$\Lambda_{t,t+1} \equiv \frac{\beta \mathbb{E}_t \mathrm{MU}_{C,t+1}}{\mathrm{MU}_{C,t}}$$

(eqn. 04/72)
$$\omega Z_t L_{1,t}^{\varphi} = \mathrm{MU}_{C,t} w_{1,t}$$

(eqn. 05/72)
$$\mathbb{E}_t \Lambda_{t,t+1} \pi_{C,t+1}^{-1} R_t^D = 1$$

(eqn. 06/72)
$$Y_t = C_{H,t} + I_t + G_t + X_t$$

(eqn. 07/72)
$$C_{H,t} = (1-\nu_1)\, p_{H,t}^{-\eta_1} C_t,$$

where

$$\begin{array}{rcll} C_t &=& \left((1-\nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1-1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} (C_{F,t})^{\frac{\eta_1-1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1-1}} & \text{if } \eta_1 \neq 1 \\ C_t &=& \frac{C_{H,t}^{1-\nu_1} C_{F,t}^{\nu_1}}{(1-\nu_1)^{1-\nu_1}\nu_1^{\nu_1}} & \text{if } \eta_1 = 1 \end{array}$$

(eqn. 08/72)
$$C_{F,t} = \nu_1 p_{F,t}^{-\eta_1} C_t$$

(eqn. 09/72)
$$\begin{array}{rcll} 1 &=& (1-\nu_1)\, p_{H,t}^{1-\eta_1} + \nu_1 p_{F,t}^{1-\eta_1} & \text{if } \eta_1 \neq 1 \\ 1 &=& p_{H,t}^{1-\nu_1} p_{F,t}^{\nu_1} & \text{if } \eta_1 = 1 \end{array}$$

B. Labor Market

Labor Unions.—

(eqn. 10/72)
$$w_{2,t}^* = \frac{\epsilon_w}{\epsilon_w - 1} \frac{f_{1,t}}{f_{2,t}}$$

(eqn. 11/72)
$$f_{1,t} = w_{1,t} w_{2,t}^{\epsilon_w} L_{2,t} + \theta_w \pi_{C,t}^{-\epsilon_w \gamma_w} \mathbb{E}_t \Lambda_{t,t+1} f_{1,t+1} \pi_{C,t+1}^{\epsilon_w}$$

(eqn. 12/72)
$$f_{2,t} = w_{2,t}^{\epsilon_w} L_{2,t} + \theta_w \pi_{C,t}^{(1-\epsilon_w)\gamma_w} \mathbb{E}_t \Lambda_{t,t+1} f_{2,t+1} \pi_{C,t+1}^{\epsilon_w - 1}$$

Labor Packer.—

(eqn. 13/72)
$$L_{1,t} = L_{2,t} v_t^w$$

(eqn. 14/72) $$\upsilon_t^w = (1-\theta_w)\left(\frac{w_{2,t}^*}{w_{2,t}}\right)^{-\epsilon_w} + \theta_w\left(\frac{\pi_{C,t}}{\pi_{C,t-1}^{\gamma_w}}\right)^{\epsilon_w}\left(\frac{w_{2,t}}{w_{2,t-1}}\right)^{\epsilon_w}\upsilon_{t-1}^w$$

(eqn. 15/72) $$w_{2,t}^{1-\epsilon_w} = \left(\frac{\pi_{C,t-1}^{\gamma_w}}{\pi_{C,t}}\right)^{1-\epsilon_w}\theta_w w_{2,t-1}^{1-\epsilon_w} + (1-\theta_w)\left(w_{2,t}^*\right)^{1-\epsilon_w}$$

C. Non-Financial Firms

Wholesale.—

(eqn. 16/72) $$Y_{2,t} = A_t(u_t K_t)^{\alpha} L_{2,t}^{1-\alpha}$$

(eqn. 17/72) $$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t}$$

(eqn. 18/72) $$K_{t+1} = \hat{I}_t + (1-\delta(u_t))K_t,$$

where $\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t-1)^2$

(eqn. 19/72) $$\psi p_{K,t}\hat{I}_t = \bar{Q}_{H,t}^{Pvt}(\bar{b}_{H,t}^{Pvt} - \kappa \bar{b}_{H,t-1}^{Pvt}\pi_{C,t}^{-1})$$

(eqn. 20/72) $$b_{H,t}^{Pvt} = b_{H,t}^{Pvt}(fi) + b_{H,t}^{Pvt}(ma)$$

(eqn. 21/72) $$w_{2,t} = (1-\alpha)p_{2,t}A_t(u_t K_t)^{\alpha} L_{2,t}^{-\alpha}$$

(eqn. 22/72) $$p_{K,t}M_{1,t}\delta'(u_t) = \alpha p_{2,t}A_t(u_t K_t)^{\alpha-1}L_{2,t}^{1-\alpha},$$

where $\frac{\partial \delta(u_t)}{\partial u_t} \equiv \delta'(u_t) = \delta_1 + \delta_2(u_t - 1)$

(eqn. 23/72) $$p_{K,t}M_{1,t} = \mathbb{E}_t\Lambda_{t,t+1}\left[\alpha p_{2,t+1}A_{t+1}K_{t+1}^{\alpha-1}u_{t+1}^{\alpha}L_{2,t+1}^{1-\alpha} + (1-\delta(u_{t+1}))p_{K,t+1}M_{1,t+1}\right]$$

(eqn. 24/72) $$\bar{Q}_{H,t}^{Pvt}M_{2,t} = \mathbb{E}_t\Lambda_{t,t+1}\pi_{C,t+1}^{-1}\left[1 + \kappa\bar{Q}_{H,t+1}^{Pvt}M_{2,t+1}\right]$$

(eqn. 25/72) $$\frac{M_{1,t}-1}{M_{2,t}-1} = \psi$$

(eqn. 26/72) $$b_{H,FC,t}^{Pvt} = \upsilon_3\left(\frac{Q_{F,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}}\right)^{\eta_3}\bar{b}_{H,t}^{Pvt}$$

(eqn. 27/72) $$b_{H,t}^{Pvt} = (1-\upsilon_3)\left(\frac{Q_{H,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}}\right)^{\eta_3}\bar{b}_{H,t}^{Pvt}$$

(eqn. 28/72) $$\bar{Q}_{H,t}^{Pvt} \equiv \left[(1-\upsilon_3)\left(Q_{H,t}^{Pvt}\right)^{1+\eta_3} + \upsilon_3\left(Q_{F,t}^{Pvt}\right)^{1+\eta_3}\right]^{\frac{1}{1+\eta_3}}$$

Retail.—

(eqn. 29/72) $$p_{H,t}^* = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}}$$

(eqn. 30/72) $$x_{1,t} = p_{2,t}p_{H,t}^{\epsilon_p}Y_t + \theta_p\pi_{C,t}^{-\gamma_p\epsilon_p}\mathbb{E}_t\Lambda_{t,t+1}x_{1,t+1}\pi_{C,t+1}^{\epsilon_p}$$

(eqn. 31/72) $$x_{2,t} = p_{H,t}^{\epsilon_p}Y_t + \theta_p\pi_{C,t}^{\gamma_p(1-\epsilon_p)}\mathbb{E}_t\Lambda_{t,t+1}x_{2,t+1}\pi_{C,t+1}^{\epsilon_p-1}$$

Final-Good Producer.—

(eqn. 32/72) $$Y_{2,t} = Y_t\upsilon_t^p,$$

where $\upsilon_t^p \equiv \int_0^1\left(\frac{p_{H,t}(f)}{p_{H,t}}\right)^{-\epsilon_p}df$

(eqn. 33/72) $$\upsilon_t^p = \left(\frac{p_{H,t-1}}{p_{H,t}}\frac{\pi_{C,t-1}^{\gamma_p}}{\pi_{C,t}}\right)^{-\epsilon_p}\theta_p\upsilon_{t-1}^p + (1-\theta_p)\left(\frac{p_{H,t}^*}{p_{H,t}}\right)^{-\epsilon_p}$$

(eqn. 34/72) $$p_{H,t}^{1-\epsilon_p} = \left(\frac{\pi_{C,t-1}^{\gamma_p}}{\pi_{C,t}}\right)^{1-\epsilon_p}\theta_p p_{H,t-1}^{1-\epsilon_p} + (1-\theta_p)\left(p_{H,t}^*\right)^{1-\epsilon_p}$$

Capital-Good Producer.—

(eqn. 35/72) $$\hat{I}_t = \left[1 - \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t$$

(eqn. 36/72) $$p_{K,t}\frac{\partial \hat{I}_t}{\partial I_t} + \mathbb{E}_t\Lambda_{t,t+1}p_{K,t+1}\frac{\partial \hat{I}_{t+1}}{\partial I_t} = p_{H,t}$$

Exporter.—

(eqn. 37/72) $$X_t = \nu_2\left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\eta_2} Y_{F,t}$$

D. Financial Intermediaries

(eqn. 38/72) $$Q_t^{Pvt}b_t^{Pvt}(fi) + Q_t^{Gov}b_t^{Gov}(fi) + s_t = d_t + n_t$$

(eqn. 39/72)
$$n_t = \vartheta\pi_{C,t}^{-1}\left[\left(R_t^{Pvt} - R_{t-1}^D\right)Q_{t-1}^{Pvt}b_{t-1}^{Pvt}(fi) + \left(R_t^{Gov} - R_{t-1}^D\right)Q_{t-1}^{Gov}b_{t-1}^{Gov}(fi) + \left(R_{t-1}^s - R_{t-1}^D\right)s_{t-1} + R_{t-1}^D n_{t-1}\right] + \chi$$

(eqn. 40/72) $$\phi_t n_t = Q_t^{Pvt}b_t^{Pvt}(fi) + \Delta Q_t^{Gov}b_t^{Gov}(fi)$$

(eqn. 41/72) $$\mathbb{E}_t\Omega_{t+1}\Lambda_{t,t+1}\left(R_{t+1}^{Pvt} - R_t^D\right)\pi_{C,t+1}^{-1} = \frac{\lambda_{1,t}}{1+\lambda_{1,t}}\theta_t$$

(eqn. 42/72) $$\mathbb{E}_t\Omega_{t+1}\Lambda_{t,t+1}\left(R_{t+1}^{Gov} - R_t^D\right)\pi_{C,t+1}^{-1} = \frac{\lambda_{1,t}}{1+\lambda_{1,t}}\Delta\theta_t$$

(eqn. 43/72) $$\mathbb{E}_t\Omega_{t+1}\Lambda_{t,t+1}\left(R_{t+1}^{s} - R_t^D\right)\pi_{C,t+1}^{-1} = -\frac{\lambda_{2,t}}{1+\lambda_{1,t}}$$

(eqn 44/72) $$\Omega_t \equiv 1 - \vartheta + \vartheta\theta_t\phi_t$$

(eqn. 45/72) $$\theta_t\phi_t = (1+\lambda_{1,t})\,\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_t^D\pi_{C,t+1}^{-1} - \frac{\lambda_{2,t}s_t}{n_t}$$

(eqn. 46/72) $$\ln\theta_t = (1-\rho_\theta)\ln\theta + \rho_\theta\ln\theta_{t-1} + s_\theta\varepsilon_{\theta,t}$$

(eqn. 47/72) $$b_{H,t}^{Pvt}(fi) = (1-\nu_4)\left(\frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} b_t^{Pvt}(fi),$$

where $b_t^{Pvt}(fi) \equiv \left[(1-\nu_4)^{1/\eta_4}\left(b_{H,t}^{Pvt}(fi)\right)^{1-1/\eta_4} + (\nu_4)^{1/\eta_4}\left(b_{F,t}^{Pvt}(fi)\right)^{1-1/\eta_4}\right]^{\eta_4/(\eta_4-1)}$ if $\eta_4 \neq 1$ and $b_t^{Pvt}(fi) \equiv \frac{1}{(1-\nu_4)^{1-\nu_4}\nu_4^{\nu_4}}\left(b_{H,t}^{Pvt}(fi)\right)^{1-\nu_4}\left(b_{F,t}^{Pvt}(fi)\right)^{\nu_4}$ if $\eta_4 = 1$

(eqn. 48/72) $$b_{F,t}^{Pvt}(fi) = \nu_4\left(\frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} b_t^{Pvt}(fi)$$

(eqn. 49/72) $$b_{H,t}^{Gov}(fi) = (1-\nu_5)\left(\frac{Q_{H,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} b_t^{Gov}(fi),$$

where $b_t^{Gov}(fi) \equiv \left[(1-\nu_5)^{1/\eta_5}\left(b_{H,t}^{Gov}(fi)\right)^{1-1/\eta_5} + (\nu_5)^{1/\eta_5}\left(b_{F,t}^{Gov}(fi)\right)^{1-1/\eta_5}\right]^{\eta_5/(\eta_5-1)}$ if $\eta_5 \neq 1$ and $b_t^{Gov}(fi) = \frac{1}{(1-\nu_5)^{1-\nu_5}\nu_5^{\nu_5}}\left(b_{H,t}^{Gov}(fi)\right)^{1-\nu_5}\left(b_{F,t}^{Gov}(fi)\right)^{\nu_5}$ if $\eta_5 = 1$

(eqn. 50/72) $$b_{F,t}^{Gov}(fi) = \nu_5\left(\frac{Q_{F,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} b_t^{Gov}(fi)$$

