# Divisia Monetary Aggregates, the Great Ratios, and Classical Money Demand Functions*

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**Abstract:**

King, Plosser, Stock, and Watson (1991) evaluate the empirical relevance of a class of real business cycle models with permanent productivity shocks by analyzing the stochastic trend properties of postwar U.S. macroeconomic data. They find a common stochastic trend in a three variable system that includes output, consumption, and investment, but the explanatory power of the common trend drops significantly when they add money balances and the nominal interest rate. In this paper we revisit the cointegration tests in the spirit of King *et al.* (1991), using improved monetary aggregates whose construction has been stimulated by the Barnett critique. We show that previous rejections of the balanced-growth hypothesis and classical money demand functions can be attributed to mis-measurement of the monetary aggregate.

*JEL classification*: C32, E52, E44.

*Keywords*: Divisia monetary aggregates; Balanced growth hypothesis; Money Demand.

# 1 Introduction

King, Plosser, Stock, and Watson (1991) evaluate the empirical relevance of a class of real business cycle models with permanent productivity shocks by analyzing the stochastic trend properties of postwar U.S. macroeconomic data. They find a common stochastic trend in a three variable system that includes output, consumption, and investment, but the explanatory power of the common trend drops significantly when they add money balances and the nominal interest rate. In this paper we revisit the cointegration tests in the spirit of King *et al.* (1991), using improved monetary aggregates whose construction has been stimulated by the Barnett critique. We show that previous rejections of the balanced-growth hypothesis and classical money demand functions can be attributed to monetary aggregation issues.

In doing so, we use the Federal Reserve's simple-sum monetary aggregates and the new Divisia monetary aggregates maintained within the Center of Financial Stability (CFS) program Advances in Monetary and Financial Measurement (AMFM), called CFS Divisia aggregates and documented in detail in Barnett *et al.* (2013). We make comparisons at the M1, M2M, M2, MZM, and ALL levels of monetary aggregation. Our sample extends from 1967:1 to 2011:3 that includes the increased volatility in money supply in the aftermath of the global financial crisis and the Great Recession.

The paper is organized as follows. Section 2 provides the theoretical background and considers empirical regularities relating to certain theoretical claims in the real business cycle literature and classical money demand literature. Section 3 discusses monetary aggregation issues. Section 4 presents the empirical results and provides a comparison between the simple-sum and Divisia methods of monetary aggregation. Section 5 addresses robustness issues and the final section concludes the paper.

# 2 Theoretical Foundations

Following King *et al.* (1991), let's consider the following simple real business cycle model. The single final good, $Y_t$, is produced via a constant-returns-to-scale Cobb-Douglas production function

$$Y_t = \lambda_t K_t^{1-\theta} L_t^{\theta} \tag{1}$$

where $K_t$ is the predetermined capital stock, chosen in period $t-1$, and $L_t$ is labor input in period $t$. Total factor productivity, $\lambda_t$, follows a logarithmic random walk

$$\log(\lambda_t) = \mu_\lambda + \log(\lambda_{t-1}) + \xi_t \tag{2}$$

where $\mu_\lambda$ represents the average productivity growth rate and $\xi_t$ is an independent and identically distributed process with mean zero and variance $\sigma^2$. In equation (2), $\mu_\lambda +$

$\log(\lambda_{t-1})$ represents the deterministic part of the productivity evolution and $\xi_t$ represents the stochastic innovations (or shocks).

Under the assumption that the intertemporal elasticity of substitution in consumption is constant and independent of the level of consumption, the basic neoclassical growth model with deterministic trends implies that the two great ratios — the log output-consumption ratio and the log output-investment ratio — are constant along the steady-state growth path, since the deterministic model's steady-state common growth rate is $\mu_\lambda/\theta$. With stochastic trends, however, there is a common stochastic trend $\log(\lambda_t)/\theta$ with a growth rate of $(\mu_\lambda + \xi_t)/\theta$, implying that the great ratios, $c_t - y_t$ and $i_t - y_t$, become stationary stochastic processes — see King *et al.* (1988) for more details.

These theoretical results can be formulated as testable hypotheses in a cointegration framework. Let $\boldsymbol{X}_t$ be the multivariate stochastic process consisting of the logarithms of real per capita consumption, investment, and output, $\boldsymbol{X}_t = [c_t, i_t, y_t]$. Each component of $\boldsymbol{X}_t$ is integrated of order one [or I(1) in the terminology of Engle and Granger (1987)] — since productivity evolves as a random walk process. The balanced growth implication of this growth model with stochastic trends is that the differences $c_t - y_t$ and $i_t - y_t$ will be I(0) variables. Thus, there should be two cointegrating vectors, $[1, 0, -1]$ and $[0, 1, -1]$.