(eqn. 51/72) $$Q_t^{Pvt} = \left[(1-\nu_4)\left(Q_{H,t}^{Pvt}\right)^{1-\eta_4} + \nu_4\left(Q_{F,t}^{Pvt}\right)^{1-\eta_4}\right]^{\frac{1}{1-\eta_4}}$$

if $\eta_4 \neq 1$ and $Q_t^{Pvt} = \left(Q_{H,t}^{Pvt}\right)^{1-\nu_4}\left(Q_{F,t}^{Pvt}\right)^{\nu_4}$ if $\eta_4 = 1$

(eqn. 52/72) $$Q_t^{Gov} = \left[(1-\nu_5)\left(Q_{H,t}^{Gov}\right)^{1-\eta_5} + \nu_5\left(Q_{F,t}^{Gov}\right)^{1-\eta_5}\right]^{\frac{1}{1-\eta_5}}$$

if $\eta_5 \neq 1$ and $Q_t^{Gov} = \left(Q_{H,t}^{Gov}\right)^{1-\nu_5}\left(Q_{F,t}^{Gov}\right)^{\nu_5}$ if $\eta_5 = 1$

(eqn. 53/72) $$R_{t+1}^{Pvt} = R_{H,t+1}^{Pvt}\frac{Q_{H,t}^{Pvt}b_{H,t}^{Pvt}}{Q_t^{Pvt}b_t^{Pvt}} + R_{F,t+1}^{Pvt}\frac{Q_{F,t}^{Pvt}b_{F,t}^{Pvt}}{Q_t^{Pvt}b_t^{Pvt}}$$

(eqn. 54/72) $$R_{t+1}^{Gov} = R_{H,t+1}^{Gov}\frac{Q_{H,t}^{Gov}b_{H,t}^{Gov}}{Q_t^{Gov}b_t^{Gov}} + R_{F,t+1}^{Gov}\frac{Q_{F,t}^{Gov}b_{F,t}^{Gov}}{Q_t^{Gov}b_t^{Gov}}$$

(eqn. 55/72) $$R_{H,t}^{Gov} = \frac{1+\kappa Q_{H,t}^{Gov}}{Q_{H,t-1}^{Gov}}$$

(eqn. 56/72) $$R_{H,t}^{Pvt} = \frac{1+\kappa Q_{H,t}^{Pvt}}{Q_{H,t-1}^{Pvt}}$$

(eqn. 57/72) $$R_{F,t}^{Gov} = \frac{E_t}{E_{t-1}}\mathrm{USD}R_{F,t}^{Gov},$$

where $\mathrm{USD}R_{F,t}^{Gov} = \frac{\mathrm{USD}1+\kappa\mathrm{USD}Q_{F,t}^{Gov}}{\mathrm{USD}Q_{F,t-1}^{Gov}}$.

(eqn. 58/72) $$R_{F,t}^{Pvt} = \frac{E_t}{E_{t-1}}\mathrm{USD}R_{F,t}^{Pvt}$$

(eqn. 59/72) $$Q_{F,t}^{Gov} = E_t\mathrm{USD}Q_{F,t}^{Gov}$$

(eqn. 60/72) $$Q_{F,t}^{Pvt} = E_t\mathrm{USD}Q_{F,t}^{Pvt}$$

E. Monetary Authority

(eqn. 61/72) $$Q_{H,t}^{Pvt}b_{H,t}^{Pvt}(ma) + Q_{H,t}^{Gov}b_{H,t}^{Gov}(ma) = s_t$$

(eqn. 62/72) $$\Pi_{ma,t}^{\text{real}} \equiv \frac{\Pi_{ma,t}}{P_{C,t}} = \pi_{C,t}^{-1}\left[R_{H,t}^{Pvt}Q_{H,t-1}^{Pvt}b_{H,t-1}^{Pvt}(ma) + R_{H,t}^{Gov}Q_{H,t-1}^{Gov}b_{H,t-1}^{Gov}(ma) - R_{t-1}^{S}s_{t-1}\right]$$

(eqn. 63/72) $$\begin{aligned}\ln R_t^{Pol} &= (1-\rho_r)\ln\left(R^{Pol}\right)^{SS} + \rho_r\ln R_{t-1}^{Pol} \\ &\quad + (1-\rho_r)\left[\phi_\pi\left(\ln\pi_{C,t} - \ln\pi_C^{SS}\right) + \phi_y\left(\ln Y_t - \ln Y_{t-1}\right)\right] + s_r\varepsilon_{r,t}\end{aligned}$$

(eqn. 64/72, Scenario 1) $$R_t^S = R_t^{Pol}$$

(eqn. 64/72, Scenario 2) $$R_t^S = \max\left\{1, R_t^{Pol}\right\}$$

(eqn. 64/72, Scenario 3) $$R_t^S = \max\left\{\underline{R}, R_t^{Pol}\right\}$$

(eqn. 65/72) $$R_t^D = \max\left\{1, R_t^S\right\}$$

(eqn. 66/72, Endo.)
$$\begin{aligned} b_{H,t}^{Pvt}(ma) &= (1-\rho_1)\left(b_H^{Pvt}(ma)\right)^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) \\ &\quad + (1-\rho_1)\Psi_1\left[\phi_\pi\left(\ln\pi_{C,t} - \ln\pi_C^{SS}\right) + \phi_y\left(\ln Y_t - \ln Y_{t-1}\right)\right] + s_1\varepsilon_{1,t} \end{aligned}$$

(eqn. 66/72, Exog.) $$b_{H,t}^{Pvt}(ma) = (1-\rho_1)\left(b_H^{Pvt}(ma)\right)^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) + s_1\varepsilon_{1,t}$$

(eqn. 67/72, Endo.)
$$\begin{aligned} b_{H,t}^{Gov}(ma) &= (1-\rho_2)\left(b_H^{Gov}(ma)\right)^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) \\ &\quad + (1-\rho_2)\Psi_2\left[\phi_\pi\left(\ln\pi_{C,t} - \ln\pi_C^{SS}\right) + \phi_y\left(\ln Y_t - \ln Y_{t-1}\right)\right] + s_2\varepsilon_{2,t} \end{aligned}$$

(eqn. 67/72, Exog.) $$b_{H,t}^{Gov}(ma) = (1-\rho_2)\left(b_H^{Gov}(ma)\right)^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) + s_2\varepsilon_{2,t}$$

F. Fiscal Authority

(eqn. 68/72) $$G_t + \bar{b}_H^{Gov}\pi_{C,t}^{-1} = T_t + \Pi_{ma,t}^{\text{real}} + Q_{H,t}^{Gov}\bar{b}_H^{Gov}\left(1 - \kappa\pi_{C,t}^{-1}\right)$$

(eqn. 69/72) $$\ln G_t = (1-\rho_G)\ln G^{SS} + \rho_G \ln G_{t-1} + s_G\varepsilon_{G,t}$$

(eqn. 70/72) $$\bar{b}_H^{Gov} = b_{H,t}^{Gov}(fi) + b_{H,t}^{Gov}(ma),$$

where $b_{H,t}^{Gov}(fi) = \int b_{H,t}^{Gov}(j)\,dj$

G. Foreign Exchange Market

(eqn. 71/72) $$E_t = \frac{p_{F,t}}{p_{F,t-1}}\frac{\pi_{C,t}}{\text{USD}\pi_{F,t}}E_{t-1}$$

(eqn. 72/72)
$$\begin{aligned} p_{H,t}X_t - p_{F,t}C_{F,t} &= E_t b_{H,FC,t-1}^{Pvt}\pi_{C,t}^{-1} - \left(b_{F,t-1}^{Pvt}(fi) + b_{F,t-1}^{Gov}(fi)\right)\pi_{C,t}^{-1}E_t \\ &\quad + Q_{F,t}^{Pvt}(b_{F,t}^{Pvt}(fi) - \kappa b_{F,t-1}^{Pvt}(fi)\pi_{C,t}^{-1}) + Q_{F,t}^{Gov}(b_{F,t}^{Gov}(fi) - \kappa b_{F,t-1}^{Gov}(fi)\pi_{C,t}^{-1}) \\ &\quad - Q_{F,t}^{Pvt}\left(b_{H,FC,t}^{Pvt} - \kappa b_{H,FC,t-1}^{Pvt}\pi_{C,t}^{-1}\right) \end{aligned}$$

Online Appendix:
"International Spillovers of Conventional versus New Monetary Policy"

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Contents

We have organized this online appendix into six sections. In Section 1, we provide details of the model and derive the non-linear equilibrium conditions that are new to our model. In Section 2, we derive steady-state expressions for all 72 endogenous variables in the model. Sections 3, 4 and 5 contain additional tables and figures that are related to Sections III, IV, and V of the paper. In Section 6, we discuss the sensitivity of our main results to changes in the parameters of the international block of our model.

1 Model Details and Derivations

The foreign economy in our model is almost identical to the closed economy in Sims and Wu (2020), SW from here on.[1] The home small open economy (SOE) in our model is an extension of the economy in SW. All in all, there are 72 non-linear equilibrium conditions for the home SOE in our model. Fifty of the 72 equilibrium conditions are identical to those in SW. The remaining 22 are new. SW derive the 50 equilibrium conditions in their paper. In this online appendix, we derive (and in some cases, just state) the equations that we add to SW's model.

1.1 Household

We make three changes to the problem of the representative household: (1) We replace log utility by CRRA utility; (2) We add a preference shock, Z_t, to the utility function; and (3) we allow the consumer to consume the foreign good in addition to the home good. We assume the log of the preference shock follows an AR(1) process:[2]

(eqn. 02/72) $$\ln Z_t = \rho_Z \ln Z_{t-1} + s_Z \varepsilon_{Z,t}.$$

The household combines home-produced good, $C_{H,t}$, and foreign-produced good (imports), $C_{F,t}$, to produce the composite consumption good, C_t, according to the following CES production function:

$$C_t \equiv \left((1-\nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1 - 1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} C_{F,t}^{\frac{\eta_1 - 1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1 - 1}}.$$

The problem of the household is to choose $C_{H,t}$ and $C_{F,t}$ that minimize the cost of composite consumption C_t subject to the CES production function above. The Lagrangian for the problem is

$$\mathcal{L}_{C,t} = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + \lambda_{C,t} \left[C_t - \left((1-\nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1 - 1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} C_{F,t}^{\frac{\eta_1 - 1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1 - 1}} \right].$$

The Langrange multiplier $\lambda_{C,t}$ is the shadow price of composite consumption C_t: If we increased C_t by one unit, the total cost of C_t would increase by $\lambda_{C,t}$. Let $P_{C,t} \equiv \lambda_{C,t}$. The solution to the consumer's problem gives

$$P_{C,t} = \left[(1-\nu_1) P_{H,t}^{1-\eta_1} + \nu_1 P_{F,t}^{1-\eta_1} \right]^{\frac{1}{1-\eta_1}}.$$

In this economy, $P_{C,t}$ is the numeraire. Hence the last equation gives us a relationship between $p_{H,t} \equiv \frac{P_{H,t}}{P_{C,t}}$ and $p_{F,t} \equiv \frac{P_{F,t}}{P_{C,t}}$. Rewrite the last equation as:

$$P_{C,t}^{1-\eta_1} = (1-\nu_1) P_{H,t}^{1-\eta_1} + \nu_1 P_{F,t}^{1-\eta_1}.$$

Divide both sides by $P_{C,t}^{1-\eta_1}$:

(eqn. 09/72) $$1 = (1-\nu_1) p_{H,t}^{1-\eta_1} + \nu_1 p_{F,t}^{1-\eta_1}.$$

[1] Sims, Eric R. and Wu, Jing Cynthia. 2019. "Evaluating Central Banks' Tool Kit: Past, Present and Future." *Journal of Monetary Economics*, 118, 135–160.

[2] The equation numbers in this online appendix that appear as eqn. 01/72, 02/72 and so on, are the same as in Appendix B of the paper.

The solution to the consumer's problem also gives

(eqn. 08/72) $$C_{F,t} = \nu_1 p_{F,t}^{-\eta_1} C_t$$

and

(eqn. 07/72) $$C_{H,t} = (1-\nu_1)\, p_{H,t}^{-\eta_1} C_t,$$

where

$$C_t = \left((1-\nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1 - 1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} (C_{F,t})^{\frac{\eta_1 - 1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1 - 1}}.$$

1.2 Non-Financial Firms

We introduce two changes to the non-financial business sector in SW: (1) We allow the wholesale firm to borrow from abroad by issuing bonds in foreign currency; and (2) We add a representative exporter who exports part of the final output according to an export demand function.