If $\boldsymbol{X}_t$ is augmented to include real per capita money balances, $(m-p)_t$, and the nominal interest rate, $R_t$, that is, if $\boldsymbol{X}_t = [c_t, i_t, (m-p)_t, y_t, R_t]$, and if $(m-p)_t$ and $R_t$ are each integrated of order one, then according to the theory we would expect to find three cointegrating vectors — the two great ratios, $[1, 0, 0, -1, 0]$ and $[0, 1, 0, -1, 0]$, and the money demand relation, $[0, 0, 1, \beta_y, \beta_R]$. In fact, according to the theory we expect $\beta_y = -1$ and $\beta_R$ to be small and positive. These coefficients in the cointegrating vector imply a one-to-one positive relation between real money balances and real output and a small but negative relation between real balances and the nominal rate of interest.

King *et al.* (1991) find a common stochastic trend in the three variable system that includes output, consumption, and investment for U.S. macroeconomic data. But the explanatory power of the common trend drops significantly when they add money balances and the nominal interest rate.

# 3 Monetary Aggregation Matters

The measure of the money supply used by King *et al.* (1991) is the official simple-sum M2 monetary aggregate (consisting of currency, demand deposits, and savings deposits). In this regard, Barnett (1980) argues that official simple-sum monetary aggregates, constructed by the Federal Reserve, produce an internal inconsistency between the implicit aggregation theory and the theory relevant to the models and policy within which the resulting data are nested and used. That incoherence has been called the Barnett Critique [see, for example, Chrystal and MacDonald (1994) and Belongia and Ireland (2013)], with emphasis on the resulting inference and policy errors and the induced appearances of function instability. Barnett (1980) applied economic aggregation and index number theory to construct monetary aggregates consistent with Diewert's (1976) class of superlative quantity index numbers. Barnett's monetary aggregates are Törnqvist-Theil discrete time Divisia quantity indices, named after Francois Divisia, who first proposed the continuous time index in 1925 for aggregating over goods; Barnett (1980) proved how the formula could be extended to include monetary assets.

To provide some perspective on the simple-sum and Divisia methods of monetary aggregation, in Figures 1 and 2 we provide graphical representations of the simple-sum and Divisia monetary aggregates at the M2 and ALL levels of aggregation, respectively. In particular, we plot quarter-to-quarter growth rates, using data over the period from 1967:1 to 2011:3, and the recent CFS Divisia series, documented in detail by Barnett *et al.* (2013). We also report the correlation coefficients between each simple-sum monetary aggregate and its CFS Divisia counterpart (in growth rates), for the full sample as well as for a restricted sample that excludes the 1983 peak. Shaded areas represent recessions. Clearly, the correlations are higher in the restricted sample and the Divisia monetary aggregates are very different from the simple-sum aggregates. We will show that these differences are of economic importance when we investigate the existence of a long-run money demand relationship in the following section.

# 4 Empirical Evidence

In this section, we apply the Johansen (1988) maximum likelihood approach for estimating long-run equilibrium relations in multivariate vector autoregressive models. Our objective is to determine whether previous rejections of the balanced growth hypothesis and classical money demand theory can be attributed to mis-measurement of the monetary aggregate.

We start with a three-variable model containing real per capita money balances, $m - p$, real output $y$ on a per capita basis, and the opportunity cost of holding money, $R$. The monetary series used are the simple-sum and CFS Divisia monetary aggregates at the M1, M2M, M2, MZM and ALL levels of aggregation and are transformed to real per capita money balances using the appropriate GNP deflator. We use private GNP as the output series and the 3-month Treasury-bill rate, $R$, as the opportunity cost for each of the simple-sum and Divisia monetary aggregates. All data are transformed to natural logs with the exception of the T-bill rate. Moreover, based on augmented Dickey and Fuller (1981) tests, Kwiatkowski *et al.* (1992) trend stationarity tests, and Elliot *et al.* (1996) point optimal tests, we find that all series are I(1). These results are not reported here but are available on request.

According to the theory, in this system we expect to find one cointegrating vector, $[1, \beta_y, \beta_R]$, which corresponds to the long-run money demand function. In fact, according to the theory we expect $\beta_y = -1$ and $\beta_R > 0$. That is, real balances should be positively related to income and negatively related to the opportunity cost of holding money. Table 1 reports the results of the Johansen maximum likelihood cointegration tests on a VAR with a lag length selected by the SIC. We also report tail areas of residual misspecification tests. In particular, J-B is the Jarque-Bera (1980) test statistic distributed as a $\chi^2(2)$ under the null hypothesis of normality and LM is a multivariate test statistic distributed as a $\chi^2$ with $K^2$ degrees of freedom (where $K$ is the number of endogenous variables in the VAR) under the null hypothesis of no serial correlation.