1.2.1 Wholesale Firms

The wholesale firm's "loan-in-advance constraint" is

$$\psi P_{k,t} \widehat{I}_t \leq \bar{Q}_{H,t}^{Pvt} (\bar{B}_{H,t}^{Pvt} - \kappa \bar{B}_{H,t-1}^{Pvt}).$$

This constraint is always binding. We can write it in real terms as:

(eqn. 19/72) $$\psi p_{k,t} \widehat{I}_t = \bar{Q}_{H,t}^{Pvt} (\bar{b}_{H,t}^{Pvt} - \kappa \bar{b}_{H,t-1}^{Pvt} \pi_{C,t}^{-1}).$$

Once the firm knows its need for new borrowing, which is given by the right-hand side of the above constraint, we allow it to issue bonds in both home and foreign markets in their respective currencies. The values of the two types of bonds aggregate according to

$$\bar{Q}_{H,t}^{Pvt} \bar{B}_{H,t}^{Pvt} = Q_{H,t}^{Pvt} B_{H,t}^{Pvt} + Q_{H,FC,t}^{Pvt} B_{H,FC,t}^{Pvt}.$$

The first term on the right, $Q_{H,t}^{Pvt} B_{H,t}^{Pvt}$, is the market value of bonds issued by the wholesale firm in the home currency. These bonds are held by the home financial intermediaries (indexed by $j \in [0,1]$) and the home monetary authority:

$$B_{H,t}^{Pvt} = B_{H,t}^{Pvt}(fi) + B_{H,t}^{Pvt}(ma),$$

where $B_{H,t}^{Pvt}(fi) \equiv \int B_{H,t}^{Pvt}(j)\, dj$ are the private bond holdings of financial intermediaries and $B_{H,t}^{Pvt}(ma)$ are the private bond holdings of the monetary authority. We can write the last equation in real terms as:

(eqn. 20/72) $$b_{H,t}^{Pvt} = b_{H,t}^{Pvt}(fi) + b_{H,t}^{Pvt}(ma).$$

The second term on the right, $Q_{H,FC,t}^{Pvt} B_{H,FC,t}^{Pvt}$, is the market value, in home currency, of the bonds issued by the wholesale firm in the foreign currency. For simplicity, we assume that these bonds are held only by foreign financial intermediaries. Because of the SOE assumption, the wholesale firm takes the price of foreign private bonds, $Q_{H,FC,t}^{Pvt}$, as given. This is the same price that the home financial intermediaries (see Section 1.3 below) pay to buy foreign currency bonds, hence $Q_{H,FC,t}^{Pvt} = Q_{F,t}^{Pvt} = E_t \mathrm{USD} Q_{F,t}^{Pvt}$.

The term on the left-hand side is the product of two CES aggregators that we define below. $\bar{Q}_{H,t}^{Pvt}$ is a CES aggregate of bond prices and $\bar{B}_{H,t}^{Pvt}$ is a CES aggregate of the quantities of the two types of bonds.

The goal of the wholesale firm is to choose $B_{H,t}^{Pvt}$ and $B_{H,FC,t}^{Pvt}$ to maximize the bond proceeds

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt} + Q_{F,t}^{Pvt} B_{H,FC,t}^{Pvt}$$

subject to

$$\bar{B}_{H,t}^{Pvt} \equiv \left[\left(\frac{1}{1-\upsilon_3} \right)^{1/\eta_3} \left(B_{H,t}^{Pvt} \right)^{1+1/\eta_3} + \left(\frac{1}{\upsilon_3} \right)^{1/\eta_3} \left(B_{H,FC,t}^{Pvt} \right)^{1+1/\eta_3} \right]^{\frac{\eta_3}{\eta_3+1}}.$$

The Lagrangian for the problem is

$$\mathcal{L}_{B,t} = Q_{H,t}^{Pvt} B_{H,t}^{Pvt} + Q_{F,t}^{Pvt} B_{H,FC,t}^{Pvt} + \lambda_{B,t} \left[\bar{B}_{H,t}^{Pvt} - \left[\left(\frac{1}{1-\upsilon_3} \right)^{1/\eta_3} \left(B_{H,t}^{Pvt} \right)^{1+1/\eta_3} + \left(\frac{1}{\upsilon_3} \right)^{1/\eta_3} \left(B_{H,FC,t}^{Pvt} \right)^{1+1/\eta_3} \right]^{\frac{\eta_3}{\eta_3+1}} \right].$$

The Lagrange multiplier $\lambda_{B,t}$ is the shadow price of the composite quantity of bonds $\bar{B}_{H,t}^{Pvt}$: If $\bar{B}_{H,t}^{Pvt}$ increases by one unit, the bond proceeds of the wholesale firm will increase by $\lambda_{B,t}$. Let $\bar{Q}_{H,t}^{Pvt} \equiv \lambda_{B,t}$. Solving the above problem gives

(eqn. 28/72) $$\bar{Q}_{H,t}^{Pvt} = \left[(1-\upsilon_3) \left(Q_{H,t}^{Pvt} \right)^{\eta_3+1} + \upsilon_3 \left(Q_{F,t}^{Pvt} \right)^{\eta_3+1} \right]^{\frac{1}{\eta_3+1}}$$

and the following 'supply' functions for the two type of bonds:

$$B_{H,t}^{Pvt} = (1-\upsilon_3) \left(\frac{Q_{H,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} \right)^{\eta_3} \bar{B}_{H,t}^{Pvt}$$

and

$$B_{H,FC,t}^{Pvt} = \upsilon_3 \left(\frac{Q_{F,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} \right)^{\eta_3} \bar{B}_{H,t}^{Pvt}.$$

We can write the supply functions in real terms as:

(eqn. 27/72) $$b_{H,t}^{Pvt} = (1-\upsilon_3) \left(\frac{Q_{H,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} \right)^{\eta_3} \bar{b}_{H,t}^{Pvt}$$

and

(eqn. 26/72) $$b_{H,FC,t}^{Pvt} = \upsilon_3 \left(\frac{Q_{F,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} \right)^{\eta_3} \bar{b}_{H,t}^{Pvt}.$$

The rest of the problem of the wholesale firm is similar to that in SW.

1.2.2 Exporter

The exporter buys final output and exports it according to the following export demand function:

(eqn. 37/72) $$X_t = \nu_2 \left(\frac{p_{H,t}}{p_{F,t}} \right)^{-\eta_2} Y_{F,t}.$$

1.3 Financial Intermediaries

The total value of private bonds held by financial intermediary (FI) j is the sum of the values of its home and foreign private bonds:

$$Q_t^{Pvt} B_t^{Pvt}(j) = Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt} B_{F,t}^{Pvt}(j),$$

where

(eqn. 60/72) $$Q_{F,t}^{Pvt} = E_t \text{USD} Q_{F,t}^{Pvt}.$$

Similarly, for government bonds:

$$Q_t^{Gov} B_t^{Gov}(j) = Q_{H,t}^{Gov} B_{H,t}^{Gov}(j) + Q_{F,t}^{Gov} B_{F,t}^{Gov}(j),$$

where

(eqn. 59/72) $$Q_{F,t}^{Gov} = E_t \text{USD} Q_{F,t}^{Gov}.$$

The financial intermediary chooses $B_{H,t}^{Pvt}(j)$ and $B_{F,t}^{Pvt}(j)$ to minimize cost of bonds

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt} B_{F,t}^{Pvt}(j)$$

subject to

$$B_t^{Pvt}(j) = \left[\left(\frac{1}{1-\nu_4}\right)^{-1/\eta_4} \left[B_{H,t}^{Pvt}(j)\right]^{\frac{\eta_4-1}{\eta_4}} + \left(\frac{1}{\nu_4}\right)^{-1/\eta_4} \left[B_{F,t}^{Pvt}(j)\right]^{\frac{\eta_4-1}{\eta_4}}\right]^{\frac{\eta_4}{\eta_4-1}}.$$

The FOC's give

$$B_{H,t}^{Pvt}(j) = (1-\nu_4)\left(\frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} B_t^{Pvt}(j)$$

and

$$B_{F,t}^{Pvt}(j) = \nu_4\left(\frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} B_t^{Pvt}(j),$$

where

(eqn. 51/72) $$Q_t^{Pvt} = \left[(1-\nu_4)\left(Q_{H,t}^{Pvt}\right)^{1-\eta_4} + \nu_4\left(Q_{F,t}^{Pvt}\right)^{1-\eta_4}\right]^{\frac{1}{1-\eta_4}}.$$

Similarly, for government bonds we can derive the demand functions as:

$$B_{H,t}^{Gov}(j) = (1-\nu_5)\left(\frac{Q_{H,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} B_t^{Gov}(j)$$

and

$$B_{F,t}^{Gov}(j) = \nu_5\left(\frac{Q_{F,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} B_t^{Gov}(j),$$

where

(eqn. 52/72) $$Q_t^{Gov} = \left[(1-\nu_5)\left(Q_{H,t}^{Gov}\right)^{1-\eta_5} + \nu_5\left(Q_{F,t}^{Gov}\right)^{1-\eta_5}\right]^{\frac{1}{1-\eta_5}}.$$

We can write these demands in real terms by dividing both sides in all four equations by $P_{C,t}$. This gives:

$$b_{H,t}^{Pvt}(j) = (1-\nu_4)\left(\frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} b_t^{Pvt}(j),$$

$$b_{F,t}^{Pvt}(j) = \nu_4\left(\frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} b_t^{Pvt}(j),$$

$$b_{H,t}^{Gov}(j) = (1-\nu_5)\left(\frac{Q_{H,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} b_t^{Gov}(j),$$

and

$$b_{F,t}^{Gov}(j) = \nu_5\left(\frac{Q_{F,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} b_t^{Gov}(j).$$

When we aggregate the demands for all financial intermediatries, $j \in [0,1]$, we get:

(eqn. 47/72)
$$b_{H,t}^{Pvt}(fi) = (1-\nu_4)\left(\frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} b_t^{Pvt},$$

(eqn. 48/72)
$$b_{F,t}^{Pvt}(fi) = \nu_4\left(\frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}}\right)^{-\eta_4} b_t^{Pvt},$$

(eqn. 49/72)
$$b_{H,t}^{Gov}(fi) = (1-\nu_5)\left(\frac{Q_{H,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} b_t^{Gov},$$

and

(eqn. 50/72)
$$b_{F,t}^{Gov}(fi) = \nu_5\left(\frac{Q_{F,t}^{Gov}}{Q_t^{Gov}}\right)^{-\eta_5} b_t^{Gov}.$$

We next derive the return equations. We start with the equation:

$$Q_t^{Pvt}B_t^{Pvt}(j) = Q_{H,t}^{Pvt}B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j).$$

The period t value of the private home bond holdings of FI j is $Q_{H,t}^{Pvt}B_{H,t}^{Pvt}(j)$. The value of the same holdings in period $t+1$ will be $R_{H,t+1}^{Pvt}Q_{H,t}^{Pvt}B_{H,t}^{Pvt}(j)$, where

(eqn. 56/72)
$$R_{H,t+1}^{Pvt} = \frac{1+\kappa Q_{H,t+1}^{Pvt}}{Q_{H,t}^{Pvt}}.$$

Similarly, $Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j)$ will become $R_{F,t+1}^{Pvt}Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j)$ where

(eqn. 58/72)
$$R_{F,t+1}^{Pvt} = \frac{E_{t+1}}{E_t}\mathrm{USD}R_{F,t+1}^{Pvt}$$

and

$$\mathrm{USD}R_{F,t+1}^{Pvt} = \frac{\mathrm{USD}1+\kappa\mathrm{USD}Q_{F,t+1}^{Pvt}}{\mathrm{USD}Q_{F,t}^{Pvt}}.$$

In foreign currency, we start from $\mathrm{USD}Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j) = Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j)/E_t$. It becomes $\mathrm{USD}R_{F,t+1}^{Pvt}\mathrm{USD}Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j) = R_{F,t+1}^{Pvt}Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j)/E_{t+1}$ in the next period. Define a composite return $R_{t+1}^{Pvt}\left(R_{H,t+1}^{Pvt}, R_{F,t+1}^{Pvt}\right)$ as:

$$R_{t+1}^{Pvt}Q_t^{Pvt}B_t^{Pvt}(j) = R_{H,t+1}^{Pvt}Q_{H,t}^{Pvt}B_{H,t}^{Pvt}(j) + R_{F,t+1}^{Pvt}Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j)$$

$\Longleftrightarrow$

(eqn. 53/72)
$$R_{t+1}^{Pvt} = R_{H,t+1}^{Pvt}\frac{Q_{H,t}^{Pvt}B_{H,t}^{Pvt}(j)}{Q_t^{Pvt}B_t^{Pvt}(j)} + R_{F,t+1}^{Pvt}\frac{Q_{F,t}^{Pvt}B_{F,t}^{Pvt}(j)}{Q_t^{Pvt}B_t^{Pvt}(j)}.$$

Similarly

(eqn. 54/72) $$R_{t+1}^{Gov} = R_{H,t+1}^{Gov} \frac{Q_{H,t}^{Gov} B_{H,t}^{Gov}(j)}{Q_t^{Gov} B_t^{Gov}(j)} + R_{F,t+1}^{Gov} \frac{Q_{F,t}^{Gov} B_{F,t}^{Gov}(j)}{Q_t^{Gov} B_t^{Gov}(j)},$$

where

(eqn. 55/72) $$R_{H,t+1}^{Gov} = \frac{1 + \kappa Q_{H,t+1}^{Gov}}{Q_{H,t}^{Gov}},$$

(eqn. 57/72) $$R_{F,t+1}^{Gov} = \frac{E_{t+1}}{E_t} \mathrm{USD} R_{F,t+1}^{Gov}$$

and

$$\mathrm{USD} R_{F,t+1}^{Gov} = \frac{\mathrm{USD1} + \kappa \mathrm{USD} Q_{F,t+1}^{Gov}}{\mathrm{USD} Q_{F,t}^{Gov}}.$$

This completes the explanation of all the new elements that we have added to the FI sector of SW. The other elements of the FI sector in our model are identical to those in SW.

2 Steady State of the Model

In this section, we derive the steady-state expressions for all 72 endogenous variables for the home SOE in our model. The serial number of each endogenous variable below corresponds to its serial number in Appendix A (Notation) sub-section A (Variables) of the paper.

The following variables are either normalized to 1 or the equilibrium condition(s) imply that they are equal to one in the steady state.

$$A^{SS} = 1 \tag{var. 01/72 in SS}$$

$$L_1^{SS} = 1 \tag{var. 10/72 in SS}$$

$$Z^{SS} = 1 \tag{var. 35/72 in SS}$$

$$E^{SS} = 1 \tag{var. 05/72 in SS}$$

$$u^{SS} = 1 \tag{var. 57/72 in SS}$$

$$\pi_C^{SS} = 1 \tag{var. 69/72 in SS}$$

$$(\upsilon^p)^{SS} = 1 \tag{var. 70/72 in SS}$$

$$(\upsilon^w)^{SS} = 1 \tag{var. 71/72 in SS}$$

Next, we list the variables whose SS values match directly to a target or parameter.

$$\left(R^{Pol}\right)^{SS} = R^{Pol} \tag{var. 26/72 in SS}$$

$$\left(R^{D}\right)^{SS} = \pi_C^{SS}/\Lambda_{0,1}^{SS} = 1/\beta \tag{var. 22/72 in SS}$$

$$\left(R_H^{Gov}\right)^{SS} = \left(R^{D}\right)^{SS} + \text{ Target government bond spread} \tag{var. 25/72 in SS}$$

$$\left(R_H^{Pvt}\right)^{SS} = \left(R^{D}\right)^{SS} + \text{ Target private bond spread} \tag{var. 29/72 in SS}$$

The next two equations are our targets for foreign government and private bond returns.

$$\left(\text{USD}R_F^{Gov}\right)^{SS} = \left(R_H^{Gov}\right)^{SS}$$

$$\left(\text{USD}R_F^{Pvt}\right)^{SS} = \left(R_H^{Pvt}\right)^{SS}$$

$$\left(R_F^{Gov}\right)^{SS} = \left(\text{USD}R_F^{Gov}\right)^{SS} \tag{var. 24/72 in SS}$$

$$\left(R_F^{Pvt}\right)^{SS} = \left(\text{USD}R_F^{Pvt}\right)^{SS} \tag{var. 28/72 in SS}$$

(var. 44/72 in SS) $$\left(b_H^{Pvt}(ma)\right)^{SS} = b_H^{Pvt}(ma) = 0$$

(var. 39/72 in SS) $$\left(b_H^{Gov}(ma)\right)^{SS} = b_H^{Gov}(ma)$$

(var. 63/72 in SS) $$\Lambda_{0,1} = \beta$$

The following variables have simple steady-state expressions.

(var. 11/72 in SS) $$L_2^{SS} = \frac{L_1^{SS}}{(\upsilon^w)^{SS}}$$

(var. 16/72 in SS) $$\left(Q_F^{Gov}\right)^{SS} = \left(\text{USD}Q_F^{Gov}\right)^{SS} = \frac{1}{\left(\text{USD}R_F^{Gov}\right)^{SS} - \kappa}$$

(var. 19/72 in SS) $$\left(Q_F^{Pvt}\right)^{SS} = \left(\text{USD}Q_F^{Pvt}\right)^{SS} = \frac{1}{\left(\text{USD}R_F^{Pvt}\right)^{SS} - \kappa}$$

The last two equations imply $E^{SS} = 1$.

(var. 17/72 in SS) $$\left(Q_H^{Gov}\right)^{SS} = \frac{1}{\left(R_H^{Gov}\right)^{SS} - \kappa}$$

(var. 20/72 in SS) $$\left(Q_H^{Pvt}\right)^{SS} = \frac{1}{\left(R_H^{Pvt}\right)^{SS} - \kappa}$$

(var. 15/72 in SS) $$\left(Q^{Gov}\right)^{SS} = \left[(1-\nu_5)\left(\left(Q_H^{Gov}\right)^{SS}\right)^{1-\eta_5} + \nu_5\left(\left(Q_F^{Gov}\right)^{SS}\right)^{1-\eta_5}\right]^{\frac{1}{1-\eta_5}}$$

If $\left(\text{USD}R_F^{Gov}\right)^{SS} = \left(R_H^{Gov}\right)^{SS}$,

$$\left(Q^{Gov}\right)^{SS} = \left(Q_H^{Gov}\right)^{SS} = \left(Q_F^{Gov}\right)^{SS} = \left(\text{USD}Q_F^{Gov}\right)^{SS}.$$

(var. 18/72 in SS) $$\left(Q^{Pvt}\right)^{SS} = \left[(1-\nu_4)\left(\left(Q_H^{Pvt}\right)^{SS}\right)^{1-\eta_4} + \nu_4\left(\left(Q_F^{Pvt}\right)^{SS}\right)^{1-\eta_4}\right]^{\frac{1}{1-\eta_4}}$$

If $\left(\text{USD}R_F^{Pvt}\right)^{SS} = \left(R_H^{Pvt}\right)^{SS}$,

$$\left(Q^{Pvt}\right)^{SS} = \left(Q_H^{Pvt}\right)^{SS} = \left(Q_F^{Pvt}\right)^{SS} = \left(\text{USD}Q_F^{Pvt}\right)^{SS}.$$

(var. 30/72 in SS) $$\left(R^S\right)^{SS} = \left(R^D\right)^{SS}$$

We now solve for the steady-state expressions for the remaining variables. From eqn. 70/72, we got

(var. 38/72 in SS) $$\left(b_H^{Gov}(fi)\right)^{SS} = \left(\bar{b}_H^{Gov}\right)^{SS} - \left(b_H^{Gov}(ma)\right)^{SS}.$$

(var. 21/72 in SS) $$\left(\bar{Q}_H^{Pvt}\right)^{SS} = \left[(1-\upsilon_3)\left(\left(Q_H^{Pvt}\right)^{SS}\right)^{1+\eta_3} + \upsilon_3\left(\left(Q_F^{Pvt}\right)^{SS}\right)^{1+\eta_3}\right]^{\frac{1}{1+\eta_3}}$$

We assume that in the SS

$$p_F^{SS} = p_H^{SS}.$$

This, together with eqn. 09/72 implies

(vars. 52-53/72 in SS) $$p_F^{SS} = p_H^{SS} = 1.$$

Also note that

$$p_F^{SS} \equiv \frac{P_F^{SS}}{P_C^{SS}} = \frac{E^{SS}\mathrm{USD}P_F^{SS}}{P_C^{SS}} = \frac{P_H^{SS}}{P_C^{SS}} = p_H^{SS}.$$

Hence

$$E^{SS}\mathrm{USD}P_F^{SS} = P_H^{SS}.$$

From eqn. 34/72

(var. 54/72 in SS) $$\left(p_H^*\right)^{SS} = p_H^{SS} = 1.$$

In the next step, we are not deriving x_1^{SS} and x_2^{SS}. Instead, we need the ratio $\frac{x_1^{SS}}{x_2^{SS}}$. Note that

(var. 61/72 in SS) $$x_1^{SS} = \frac{p_2^{SS}\left(p_H^{SS}\right)^{\epsilon_p}Y^{SS}}{1-\theta_p\Lambda_{0,1}^{SS}}.$$

and

(var. 62/72 in SS) $$x_2^{SS} = \frac{\left(p_H^{SS}\right)^{\epsilon_p}Y^{SS}}{1-\theta_p\Lambda_{0,1}}.$$

The ratio of the two is:

$$\frac{x_1^{SS}}{x_2^{SS}} = p_2^{SS}.$$

Substitute this in

(eqn. 29/72) $$p_{H,t}^* = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}}$$

to get

$$\left(p_H^*\right)^{SS} = \frac{\epsilon_p}{\epsilon_p-1}p_2^{SS}$$

$\Longrightarrow$

(var. 51/72) $$p_2^{SS} = \frac{\epsilon_p-1}{\epsilon_p}\left(p_H^*\right)^{SS} = \frac{\epsilon_p-1}{\epsilon_p}.$$

In the SS

$$\frac{\partial\hat{I}_t}{\partial I_t} = 1 - \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \kappa_I\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-1\right)$$

becomes

$$\left(\frac{\partial\hat{I}_t}{\partial I_t}\right)^{SS} = 1$$

and

$$\frac{\partial\hat{I}_{t+1}}{\partial I_t} = \kappa_I\left(\frac{I_{t+1}}{I_t}-1\right)\left(\frac{I_{t+1}}{I_t}\right)^2.$$

becomes

$$\left(\frac{\partial \hat{I}_{t+1}}{\partial I_t}\right)^{SS} = 0.$$

Substitute the SS values of these two derivatives in the SS version of eqn. 36/72 to get

(eqn. 36/72 in SS) $$p_K^{SS}\left(\frac{\partial \hat{I}_t}{\partial I_t}\right)^{S} + \Lambda_{0,1}^{SS} p_K^{SS}\left(\frac{\partial \hat{I}_{t+1}}{\partial I_t}\right)^{SS} = p_H^{SS}$$

$\Longleftrightarrow$

$$p_K^{SS}(1) + \Lambda_{0,1}^{SS} p_K^{SS}(0) = p_H^{SS}$$

$\Longrightarrow$

(var. 55/72 in SS) $$p_K^{SS} = p_H^{SS}.$$

From eqn. 24/72, we get

(var. 13/72 in SS) $$M_2^{SS} = \frac{\Lambda_{0,1}^{SS}}{\pi_C^{SS} - \kappa \Lambda_{0,1}^{SS}} \frac{1}{\left(\bar{Q}_H^{Pvt}\right)^{SS}}.$$

From eqn. 25/72, we get

(var. 12/72 in SS) $$M_1^{SS} = 1 + \psi(M_2^{SS} - 1).$$

From eqn. 23/72, we get

(var. 09/72 in SS) $$K^{SS} = \left[\frac{\alpha p_2^{SS}}{p_k^{SS} M_1^{SS}} \frac{\Lambda_{0,1}^{SS}}{1 - \Lambda_{0,1}^{SS}(1-\delta_0)}\right]^{\frac{1}{1-\alpha}} L_2^{SS}.$$

If we substitute K^{SS} into the steady-state version of eqn. 22/72 (the FOC w.r.t. u_t) we get

$$\delta_1 = \frac{1}{\beta} - (1 - \delta_0).$$

This shows that δ_1 depends on δ_0 and β, and is not a free parameter.

In the SS

(eqn. 21/72) $$w_{2,t} = (1-\alpha) p_{2,t} A_t (u_t K_t)^{\alpha} L_{2,t}^{-\alpha}$$

becomes

(var. 59/72 in SS) $$w_2^{SS} = (1-\alpha) p_2^{SS} (K^{SS})^{\alpha},$$

where we have used $A^{SS} = u^{SS} = L_2^{SS} = 1$.

From eqn. 15/72, we get

(var. 60/72 in SS) $$(w_2^*)^{SS} = w_2^{SS}.$$

Just like the ratio of the x's above, here we need the ratio of the f's. From eqn. 11/72, we get

(var. 48/72 in SS) $$f_1^{SS} = \frac{w_1^{SS}\left(w_2^{SS}\right)^{\epsilon_w} L_2^{SS}}{\left[1 - \theta_w \Lambda_{0,1}^{SS}\left(\pi_C^{SS}\right)^{\epsilon_w(1-\gamma_w)}\right]}$$

and from eqn. 12/72 we get

(var. 49/72 in SS) $$f_2^{SS} = \frac{\left(w_2^{SS}\right)^{\epsilon_w} L_2^{SS}}{\left[1 - \theta_w \Lambda_{0,1}^{SS} \left(\pi_C^{SS}\right)^{(1-\epsilon_w)(\gamma_w - 1)}\right]}.$$

Because $\pi_C^{SS} = 1$, the ratio of the last two equations gives

$$\frac{f_1^{SS}}{f_2^{SS}} = w_1^{SS}.$$

Substitute it in the SS version of

(eqn. 10/72) $$w_{2,t}^* = \frac{\epsilon_w}{\epsilon_w - 1} \frac{f_{1,t}}{f_{2,t}}$$

to get

$$w_{2,t}^* = \frac{\epsilon_w}{\epsilon_w - 1} w_1^{SS}$$

$\Longleftrightarrow$

(var. 58/72 in SS) $$w_1^{SS} = \frac{\epsilon_w - 1}{\epsilon_w} w_{2,t}^*.$$

From eqn. 13/72

(var. 11/72 in SS) $$L_2^{SS} = \frac{L_1^{SS}}{(\upsilon^w)^{SS}} = \frac{1}{1} = 1.$$

The SS version of eqn. 16/72 is

$$Y_2^{SS} = A^{SS} \left(u^{SS} K^{SS}\right)^{\alpha} \left(L_2^{SS}\right)^{1-\alpha},$$

which, when we substitute $A^{SS} = u^{SS} = L_2^{SS} = 1$, becomes

(var. 34/72 in SS) $$Y_2^{SS} = \left(K^{SS}\right)^{\alpha}.$$

From eqn. 32/72

(var. 33/72 in SS) $$Y^{SS} = \frac{Y_2^{SS}}{(\upsilon^p)^{SS}} = Y_2^{SS}.$$

In the SS, we have

(var. 06/72 in SS) $$G^{SS} = \bar{g} Y^{SS}.$$

From eqn. 18/72, we get

(var. 08/72 in SS) $$\hat{I}^{SS} = \delta_0 K^{SS}.$$

From eqn. 35/72, we get

(var. 07/72 in SS) $$I^{SS} = \hat{I}^{SS}.$$

From eqn. 37/72, we get

$$X^{SS} = \nu_2 \left(\frac{p_H^{SS}}{p_F^{SS}}\right)^{-\eta_2} Y_F^{SS},$$

which becomes

(var. 32/72 in SS) $$X^{SS} = \nu_2 Y_F^{SS}$$

because $p_H^{SS} = p_F^{SS}$.