Two test statistics are used to test for the number of cointegrating vectors, the trace ($\lambda_{\text{trace}}$) and maximum eigenvalue ($\lambda_{\max}$) test statistics, and are presented in Table 1. In the trace test the null hypothesis that there are at most $r$ cointegrating vectors is tested against a general alternative whereas in the maximum eigenvalue test the alternative is explicit. Using 99% critical values, we see that the $\lambda_{\text{trace}}$ and $\lambda_{\max}$ test statistics provide evidence of one cointegrating relation in the cases of the Sum M1 aggregate and the Divisia aggregates at the M1, M2M, M2, and MZM levels of aggregation. In Table 2, the restriction $\beta = [1, -1, \beta_R]$ that identifies the money demand function is rejected for the Sum M1 monetary aggregate with a $p$-value of 0.000 and for the Divisia M1 aggregate with a $p$-value of 0.001. This restriction cannot be rejected for the Divisia aggregates at the M2M, M2, and MZM levels with $p$-values of 0.091, 0.239, and 0.043, respectively. Moreover, we get $\beta_R > 0$ with all the aggregates, consistent with what is expected according to the theory.

We now turn to the multivariate stochastic process, $\boldsymbol{X}_t = [c_t, i_t, (m-p)_t, y_t, R_t]$ that includes five variables. The variables $m - p$, $y$, and $R$ are defined as in the three-variable system and $c$ and $i$ are logged real per capita personal consumption expenditure and private fixed investment, respectively. According to the theory, in this system we expect to find three cointegrating vectors, $\beta_1 = [1, 0, 0, -1, 0]$ and $\beta_2 = [0, 1, 0, -1, 0]$ that correspond to the consumption-output and investment-output great ratios and $\beta_3 = [0, 0, 1, -1, \beta_R]$, the money demand function. The results of the Johansen maximum likelihood cointegration tests are reported in Table 3. According to the $\lambda_{\text{trace}}$ and $\lambda_{\max}$ test statistics at the 99% confidence level, we cannot reject the null hypothesis of one cointegrating vector with the Sum M1, Sum M2M, Sum MZM, and Sum ALL monetary aggregates. We cannot reject the null of two cointegrating vectors with the Divisia monetary aggregates at the M1 and M2M levels. Finally, the null of three cointegrating relations cannot be rejected in the case of the Divisia aggregates at the M2, MZM, and ALL levels of aggregation.

The next step is to identify the cointegrating vectors in the systems for which the Johansen maximum likelihood cointegration test provided evidence of one or more cointegrating vectors. Clearly, the evidence in support of one or two cointegrating vectors does not provide any direction as to which one of the three vectors expected by economic theory are picked up by the Johansen procedure. In Table 4, we test the over-identifying restrictions on the corresponding VAR for the existence of each of the three cointegrating vectors separately and provide the respective probabilities in the last column. The restrictions that identify the consumption-output and investment-output great ratios and the money demand function are rejected for all the simple-sum monetary aggregates and the Divisia aggregates at the M1 and ALL levels of aggregation.

With the Divisia aggregates at the M2M, M2, and MZM levels of aggregation, we cannot reject the consumption-output great ratio and the money demand function. The $i-y$ ratio, nonetheless is not identified. Moreover, all the coefficients on the opportunity cost variable have the correct sign. In fact, in the case of the Divisia M2M, M2, and MZM aggregates we also cannot reject the joint restriction that identifies both the consumption-output ratio and the money demand function with $p$-values of 0.243, 0.192, and 0.054, respectively. Thus, our data provide evidence that simultaneously identifies the consumption-output great ratio and the money demand function when the Divisia monetary aggregates are used at the M2M, M2, and MZM levels of aggregation. We also identify the consumption-output great ratio and the money demand function when the Divisia MZM aggregate is used.

It is to be noted that in this paper we do not report evidence using the St. Louis Fed's Divisia monetary aggregates, called MSI (monetary services indices), the new vintage of which is documented in Anderson and Jones (2011). In fact, we get qualitatively similar but weaker results when we use the MSI Divisia aggregates instead of the CFS Divisia monetary aggregates. These results are available on request. See also Serletis *et al.* (2013) for a comparison among the simple-sum, CFS Divisia, and MSI Divisia monetary aggregates at the M1, M2M, M2, MZM, and ALL levels of monetary aggregation.

# 5 Robustness

Our results for investment provide little support for the balanced growth hypothesis in our updated data set, in contrast to the original work by King *et al.* (1991). In this regard, Whelan (2005), building on earlier work by Whelan (2003), argues this could reflect the fact that investment-specific technological change over the past 20 years has led to a noticeable decline in the relative price of investment, which has implied that real investment has grown at a sharply faster rate than real consumption (or the output aggregate) even though nominal shares of consumption and investment in output have remained fairly stable. If this is the case, the investment relative price decline is reflected in the nominal amounts of investment, consumption and output, but not in the real prices as the process of deflating the variables with the same average price index alters the relative time series dynamics of the variables. In fact, Whelan (2005) puts forward an alternative balanced growth hypothesis, which is that the ratio of nominal consumption to nominal investment is stationary. He tests this hypothesis and presents evidence that this hypothesis is consistent with U.S. macroeconomic data.

To investigate the robustness of our results and test Whelan's (2005) alternative balanced growth hypothesis, we rerun our five-variable system, using nominal consumption, investment, output, and money balances. Imposing the restriction $\beta_1 = [1, -1, 0, 0, 0]$ that identifies Whelan's (2005) nominal consumption-investment ratio in the five-variable system, we strongly rejected it for all simple-sum and Divisia monetary aggregates. Finally, the restriction that identifies the money demand function is rejected for all simple sum aggregates. We cannot reject the money demand function for all the Divisia aggregates.

In this regard, Ahmed and Rogers (2000), also following closely King *et al.* (1991) use annual U.S. data (over the period from 1889 to 1995) to investigate the relationship between inflation and the great ratios, in an attempt to address a number of theoretical results in the monetary optimal growth literature including the Tobin (1965) effect and the Sidrauski (1967) monetary superneutrality result and the Fisherian link between the nominal interest rate and the inflation rate. They find evidence of a positive Tobin effect that is equivalently regarded as evidence against the Fisher effect. Their evidence regarding the presence of a Tobin effect is inconsistent with a large part of the empirical literature on the neutrality and superneutrality of money, including Fisher and Seater (1993), King and Watson (1997), and Serletis and Koustas (1998); although their result regarding the Fisherian link is consistent with most of the empirical literature on the Fisher effect — see, for example, Koustas and Serletis (1999). Moreover, their results are inconsistent with a variety of monetary optimal growth models, and as Ahmed and Rogers (2000, p. 29) put it, "our empirical approach does not tell us the exact mechanism that generates a Tobin-effect and we leave this as an open question."

More recently, however, Rappach (2003) and Gillman and Nakov (2003) also report results in support of the Tobin effect of inflation. Also, Gillman and Kejak (2011) calibrate a monetary model of endogenous growth and show that in a balanced growth path equilibrium, inflation lowers the great ratios, as found by Ahmed and Rogers (2000), when the monetary framework is a Stockman (1981)-type cash-in-advance constraint applied only to the purchases of consumption goods. They also show that when the cash-in-advance constraint applies to purchases of consumption as well as investment, the inflation tax falls on investment as well as consumption, and the investment-output great ratio falls on the balanced growth path with a higher stationary monetary growth rate. Regarding inflation rates within our sample, we note that the Federal Reserve began fighting inflation in 1979 by reducing the growth rate of the money supply. In fact, under Volcker's leadership, the inflation rate was reduced from more than 11% in 1979 to 6% in 1982, and the inflation rate has generally remained below 5% ever since. Thus, we cannot explain the nonstationarity of the investment-output great ratio by an increase in the average money supply growth rate and we leave this as an open question.

# 6 Conclusion

We have tested the balanced growth hypothesis and classical money demand theory in the context of a multivariate stochastic process consisting of the logarithms of real per capita consumption, investment, money balances, output, and the opportunity cost of holding money. In doing so, we have made comparisons among traditional simple-sum monetary aggregates and the Divisia monetary aggregates recently produced within the Center of Financial Stability (CFS) program, Advances in Monetary and Financial Measurement (AMFM). We provide evidence that simultaneously identifies the consumption-output great ratio and the money demand function when CFS Divisia monetary aggregates are used, but we do not find convincing evidence of a stationary investment-output great ratio.

Our improved results concerning the empirical validity of the long-run relationship between major real and nominal macro variables provide a confirmation that Divisia monetary aggregates can and should play an important role in monetary growth theory and money demand theory. Although we are not able to nail down the choice of the specific level of aggregation for the monetary aggregate, our results suggest answers to this question and also to a number of questions raised over previous studies of the role of money in the economy. Most important is the idea that a meaningful comparison of alternative monetary aggregates requires the discovery of the structure of preferences over monetary assets by testing for weakly separable subgroupings. The typical applied study starts from a structure specified *a priori* and never exploits the sample to find other groupings of monetary assets consistent with the optimizing behavior of the representative economic agent. We believe that separability-based Divisia measures of money, using the new Center for Financial Stability quantity and user cost component data, will improve our understanding of how money affects the economy, as noted by Barnett (1982) and investigated (among others) by Serletis (1991).