From eqn. 06/72, we get

(var. 04/72 in SS) $$C_H^{SS} = Y^{SS} - I^{SS} - G^{SS} - X^{SS}.$$

From eqns. 07/72 and 08/72, we get

(var. 03/72 in SS) $$C_F^{SS} = \frac{\nu_1}{1-\nu_1}\left(\frac{p_F^{SS}}{p_H^{SS}}\right)^{-\eta_1} C_H^{SS},$$

which becomes

(var. 03/72 in SS) $$C_F^{SS} = \frac{\nu_1}{1-\nu_1} C_H^{SS}$$

because $p_H^{SS} = p_F^{SS}$.

From eqn. 07/72, we get

(var. 02/72 in SS) $$C^{SS} = \frac{C_H^{SS}}{1-\nu_1}$$

because $p_H^{SS} = 1$.

From eqn. 01/72, we get

(var. 14/72 in SS) $$\mathrm{MU}_C^{SS} = (1-\beta h)\left[(1-h)\, C^{SS}\right]^{-\sigma}.$$

From eqn. 11/72, we get

(var. 48/72 in SS) $$f_1^{SS} = \frac{w_1^{SS}\left(w_2^{SS}\right)^{\epsilon_w} L_2^{SS}}{\left[1-\theta_w \Lambda_{0,1}^{SS}\left(\pi_C^{SS}\right)^{\epsilon_w(1-\gamma_w)}\right]}.$$

From eqn. 12/72, we get

(var. 49/72 in SS) $$f_2^{SS} = \frac{\left(w_2^{SS}\right)^{\epsilon_w} L_2^{SS}}{\left[1-\theta_w \Lambda_{0,1}^{SS}\left(\pi_C^{SS}\right)^{(1-\epsilon_w)(\gamma_w-1)}\right]}.$$

From eqn. 30/72, we get

(var. 61/72 in SS) $$x_1^{SS} = \frac{p_2^{SS}\left(p_H^{SS}\right)^{\epsilon_p} Y^{SS}}{1-\theta_p \Lambda_{0,1}^{SS}}.$$

From eqn. 31/72, we get

(var. 62/72 in SS) $$x_2^{SS} = \frac{\left(p_H^{SS}\right)^{\epsilon_p} Y^{SS}}{1-\theta_p \Lambda_{0,1}}.$$

From eqn. 19/72, we get

(var. 46/72 in SS) $$\left(\bar{b}_H^{Pvt}\right)^{SS} = \frac{\psi p_k^{SS} \hat{I}^{SS}}{(1-\kappa)\left(\bar{Q}_H^{Pvt}\right)^{SS}}.$$

From eqn. 61/72, we get

(var. 56/72 in SS) $$s^{SS} = \left(Q_H^{Pvt}\right)^{SS} \left(b_H^{Pvt}\left(ma\right)\right)^{SS} + \left(Q_H^{Gov}\right)^{SS} \left(b_H^{Gov}\left(ma\right)\right)^{SS}.$$

From eqn. 26/72, we get

(var. 45/72 in SS) $$\left(b_{H,FC}^{Pvt}\right)^{SS} = \upsilon_3 \left(\frac{\left(Q_F^{Pvt}\right)^{SS}}{\left(\bar{Q}_H^{Pvt}\right)^{SS}}\right)^{\eta_3} \left(\bar{b}_H^{Pvt}\right)^{SS}.$$

From eqn. 27/72, we get

(var. 42/72 in SS) $$\left(b_H^{Pvt}\right)^{SS} = \left(1-\upsilon_3\right) \left(\frac{\left(Q_H^{Pvt}\right)^{SS}}{\left(\bar{Q}_H^{Pvt}\right)^{SS}}\right)^{\eta_3} \left(\bar{b}_H^{Pvt}\right)^{SS}.$$

From eqn. 49/72, we get

(var. 36/72 in SS) $$\left(b^{Gov}\left(fi\right)\right)^{SS} = \frac{1}{1-\nu_5} \left(b_H^{Gov}\left(fi\right)\right)^{SS} \left(\frac{\left(Q_H^{Gov}\right)^{SS}}{\left(Q^{Gov}\right)^{SS}}\right)^{\eta_5}.$$

From eqn. 50/72, we get

(var. 37/72 in SS) $$\left(b_F^{Gov}\left(fi\right)\right)^{SS} = \nu_5 \left(\frac{\left(Q_F^{Gov}\right)^{SS}}{\left(Q^{Gov}\right)^{SS}}\right)^{-\eta_5} \left(b^{Gov}\left(fi\right)\right)^{SS}.$$

From eqn. 20/72, we get

(var. 43/72 in SS) $$\left(b_H^{Pvt}\left(fi\right)\right)^{SS} = \left(b_H^{Pvt}\right)^{SS} - \left(b_H^{Pvt}\left(ma\right)\right)^{SS}.$$

From eqn. 47/72, we get

(var. 40/72 in SS) $$\left(b^{Pvt}\left(fi\right)\right)^{SS} = \frac{1}{1-\nu_4} \left(b_H^{Pvt}\left(fi\right)\right)^{SS} \left(\frac{\left(Q_H^{Pvt}\right)^{SS}}{\left(Q^{Pvt}\right)^{SS}}\right)^{\eta_4}.$$

From eqn. 48/72, we get

(var. 41/72 in SS) $$\left(b_F^{Pvt}\left(fi\right)\right)^{SS} = \nu_4 \left(\frac{\left(Q_F^{Pvt}\right)^{SS}}{\left(Q^{Pvt}\right)^{SS}}\right)^{-\eta_4} \left(b^{Pvt}\left(fi\right)\right)^{SS}.$$

From eqn. 54/72, we get

(var. 23/72 in SS) $$\left(R^{Gov}\right)^{SS} = \left(R_H^{Gov}\right)^{SS} \frac{\left(Q_H^{Gov}\right)^{SS}\left(b_H^{Gov}\left(fi\right)\right)^{SS}}{\left(Q^{Gov}\right)^{SS}\left(b^{Gov}\left(fi\right)\right)^{SS}} + \left(R_F^{Gov}\right)^{SS} \frac{\left(Q_F^{Gov}\right)^{SS}\left(b_F^{Gov}\left(fi\right)\right)^{SS}}{\left(Q^{Gov}\right)^{SS}\left(b^{Gov}\left(fi\right)\right)^{SS}}.$$

From eqn. 53/72, we get

(var. 27/72 in SS) $$\left(R^{Pvt}\right)^{SS} = \left(R_H^{Pvt}\right)^{SS} \frac{\left(Q_H^{Pvt}\right)^{SS}\left(b_H^{Pvt}\left(fi\right)\right)^{SS}}{\left(Q^{Pvt}\right)^{SS}\left(b^{Pvt}\left(fi\right)\right)^{SS}} + \left(R_F^{Pvt}\right)^{SS} \frac{\left(Q_F^{Pvt}\right)^{SS}\left(b_F^{Pvt}\left(fi\right)\right)^{SS}}{\left(Q^{Pvt}\right)^{SS}\left(b^{Pvt}\left(fi\right)\right)^{SS}}.$$

From eqn. 62/72, we get
(var. 64/72 in SS)
$$\Pi_{ma,t}^{\text{real}} = \left(R_H^{Pvt}\right)^{SS} \left(Q_H^{Pvt}\right)^{SS} \left(b_H^{Pvt}\left(ma\right)\right)^{SS} + \left(R_H^{Gov}\right)^{SS} \left(Q_H^{Gov}\right)^{SS} \left(b_H^{Gov}\left(ma\right)\right)^{SS} - \left(R^S\right)^{SS} s^{SS}.$$

From eqn. 68/72, we get

$$T^{SS} = G^{SS} + \left(\bar{b}_H^{Gov}\right)^{SS} \left[\left(\pi_C^{SS}\right)^{-1} - \left(Q_H^{Gov}\right)^{SS} \left(1 - \kappa \left(\pi_C^{SS}\right)^{-1}\right)\right] - \left(\Pi_{ma,t}^{\text{real}}\right)^{SS},$$

which simplifies to

(var. 31/72 in the SS) $$T^{SS} = G^{SS} + \left(\bar{b}_H^{Gov}\right)^{SS} \left[1 - \left(Q_H^{Gov}\right)^{SS} (1 - \kappa)\right] - \left(\Pi_{ma,t}^{\text{real}}\right)^{SS}.$$

From eqn. 38/72, the balance sheet of an FI is

$$\text{Assets} = d^{SS} + n^{SS},$$

where

$$\text{Assets} = \left(Q^{Pvt}\right)^{SS} \left(b^{Pvt}(fi)\right)^{SS} + \left(Q^{Gov}\right)^{SS} \left(b^{Gov}(fi)\right)^{SS} + s^{SS}.$$

We define the leverage ratio (LR) for an FI to be the ratio of its assets to networth and match it to data:

$$\text{LR} = \frac{\text{Assets}}{n^{SS}},$$

which implies

(var. 50/72 in SS) $$n^{SS} = \frac{\text{Assets}}{\text{Target LR}}.$$

From

$$\text{Assets} = d^{SS} + n^{SS},$$

(var. 47/72 in SS) $$d^{SS} = \text{Assets} - n^{SS}.$$

From eqn. 40/72

(var. 72/72 in SS) $$\phi^{SS} = \frac{\left(Q^{Pvt}\right)^{SS} \left(b^{Pvt}(fi)\right)^{SS} + \Delta \left(Q^{Gov}\right)^{SS} \left(b^{Gov}(fi)\right)^{SS}}{n^{SS}}.$$

In the SS,

(var. 68/72 in SS) $$\lambda_2^{SS} = 0.$$

Next, we use eqns. 41/72, 44/72 and 45/72 to solve for λ_1^{SS}, θ^{SS} and Ω^{SS}. The SS versions of the three equations are:

(eqn. 41/72) $$\Omega^{SS} \beta \sigma^{Pvt} = \frac{\lambda_1^{SS}}{1 + \lambda_1^{SS}} \theta^{SS},$$

(eqn 44/72) $$\Omega^{SS} = 1 - \vartheta + \vartheta \theta^{SS} \phi^{SS}$$

and

(eqn. 45/72) $$\theta^{SS} \phi^{SS} = \left(1 + \lambda_1^{SS}\right) \Omega^{SS}.$$

From eqn. 45/72:

$$1 + \lambda_1^{SS} = \frac{\theta^{SS} \phi^{SS}}{\Omega^{SS}}$$

and

$$\lambda_1^{SS} = \frac{\theta^{SS}\phi^{SS}}{\Omega^{SS}} - 1 = \frac{\theta^{SS}\phi^{SS} - \Omega^{SS}}{\Omega^{SS}}.$$

Take the ratio of the two:

$$\frac{\lambda_1^{SS}}{1+\lambda_1^{SS}} = \frac{\theta^{SS}\phi^{SS} - \Omega^{SS}}{\Omega^{SS}} \frac{\Omega^{SS}}{\theta^{SS}\phi^{SS}} = \frac{\theta^{SS}\phi^{SS} - \Omega^{SS}}{\theta^{SS}\phi^{SS}}.$$

Substitute the last equation into eqn. 41/72

$$\Omega^{SS}\beta\sigma^{Pvt} = \frac{\theta^{SS}\phi^{SS} - \Omega^{SS}}{\theta^{SS}\phi^{SS}}\theta^{SS}$$

$\Longleftrightarrow$

$$\Omega^{SS}\beta\sigma^{Pvt} = \frac{\theta^{SS}\phi^{SS} - \Omega^{SS}}{\phi^{SS}}$$

$\Longleftrightarrow$

$$\Omega^{SS}\beta\phi^{SS}\sigma^{Pvt} = \theta^{SS}\phi^{SS} - \Omega^{SS}$$

$\Longleftrightarrow$

$$\Omega^{SS}\beta\phi^{SS}\sigma^{Pvt} + \Omega^{SS} = \theta^{SS}\phi^{SS}$$

$\Longleftrightarrow$

$$\Omega^{SS}\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right) = \theta^{SS}\phi^{SS}.$$

Substitute eqn. 44/72 into the last equation

$$\left(1 - \vartheta + \vartheta\theta^{SS}\phi^{SS}\right)\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right) = \theta^{SS}\phi^{SS}$$

$\Longleftrightarrow$

$$(1-\vartheta)\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right) + \vartheta\theta^{SS}\phi^{SS}\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right) = \theta^{SS}\phi^{SS}$$

$\Longleftrightarrow$

$$(1-\vartheta)\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right) = \theta^{SS}\phi^{SS}\left[1 - \vartheta\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right)\right]$$

$\Longleftrightarrow$

(var. 66/72 in SS)

$$\theta^{SS} = \frac{(1-\vartheta)\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right)}{\phi^{SS}\left[1 - \vartheta\left(\beta\phi^{SS}\sigma^{Pvt} + 1\right)\right]}.$$

From eqn. 44/72:

(var. 65/72 in SS)

$$\Omega^{SS} = 1 - \vartheta + \vartheta\theta^{SS}\phi^{SS}.$$

And from eqn 45/72:

(eqn. 45/72)

$$\frac{\theta^{SS}\phi^{SS}}{\Omega^{SS}} = 1 + \lambda_1^{SS}$$

$\Longleftrightarrow$

(var. 67/72 in SS)

$$\lambda_1^{SS} = \frac{\theta^{SS}\phi^{SS}}{\Omega^{SS}} - 1.$$

This completes our description of the steady state of the model.