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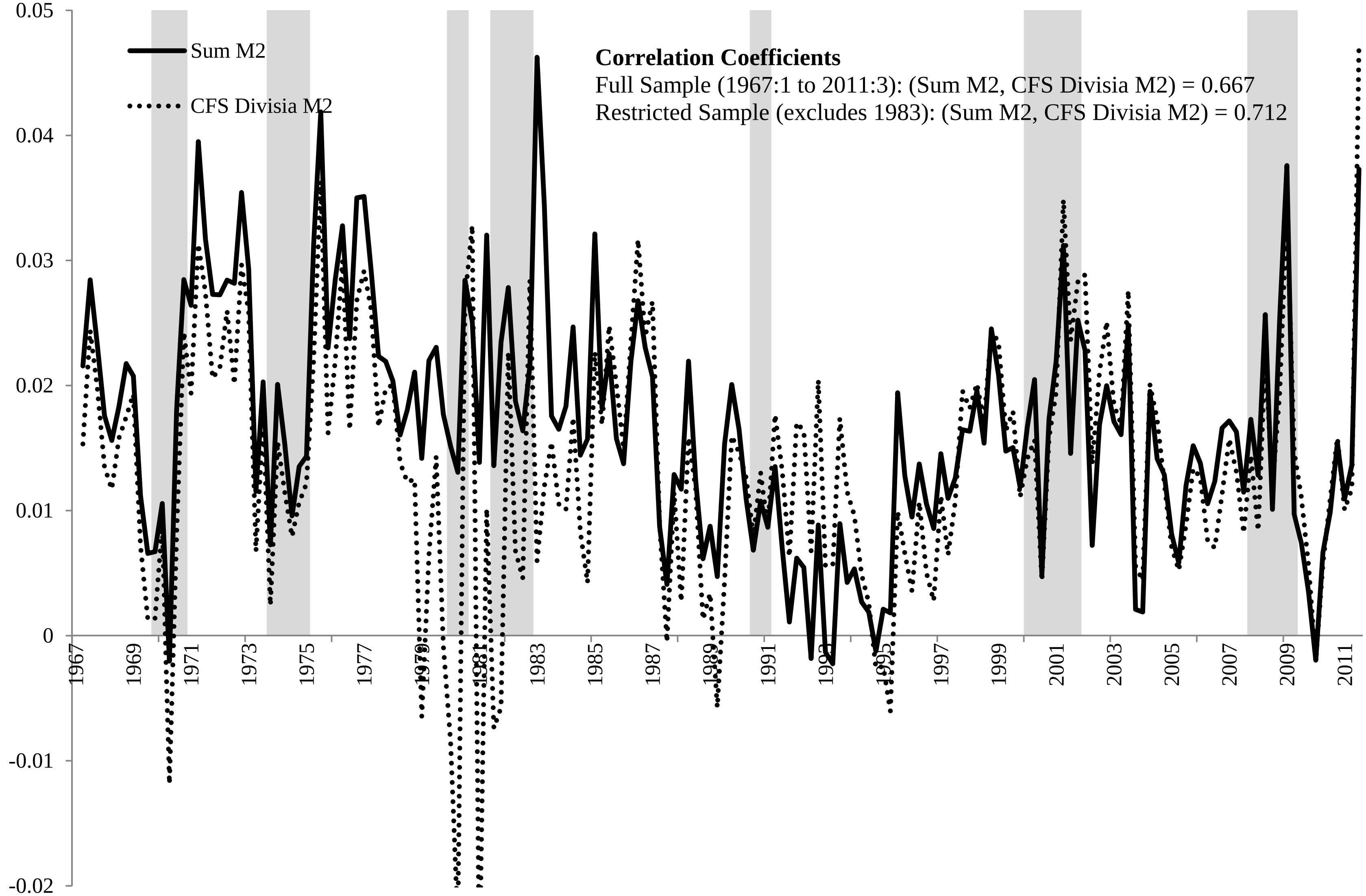
# Figure 1. M2 Monetary Aggregate Growth Rates



## Figure 2. ALL Monetary Aggregate Growth Rates

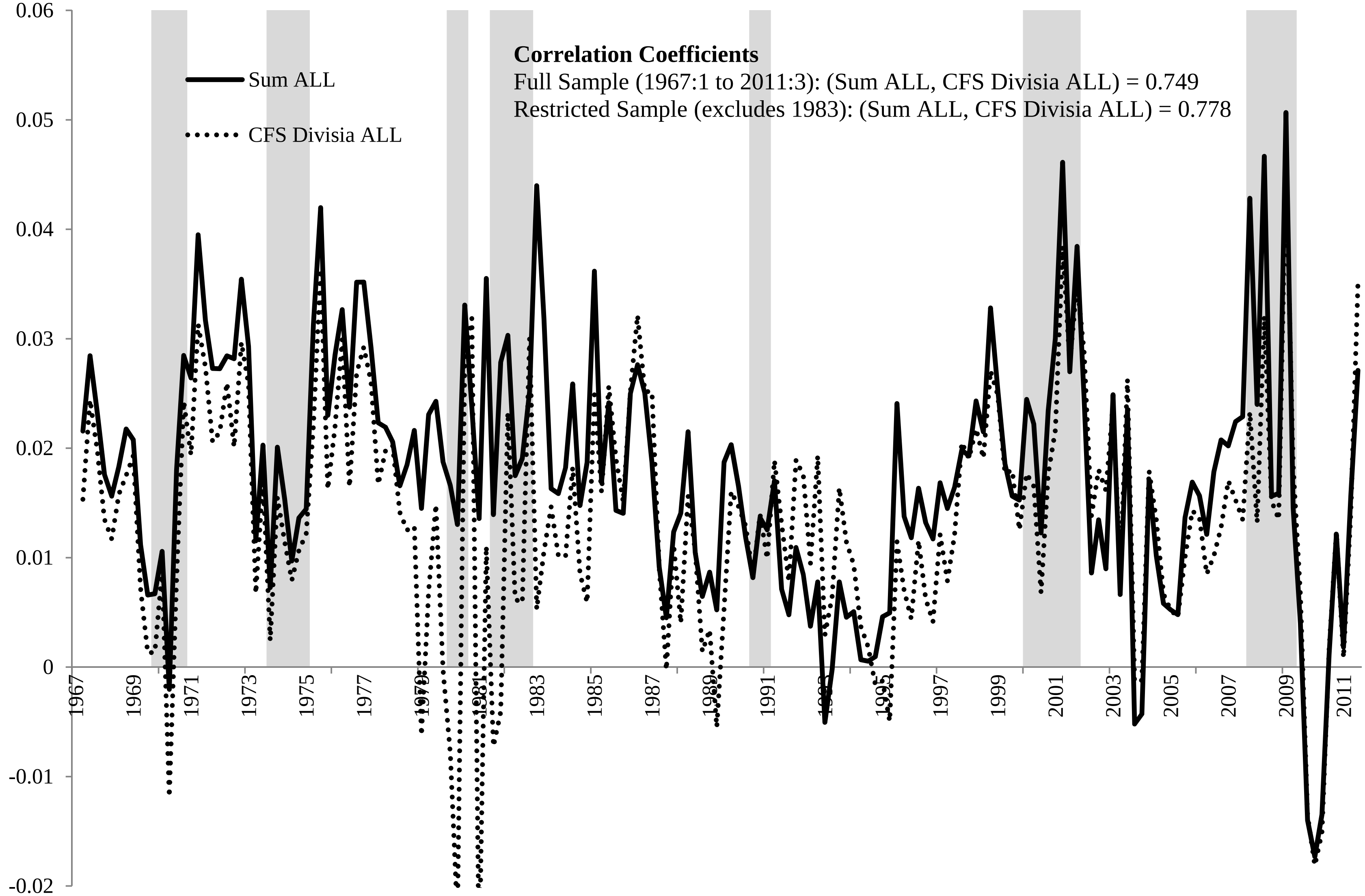


Table 1. Johansen ML Cointegration Tests in the 3-Variable Money Demand System $\{m-p, y, R\}$

| System | VAR lag | J-B | LM test | Cointegration tests: Null | $\lambda_{\text{trace}}$ | $\lambda_{\max}$ | Coint. vectors |
|---|---|---|---|---|---|---|---|
| Sum M1, $y$, $R$ | 2 | .000 | .130 | $r=0$ | .010** | .008*** | 1 |
| | | | | $r \leq 1$ | .374 | .364 | |
| | | | | $r \leq 2$ | .383 | .383 | |
| Sum M2M, $y$, $R$ | 2 | .000 | .022 | $r=0$ | .236 | .389 | 0 |
| | | | | $r \leq 1$ | .321 | .273 | |
| | | | | $r \leq 2$ | .560 | .560 | |
| Sum M2, $y$, $R$ | 2 | .000 | .265 | $r=0$ | .951 | .782 | 0 |
| | | | | $r \leq 1$ | .997 | .996 | |
| | | | | $r \leq 2$ | .725 | .725 | |
| Sum MZM, $y$, $R$ | 2 | .000 | .035 | $r=0$ | .072* | .087* | 0 |
| | | | | $r \leq 1$ | .363 | .299 | |
| | | | | $r \leq 2$ | .676 | .676 | |
| Sum ALL, $y$, $R$ | 2 | .000 | .244 | $r=0$ | .523 | .417 | 0 |
| | | | | $r \leq 1$ | .787 | .728 | |
| | | | | $r \leq 2$ | .770 | .770 | |
| CFS Divisia M1, $y$, $R$ | 2 | .000 | .140 | $r=0$ | .001*** | .001*** | 1 |
| | | | | $r \leq 1$ | .183 | .351 | |
| | | | | $r \leq 2$ | .073 | .073 | |
| CFS Divisia M2M, $y$, $R$ | 2 | .000 | .008 | $r=0$ | .000*** | .000*** | 1 |
| | | | | $r \leq 1$ | .066* | .201 | |
| | | | | $r \leq 2$ | .033** | .033 | |
| CFS Divisia M2, $y$, $R$ | 2 | .000 | .021 | $r=0$ | .001*** | .007*** | 1 |
| | | | | $r \leq 1$ | .059* | .180 | |
| | | | | $r \leq 2$ | .034** | .034** | |
| CFS Divisia MZM, $y$, $R$ | 2 | .000 | .020 | $r=0$ | .003*** | .016*** | 1 |
| | | | | $r \leq 1$ | .062* | .153 | |
| | | | | $r \leq 2$ | .050* | .050* | |
| CFS Divisia ALL, $y$, $R$ | 2 | .000 | .038 | $r=0$ | .045** | .162 | 0 |
| | | | | $r \leq 1$ | .114 | .288 | |
| | | | | $r \leq 2$ | .044* | .044* | |