3 Additional Tables for Section III

Table 1: Standard Deviation and Persistence of Shocks

Parameter	Symbol	Value
SD of shock to MA's pvt. bond holdings	s_1	0.01
SD of shock to MA's govt. bond holdings	s_2	0.01
SD of shock to technology	s_A	0.0065
SD of shock to government expenditure	s_G	0.01
SD of shock to Taylor-rule policy rate	s_R	0.0025
SD of preference shock	s_Z	0.01
SD of liquidity shock	s_θ	0.04
Degree of persistence in the MA's purchase of pvt. bonds	ρ_1	0.8
Degree of persistence in the MA's purchase of govt. bonds	ρ_2	0.8
Degree of persistence of technology shock	ρ_A	0.95
Degree of persistence of government expenditure shock	ρ_G	0.95
Degree of persistence of the Taylor-rule rate	ρ_R	0.8
Degree of persistence of preference shock	ρ_Z	0.8
Degree of persistence of a liquidity shock	ρ_θ	0.98

Table 2: Calibrated Parameters or Targets

Parameter	Symbol	Value/Target
SS government expenditure	$\bar{G}$	0.2213
Target: Govt. expenditure to GDP ratio		
Neutral policy rate	R^{Pol}	1.005
SS real govt. bond holdings of MA	$b_H^{Gov}\,(ma)$	0.0334
Target: Bond value to GDP ratio		
SS real pvt. bond holdings of MA	$b_H^{Pvt}\,(ma)$	0
Target: Bond value to GDP ratio		
Fixed real govt. debt	$\bar{b}_H^{Gov}$	0.4054
Target: Govt. debt to GDP ratio		
Habit parameter	h	0.7
Govt. bond recovery	Δ	1/3
Target: Excess return, govt. vs pvt. bonds		
Parameter on endogenous component of QE (pvt. bonds)	Ψ_1	-2
Parameter on endogenous component of QE (govt. bonds)	Ψ_2	-2
Share of capital in output	α	0.33
Discount factor	β	1/1.005
Backward price indexation parameter	γ_p	0
Backward wage indexation parameter	γ_w	0
Quarterly depreciation of capital in the steady state	δ_0	0.025
Coefficient of linear term in depreciation function	δ_1	1
Target: u^{SS}		
Coefficient of squared term in depreciation function	δ_2	0.01
Elasticity of substitution between any two retail goods	ϵ_p	11
Elasticity of substitution between any two labor types	ϵ_w	11
Fraction of pvt. bonds in total bonds held by FI's	θ	0.0075
Target: Pvt. bond excess return		
Degree of price rigidity	θ_p	0.75
Degree of wage rigidity	θ_w	0.75
Fraction of FI's that survive each period	ϑ	0.95
Depreciation rate of coupon payment on bonds	κ	40
Target: Bond duration (in quarters)		
Adjustment cost of investment parameter	κ_I	2
Inverse of the intertemporal elasticity of substitution	σ	1.38
Parameter on output growth in the Taylor rule	ϕ_y	0.25
Parameter on inflation gap in the Taylor rule	ϕ_π	1.25
Inverse Frisch elasticity	φ	1
Minimum share of borrowing to finance investment	ψ	0.7582
Lumpsum transfer from household to entering FI's	χ	4
Target: Leverage ratio		
Weight on disutility of work	ω	1
Target: L_1^{SS}		

4 Additional Figures for Section IV

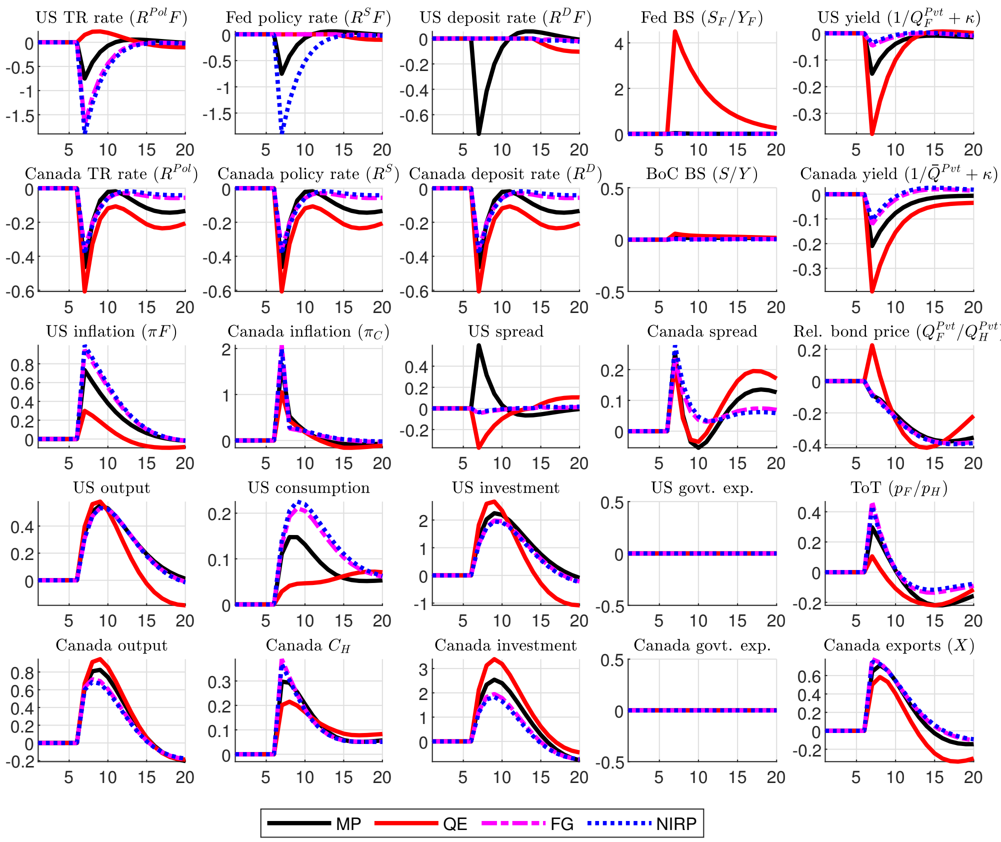

Figure 1: Exogenous monetary-policy shocks to US and Canadian economies

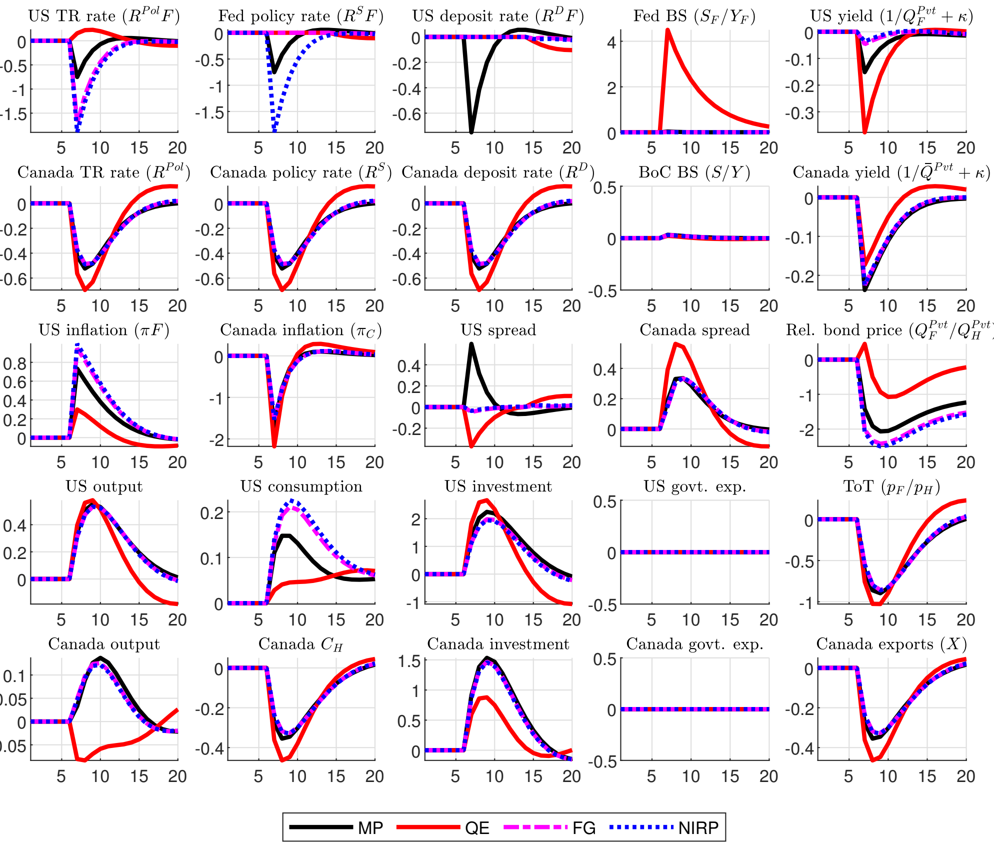

Figure 2: Exogenous monetary-policy shocks to US economy when there is no trade in international assets

5 Additional Figures for Section V

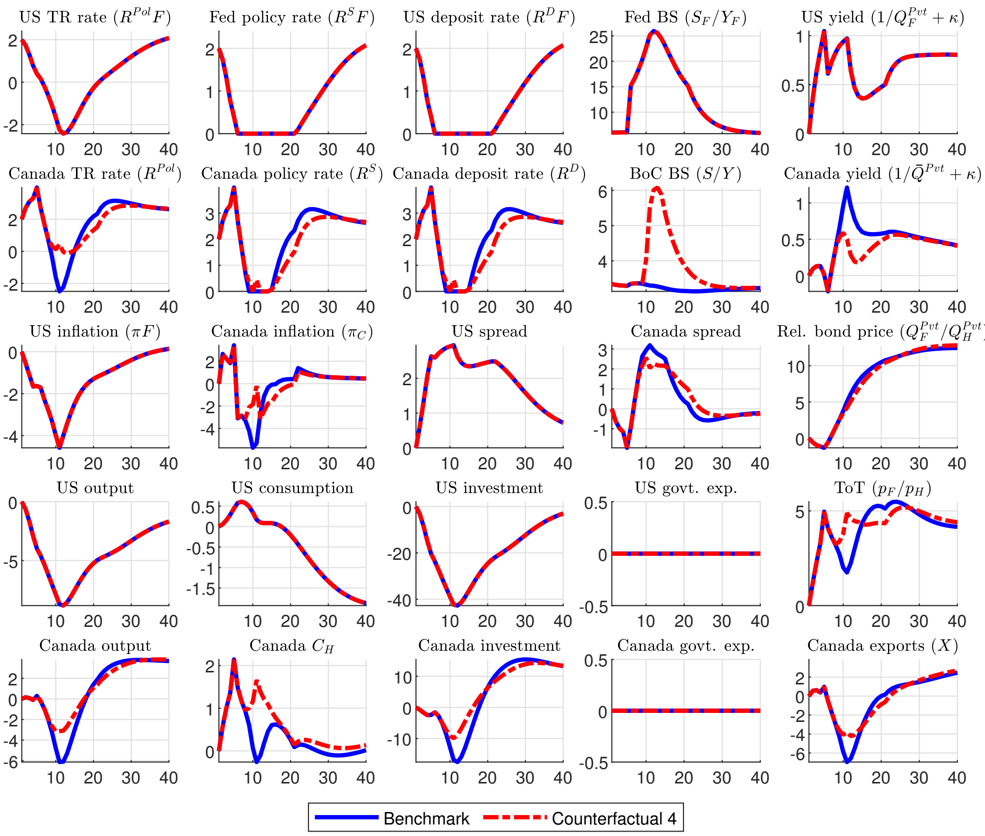

Figure 3: Benchmark vs. Bank of Canada doing QE (Counterfactual 4)

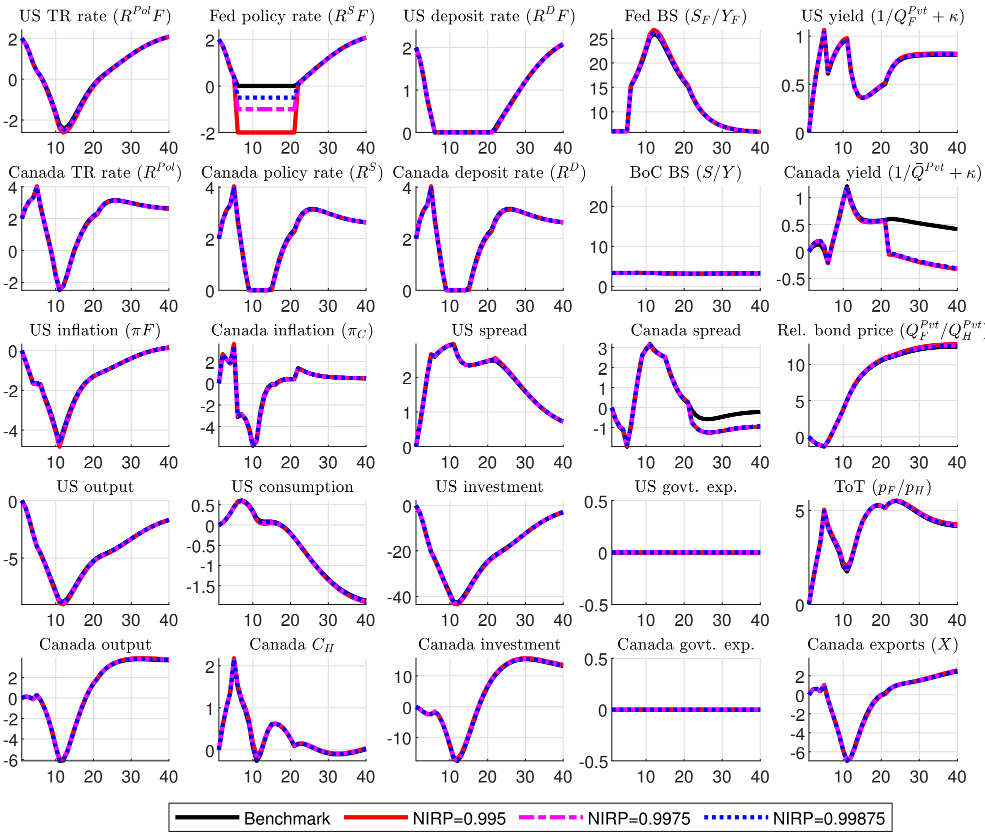

Figure 4: Negative Policy Interest Rates of -0.5%, -1% and -2% (NIRP Counterfactual 1)

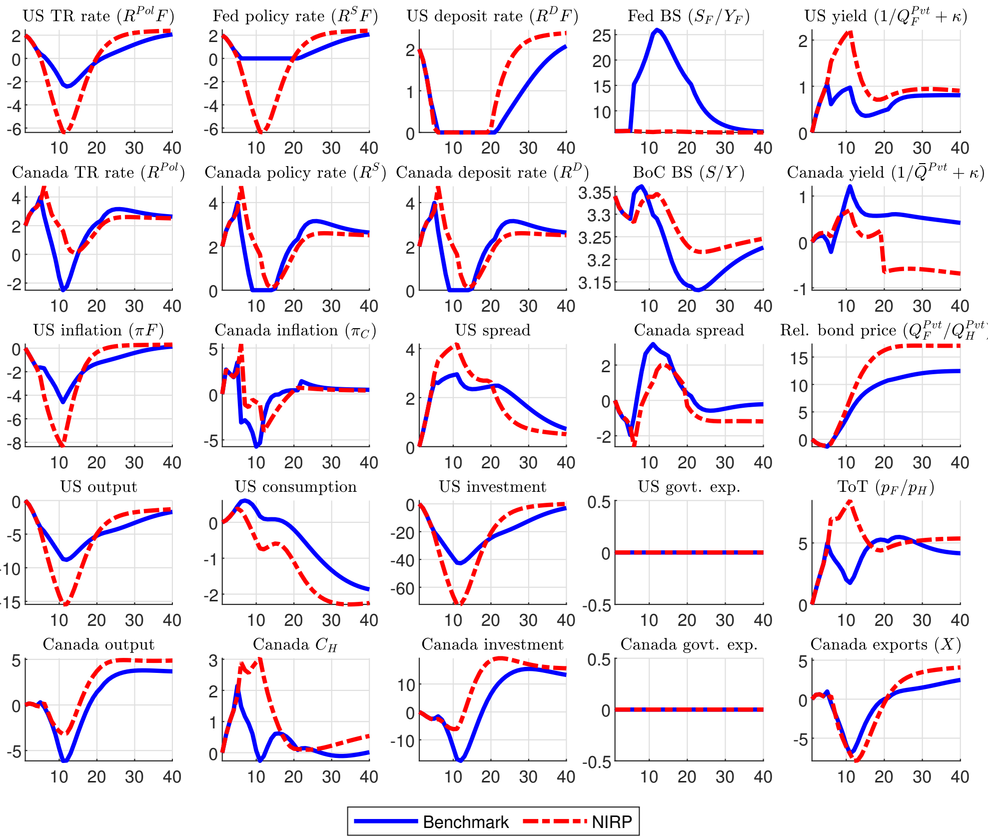

Figure 5: No Lower Bound on Policy Rate and No QE (NIRP Counterfactual 2)

6 Sensitivity Analysis

In our sensitivity analysis, we focus on the ten openness parameters that are new to our model compared to the model in SW. Among the ten, five, namely η_1, η_2, η_3, η_4 and η_5, are elasticity parameters and the other five, namely ν_1, ν_2, ν_3, ν_4 and ν_5, are share parameters, which we may also call openness parameters or home-bias parameters.

6.1 Elasticity Parameters

In the benchmark scenario, following Gali and Monacelli (2016)[3], we assumed that all elasticity parameters were equal to one. We now change one of these parameters at a time and compare the results with the benchmark. For the parameter that we change, we try two alternative values: 0.5 and 2.0.

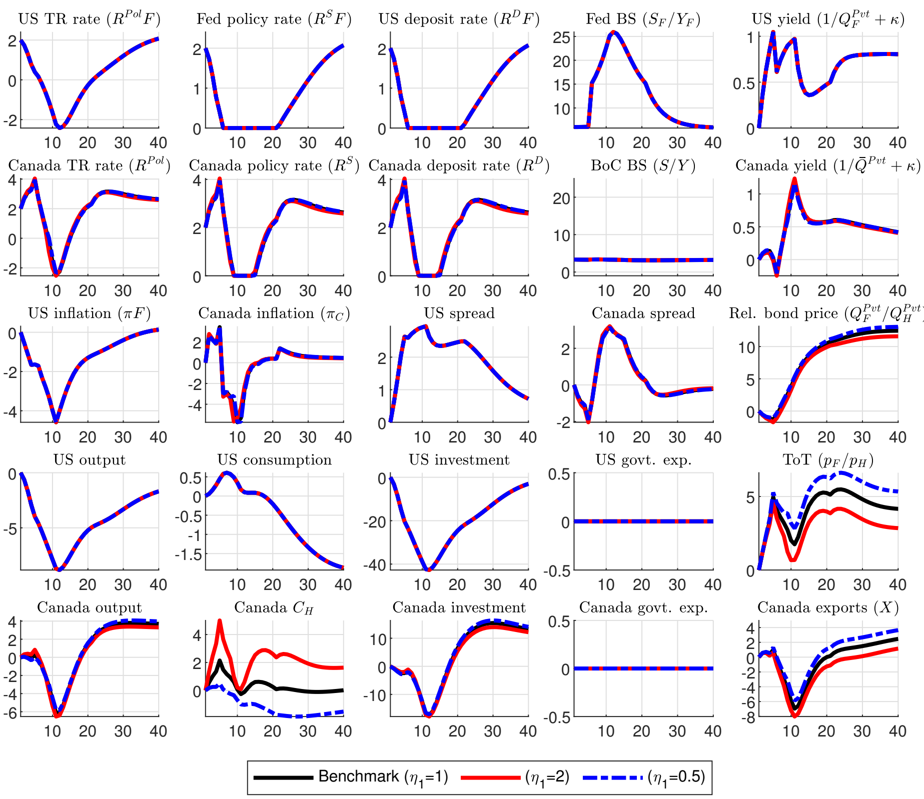

Figure 6: Sensitivity of spillovers to changes in η_1

Parameter η_1 is the elasticity of C_F/C_H with respect to p_F/p_H. To see this, divide eqn. 08/72 by eqn. 07/72 and ignore time subscripts to get:

$$\frac{C_F}{C_H} = \frac{\nu_1}{1-\nu_1}\left(\frac{p_F}{p_H}\right)^{-\eta_1}.$$

When we change η_1 (see Figure 6), the main effects are on equilibrium terms of trade (panel [4,5]), consumption of home good (panel [5,2]) and exports (panel [5,5]). For example, when η_1 is higher, i.e. equal to 2, the consumption of home good increases by more (panel [5,2]) and the equilibrium terms of trade (panel

[3] Gali, Jordi and Monacelli, Tommaso. 2016. "Understanding the Gains from Wage Flexibility: The Exchange Rate Connection." *American Economic Review*, 106(12), 3829–3868.

[4,5]) deviates less from their steady-state path. The effect on Canada's bond yield (panel [2,5]), and hence on investment (panel [5,3]), is negligible, and so is the effect on Canada's output (panel [5,1]).

Parameter η_2 is the elasticity of exports with respect to p_H/p_F :

$$X = \nu_2 \left(\frac{p_H}{p_F}\right)^{-\eta_2} Y_F.$$

When we change η_2 (see Figure 7), the main effects are on equilibrium terms of trade (panel [4,5]), consumption of home good (panel [5,2]) and exports (panel [5,5]). For example, when η_2 is higher, exports change by more. The effects on Canada's relative bond yield (panel [3,5]), investment (panel [5,3]) and output (panel [5,1]) are slightly more pronounced when we change η_2 (Figure 7) compared to when we change η_1 (Figure 6).

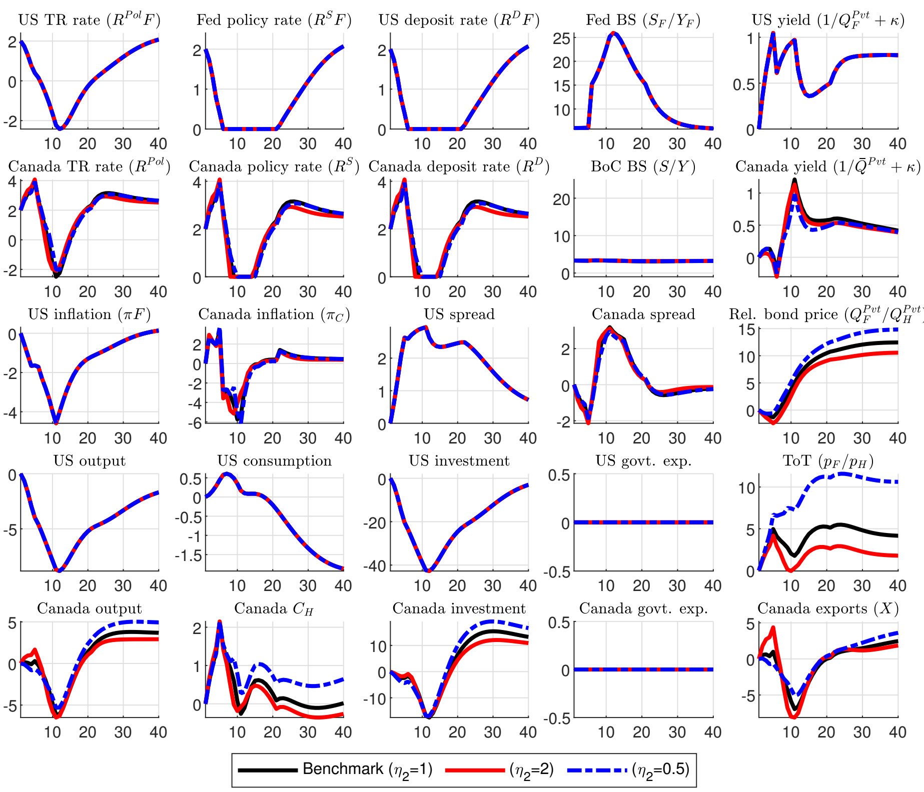

Figure 7: Sensitivity of spillovers to changes in η_2

Parameter η_3 determines the response of the wholesale firm's relative supply of foreign-currency bonds to changes in their relative price. To see this, combine eqns. 26/72 and 27/72, and ignore time subscripts to get:

$$\frac{b_{H,FC}^{Pvt}}{b_H^{Pvt}} = \frac{\upsilon_3}{1-\upsilon_3}\left(\frac{Q_F^{Pvt}}{Q_H^{Pvt}}\right)^{\eta_3}.$$

When η_3 is higher (see Figure 8, top panel), the equilibrium relative bond price (panel [3,5]) changes by less. But the larger change in relative bond quantities affects the terms of trade more (panel [4,5]), which is also reflected in larger changes in the consumption of home good (panel [5,2]) and exports (panel [5,5]). There are smaller effects on Canada's bond yield (panel [2,5]), investment (panel [5,3]) and output (panel [5,1]).

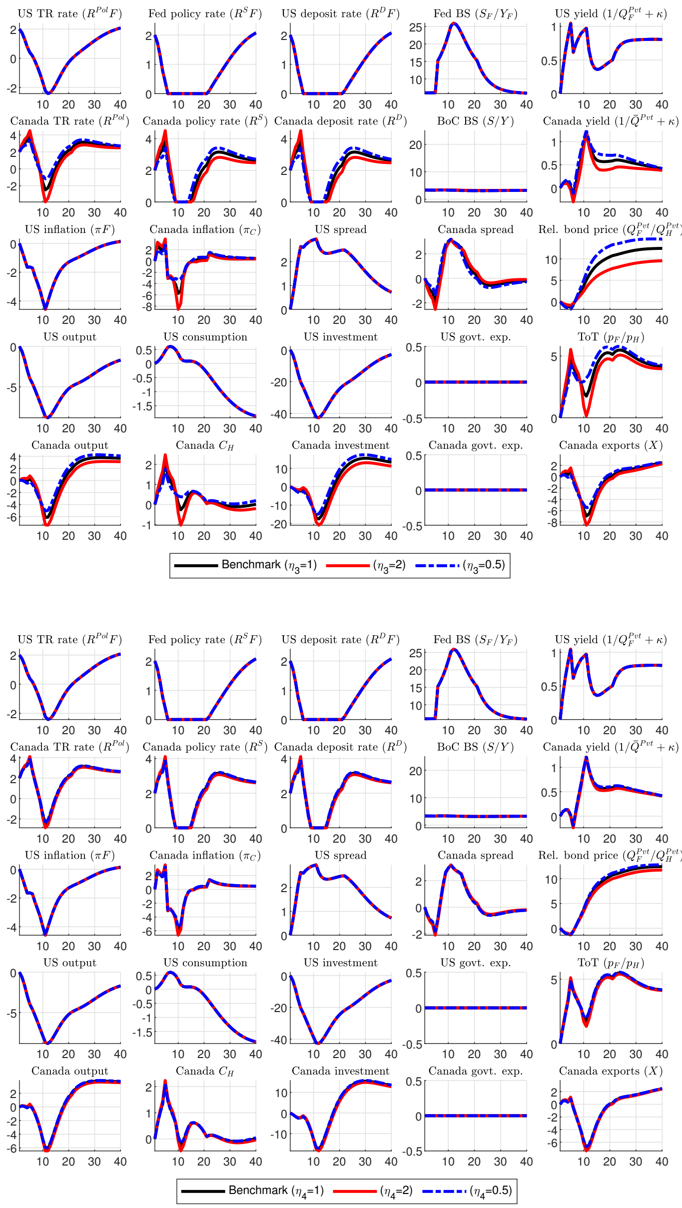

Figure 8: Sensitivity of spillovers to changes in η_3 (top) and η_4 (bottom)

Parameter η_4 determines the response of the financial intermediaries' relative demand for foreign private bonds (relative to home private bonds) to their relative price. Taking the ratio of eqns. 48/72 and 47/72, and ignoring time subscripts, we get:

$$\frac{b_F^{Pvt}(fi)}{b_H^{Pvt}(fi)} = \frac{\nu_4}{1-\nu_4}\left(\frac{Q_F^{Pvt}}{Q_H^{Pvt}}\right)^{-\eta_4}.$$

In our baseline calibration, the financial intermediaries hold only 9% of their private bond holdings in the form of foreign bonds, i.e. $\nu_4 = 0.09$. Because the foreign bond share is so small, we see in Figure 8 (bottom panel) that changing the value of η_4 has hardly any effect on spillovers.