*Notes*: *, **, and *** indicate rejection at the 10%, 5%, and 1% levels, respectively. Cointegration tests in the 3-variable system including a monetary aggregate and output in real per capita terms and the interest rate, for the period 1967Q1-2011Q3. Based on 99% critical values, the $\lambda_{\text{trace}}$ and $\lambda_{\max}$ test statistics provide evidence of one cointegrating relation in the cases of the Sum M1 aggregate and the CFS Divisia aggregates at the M1, M2M, M2, and MZM levels of aggregation.

Table 2. Estimates of the Cointegration Vector(s) in the 3-Variable Money Demand System $\{m-p, y, R\}$

| Monetary aggregate | Coint. vectors | System $m-p$ | System $y$ | System $R$ | Restrictions | $p$-value |
|---|---|---|---|---|---|---|
| Sum M1 | 1 | 1.000 | $-1.000$ | 4.746 | $[1,-1,R]$ | .000 |
| Sum M2M | 0 | | | | | |
| Sum M2 | 0 | | | | | |
| Sum MZM | 0 | | | | | |
| Sum ALL | 0 | | | | | |
| CFS Divisia M1 | 1 | 1.000 | $-1.000$ | 12.684 | $[1,-1,R]$ | .001 |
| CFS Divisia M2M | 1 | 1.000 | $-1.000$ | 26.787 | $[1,-1,R]$ | .091 |
| CFS Divisia M2 | 1 | 1.000 | $-1.000$ | 49.060 | $[1,-1,R]$ | .239 |
| CFS Divisia MZM | 1 | 1.000 | $-1.000$ | 17.012 | $[1,-1,R]$ | .043 |
| CFS Divisia ALL | 0 | | | | | |

*Notes*: The restriction $[1,-1,R]$ that identifies the money demand function cannot be rejected in the case of the CFS Divisia aggregates at the M2M, M2, and MZM levels with $p$-values of 0.091, 0.239, and 0.043, respectively. Moreover, we get $R>0$ with all the aggregates, consistent with what is expected according to the theory.

Table 3. Johansen ML Cointegration Tests in the 5-Variable System $\{c, i, m-p, y, R\}$