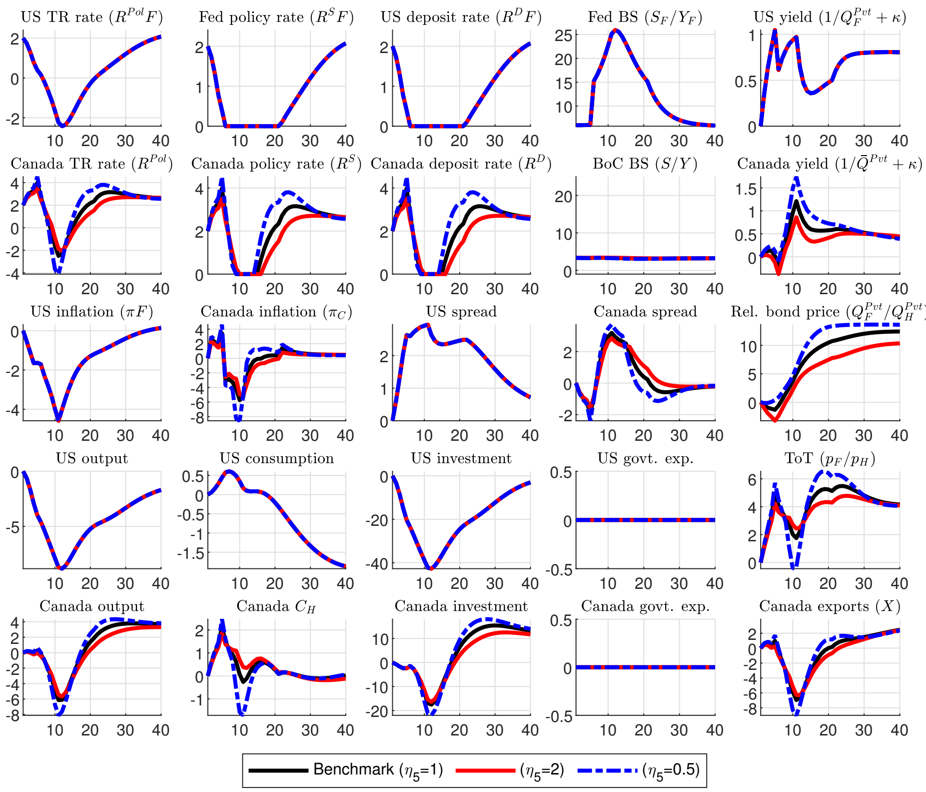

Figure 9: Sensitivity of spillovers to changes in η_5

Parameter η_5 determines the response of the financial intermediaries' demand for foreign government bonds (relative to home government bonds) to their relative price. Take the ratio of eqns. 50/72 and 49/72, and ignore the time subscripts to get:

$$\frac{b_F^{Gov}(fi)}{b_H^{Gov}(fi)} = \frac{\nu_5}{1-\nu_5}\left(\frac{Q_{F,t}^{Gov}}{Q_{H,t}^{Gov}}\right)^{-\eta_5}.$$

The effects of changing η_5 (see Figure 9) are similar to those of changing η_3 (see Figure 8, top panel) but in some ways in the opposite direction. For example, when η_5 is higher, the variations in Canada's bond yield (panel [2,5]), terms of trade (panel [4,5]), consumption (panel [5,2]), investment (panel [5,3]), exports (panel [5,5]) and output (panel [5,1]) are all milder.

Our broad conclusion from the sensitivity analysis of elasticity parameters is that changes in these parameters affect spillovers in predictable ways without overturning our broad qualitative results.

6.2 Openness Parameters

We use the observed shares of imports and exports in GDP to pin down ν_1 and ν_2. To test the sensitivity of our results to changes in these parameters, we consider a counterfactual scenario in which we reduce Canada's export-to-GDP and import-to-GDP ratios to half their observed values. We call it the *low-trade scenario* and report the values of openness parameters for this experiment in the second row of Table 3 (the benchmark parameter values are in the first row).[4]

[4]When we change ν_1 and ν_2, we need to adjust ν_4 to satisfy the balance-of-payment equilibrium condition.

Table 3: Benchmark and Sensitivity Values of Openness Parameters

Parameter→ Scenario↓	ν_1	ν_2	ν_3	ν_4	ν_5
Benchmark	0.62	0.05	0.48	0.09	0.43
Low-Trade	0.30	0.02	0.48	0.29	0.43
High Home Bias	0.62	0.05	0.32	0.00	0.00
Low Home Bias	0.62	0.05	0.99	0.00	0.82

In Figure 10, we compare the low-trade IRFs with the benchmark IRFs. As one would expect, if Canada were half as open as it actually was in 2006, the spillovers from the US would be much smaller. Canada's bond yield (panel [2,5]) would fluctuate much less and the volatility in GDP (panel [5,1]), home consumption (panel [5,2]), investment (panel [5,3]) and net exports (panel [5,5]) would all be much lower.

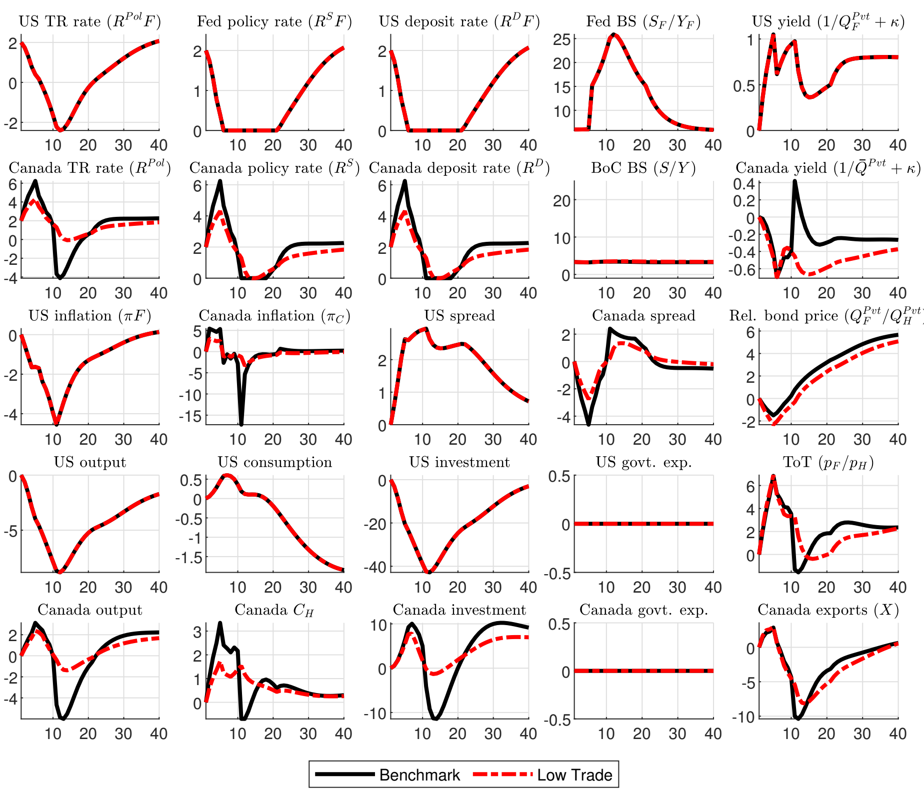

Figure 10: Sensitivity Analysis: Low-Trade Counterfactual Scenario

In the next two sensitivity experiments, we leave the export- and import-to-GDP ratios at their benchmark targets, i.e. we do not change parameters ν_1 and ν_2. Instead, we change the openness parameters on the financial side of the economy. In the first experiment, which we call the *high-home-bias scenario*, we set both ν_4 and ν_5 equal to zero (we cannot simultaneously set ν_3 equal to zero because with ν_1 and ν_2 fixed at their benchmark values, we adjust ν_3 to satisfy the balance of payments equation in the steady state). This effectively means that the home financial intermediaries do not want to hold foreign bonds (hence the name 'high-home-bias scenario'). Although $\nu_3 > 0$ because of the balance of payments equilibrium, its value is smaller than the benchmark value. We report the values of openness parameters for this experiment in the third row of Table 3 above. In the top panel of Figure 11, we compare the high-home-bias IRFs with the benchmark IRFs. The main difference is the relative bond price (panel [3,5]) which increases by more in the high-home-bias scenario. The GDP (panel [5,1]), consumption of home good (panel [5,2]), investment (panel [5,3]) and exports (panel [5,4]) IRFs look very similar except that in the high-home-bias scenario they fluctuate a little less. Because of the higher relative bond price (panel [3,5]), it is optimal for the wholesaler to issue more foreign bonds relative to home-currency bonds and the net foreign assets decrease by more (not shown in the figure).

In the second experiment, which we call the *low-home-bias scenario*, we set both ν_3 and ν_3 as high as possible without violating the balance of payment equilibrium condition and, at the same time, making sure that ν_4 is not negative. This effectively means that the home financial intermediaries hold a lot of foreign bonds (82% of total) and the wholesaler issues mostly foreign-currency bonds. We report the values of openness parameters for this experiment in the fourth row of Table 3 above. In the bottom panel of Figure 11, we compare the low-home-bias IRFs with the benchmark IRFs. The main difference is in the relative bond price (panel [3,5]), which decreases a lot more in the low-home-bias scenario. The terms of trade (panel [4,5]) fluctuates less and, as a result, so do the consumption of home good (panel [5,2]) and exports (panel [5,5]). Canada's bond yield (panel [2,5]) decreases a lot more (despite an initial small increase) and hence investment increases more (panel [5,3]).

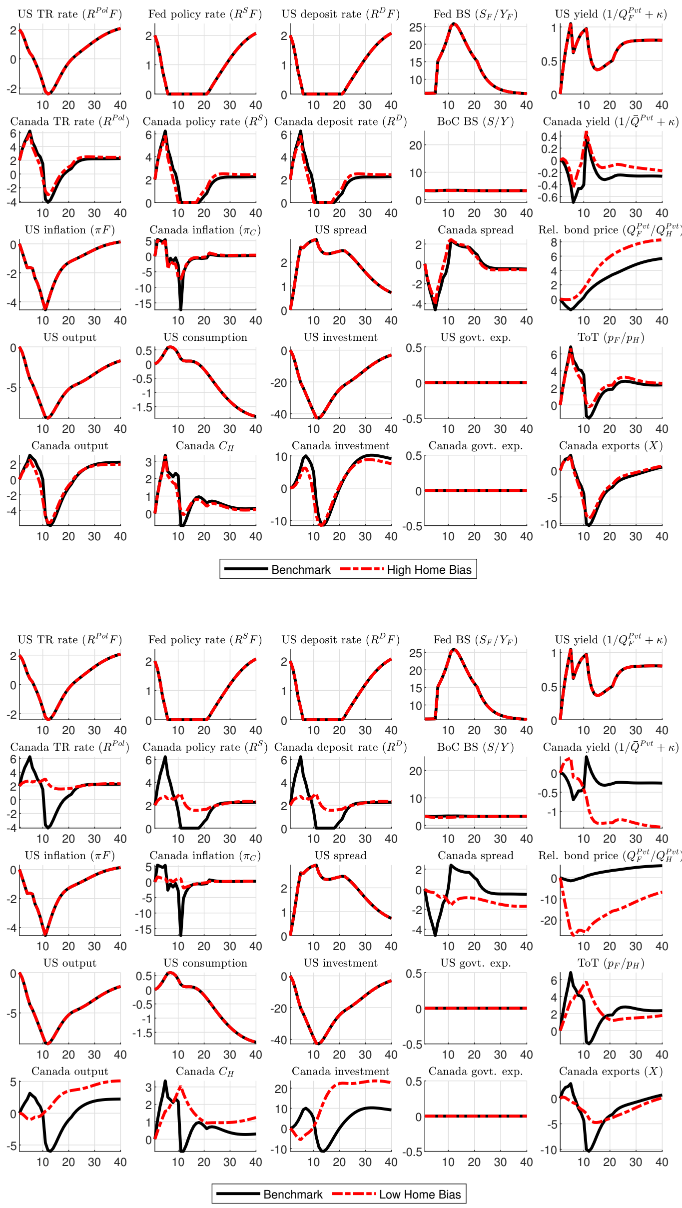

Figure 11: Sensitivity Analysis: High- (top panel) and Low-Home-Bias (bottom panel) Counterfactual Scenarios