| System | J-B | LM test | Cointegration tests: Null | $\lambda_{\text{trace}}$ | $\lambda_{\max}$ | Coint. vectors |
|---|---|---|---|---|---|---|
| $c$, $i$, Sum M1, $y$, $R$ | .000 | .186 | $r = 0$ | .000*** | .001*** | 1 |
| | | | $r \leq 1$ | .023** | .013** | |
| | | | $r \leq 2$ | .459 | .494 | |
| | | | $r \leq 3$ | .587 | .602 | |
| | | | $r \leq 4$ | .363 | .363 | |
| $c$, $i$, Sum M2M, $y$, $R$ | .000 | .095 | $r = 0$ | .008*** | .008*** | 1 |
| | | | $r \leq 1$ | .269 | .453 | |
| | | | $r \leq 2$ | .406 | .362 | |
| | | | $r \leq 3$ | .669 | .774 | |
| | | | $r \leq 4$ | .224 | .224 | |
| $c$, $i$, Sum M2, $y$, $R$ | .000 | .488 | $r = 0$ | .014** | .016** | 0 |
| | | | $r \leq 1$ | .288 | .225 | |
| | | | $r \leq 2$ | .683 | .891 | |
| | | | $r \leq 3$ | .443 | .678 | |
| | | | $r \leq 4$ | .099 | .099 | |
| $c$, $i$, Sum MZM, $y$, $R$ | .000 | .092 | $r = 0$ | .003*** | .008*** | 1 |
| | | | $r \leq 1$ | .117 | .326 | |
| | | | $r \leq 2$ | .225 | .332 | |
| | | | $r \leq 3$ | .364 | .429 | |
| | | | $r \leq 4$ | .221 | .221 | |
| $c$, $i$, Sum ALL, $y$, $R$ | .000 | .316 | $r = 0$ | .002*** | .009*** | 1 |
| | | | $r \leq 1$ | .098* | .126 | |
| | | | $r \leq 2$ | .398 | .798 | |
| | | | $r \leq 3$ | .214 | .386 | |
| | | | $r \leq 4$ | .082 | .082 | |
| $c$, $i$, CFS Divisia M1, $y$, $R$ | .000 | .246 | $r = 0$ | .000*** | .004*** | 2 |
| | | | $r \leq 1$ | .005*** | .009*** | |
| | | | $r \leq 2$ | .189 | .222 | |
| | | | $r \leq 3$ | .442 | .706 | |
| | | | $r \leq 4$ | .086* | .086* | |
| $c$, $i$, CFS Divisia M2M, $y$, $R$ | .000 | .193 | $r = 0$ | .000*** | .001*** | 2 |
| | | | $r \leq 1$ | .000*** | .003*** | |
| | | | $r \leq 2$ | .026** | .054* | |
| | | | $r \leq 3$ | .192 | .476 | |
| | | | $r \leq 4$ | .040* | .040** | |
| $c$, $i$, CFS Divisia M2, $y$, $R$ | .000 | .401 | $r = 0$ | .000*** | .009*** | 3 |
| | | | $r \leq 1$ | .000*** | .002*** | |
| | | | $r \leq 2$ | .006*** | .014** | |
| | | | $r \leq 3$ | .146 | .413 | |
| | | | $r \leq 4$ | .033** | .033** | |
| $c$, $i$, CFS Divisia MZM, $y$, $R$ | .000 | .115 | $r = 0$ | .000*** | .006*** | 3 |
| | | | $r \leq 1$ | .000*** | .013** | |
| | | | $r \leq 2$ | .009*** | .028** | |
| | | | $r \leq 3$ | .123 | .328 | |
| | | | $r \leq 4$ | .039** | .039** | |
| $c$, $i$, CFS Divisia ALL, $y$, $R$ | .000 | .325 | $r = 0$ | .000*** | .009*** | 3 |
| | | | $r \leq 1$ | .000*** | .033** | |
| | | | $r \leq 2$ | .006*** | .023** | |
| | | | $r \leq 3$ | .098* | .314 | |
| | | | $r \leq 4$ | .028** | .028** | |

*Notes*: *,**,and ***indicate rejection at the 10%,5%, and 1%levels. According to the $\lambda_{\text{trace}}$and $\lambda_{\max}$ test statistics at the 99% confidence level, we cannot reject the null hypothesis of one cointegrating vector with Sum M1, Sum M2M, Sum MZM, and Sum ALL aggregates. With the CFS Divisia aggregates we cannot reject the null of two cointegrating vectors at the M1 and M2M levels. The null of three cointegrating relations cannot be rejected in the case of the CFS Divisia aggregates at the M2, MZM, and ALL levels of aggregation.

Table 4. Estimates of the Cointegration Vector(s) in the 5-Variable System $\{c, i, m-p, y, R\}$

| Monetary aggregate | Coint. vectors | System $c$ | $i$ | $m-p$ | $y$ | $R$ | Restrictions | $p$-value |
|---|---|---|---|---|---|---|---|---|
| Sum M1 | 1 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .000 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 4.666 | $[0,0,1,-1,R]$ | .000 |
| Sum M2M | 1 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .000 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 5.663 | $[0,0,1,-1,R]$ | .000 |
| Sum M2 | 0 | | | | | | | |
| Sum MZM | 1 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .000 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 6.921 | $[0,0,1,-1,R]$ | .000 |
| Sum ALL | 1 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .000 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 1.230 | $[0,0,1,-1,R]$ | .000 |
| CFS Divisia M1 | 2 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .000 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 11.457 | $[0,0,1,-1,R]$ | .001 |
| CFS Divisia M2M | 2 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .032 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 24.946 | $[0,0,1,-1,R]$ | .787 |
| CFS Divisia M2 | 2 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .422 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 38.429 | $[0,0,1,-1,R]$ | .922 |
| CFS Divisia MZM | 2 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .017 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 16.921 | $[0,0,1,-1,R]$ | .020 |
| CFS Divisia ALL | 2 | 1.000 | .000 | .000 | $-1.000$ | .000 | $[1,0,0,-1,0]$ | .009 |
| | | .000 | 1.000 | .000 | $-1.000$ | .000 | $[0,1,0,-1,0]$ | .000 |
| | | .000 | .000 | 1.000 | $-1.000$ | 14.390 | $[0,0,1,-1,R]$ | .000 |

*Notes*: The restrictions that identify the consumption-output and investment-output great ratios and the money demand function are rejected for all of the simple-sum monetary aggregates and the CFS Divisia aggregates at the M1 and ALL levels of aggregation. For the CFS Divisia aggregates the consumption-output and the money demand function cannot be rejected at the M2M, M2 and MZM levels of aggregation.