A Hotelling Style Model of Spatial Competition for Convenience Goods [1]

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Abstract: Ordinarily people do not make special purpose trips to acquire goods like gasoline or groceries, but instead buy them as the need arises in the course of their daily lives. Such goods are commonly called convenience goods. We modify Hotelling's model of spatial competition so that we can analyze the price equilibrium of duopolists that retail a convenience good. Certain features of the duopolists' demand functions suggest that price competition is more severe in the convenience goods model than in the Hotelling model. The same features complicate the analysis because they mean that a pure strategy price equilibrium does not exists for many locational configurations. Although we are not able to find the mixed strategy price equilibrium analytically, we do present some numerical results on equilibrium prices that broadly confirm this suggestion. We also provide a more general product differentiation interpretation of the convenience good model.

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1 Introduction

The literature on spatial competition initiated by Harold Hotelling's seminal article, *Stability in Competition* (Hotelling 1929), focuses on the phenomenon of spatial differentiation of retail firms and the implications of differentiation for equilibrium prices. In Hotelling's model, identical goods offered by firms located at different points in the landscape are not perfect substitutes because travel to and from firms to buy their goods is costly for their customers. Consequently, with respect to customers that are located closer to it than to any other firm, every firm is in a position that is something like a natural monopoly. This is especially true when the typical customer's transportation cost is a significant portion of the total cost of the good to the customer. Kaldor (1935) famously described the situation as one of *overlapping oligopolies*.

A large literature on the more general phenomenon of product differentiation grew out of the insights offered by Hotelling. See for example the survey by Eaton and Lipsey (1989). This literature argues that markets for differentiated goods are stubbornly oligopolistic, more akin to natural monopoly than to perfect competition. In this paper we develop a modified version of Hotelling's model that incorporates the phenomenon of convenience shopping. For many applications, the convenience goods (CG) model seems to us to more accurately capture the dominant features of the competitive environment in retailing. We show that in most circumstances the model of convenience shopping, while far from being perfectly competitive, is significantly more competitive that the Hotelling model.

Standard models of spatial competition among retailers, like Salop (1979) and Eaton and Wooders (1985), operate on the assumption that each customer consults the prices of all retailers, identifies the lowest price inclusive of the customer's transportation costs, gets in her car, drives to the firm with the lowest price, purchases the good, and then drives back home. This assumption is, we argue, inappropriate for goods like gasoline, the quintessential convenience good. To buy gasoline for their cars, customers do not ordinarily make special purpose trips. Instead, they buy gasoline as the need arises in the course of their normal activities – on the way to and from work or school, for example. Clearly a model of spatial competition that incorporates this convenience aspect of retailing will have somewhat different properties than do standard models. In particular, the CG model will tend to be somewhat more price competitive because the convenience goods offered by different retailers that a particular customer passes in the course of her normal activities will be perfect substitutes for that customer.

Standard models also operate on the assumption that a person who wants to go from some point X in space (his or her home, for example) to another point Y (his or her place of work, for example) travels along the straight line that connects the two points, so the distance travelled in the course of a journey from X to Y is the Euclidean distance between the points. Real people, of course, use the existing street network. When this network includes higher speed arteries – highways for short – traffic and therefore the demand for convenience goods is naturally concentrated on them, because it is economical for people to use these higher speed highways as they go about their daily activities. Responding to this concentration of demand, retailers of convenience goods tend to locate on highways. Notice that identical goods offered by different retailers along the highway are perfect substitutes for people who pass the different retailers on their daily journey – the firm that offers the lowest price will capture all of these customers. Clearly, this feature of the market for convenience goods is pro-competitive.

The CG model is an elaboration of Hotelling's model. There is a small town with a highway that runs east and west through it and a rectangular network of secondary streets running east and west and north and south. People live, work and buy convenience goods in the town. Each person makes the same journey by car every day – to and from work, for clarity – and uses the highway on this journey. There are two firms that retail the convenience good, and both are located on the highway. Customers, who have full knowledge of the prices and locations of firms, buy one unit of the convenience good in each period.

In Hotelling's original model, firms played a two stage game. In the first stage they chose their locations on the highway and in the second they chose their prices. We use the same sequential decision structure in our model, although we are primarily interested is the second stage game in which prices are chosen given fixed locations. The structure of the CG model in such that we can meaningfully compare results for it with those for the Hotelling model. We show that in the CG model there is a discontinuity in the demand function of either firm at the point where its price is equal to that of its competitor, and also that there is no such discontinuity in the Hotelling model. As D'Aspremont, Gabszewicz and Thisse (1979) showed, for many locational configurations of the firms in Hotelling's model there is no equilibrium in pure price strategies. A similar non-existence issue arises in the CG model.

We are unable to find by analytical means the stage two price equilibrium of the CG model. We do, however, present some comparative results for discretized versions of the Hotelling and CG models. These results are generated using a program called Gambit, developed by McKelvey, McLennan and Turocy (2007). They indicate that in most circumstances the CG model is substantially more price competitive than the Hotelling model.

Although we restrict attention to goods like gasoline that clearly are convenience goods (rarely does anyone make a special purpose trip to get gasoline), it seems to us that to some degree the features and therefore results of our model of convenience shopping are applicable to the retailing of a wide range of consumer goods. So, in the concluding section of the paper, we develop an alternative interpretation of the CG model as a more general model of horizontal differentiation, something we call the NTP *not-too-picky* or NTP model. The take away point of the paper is the suggestion that markets for differentiated goods are significantly more price competitive than most models of horizontal product differentiation would indicate.

2 The CG Model

For clarity, it is useful to picture a small town with a highway of unit length that runs east and west through it, a residential area north of the highway, an industrial area south of the highway, and a dense network of secondary streets that run north and south, and east and west. We can represent the highway by the line segment $[0, 1]$, and any north/south secondary street by the number $y \in [0, 1]$ where the street intersects the highway. People live in the residential area north of the highway and work in the industrial area south of the highway. For purposes of the model, an individual worker/customer can be described by $\mathbf{y} = (y_1, y_2)$, where $y_1 \in [0, 1]$ is the secondary street on which she lives and $y_2 \in [0, 1]$ is the secondary street on which she works. The density of customers, $D(y_1, y_2)$ is uniform: $D(y_1, y_2) = d > 0, \forall (y_1, y_2) \in [0, 1]^2$.

Travel costs on the highway are lower than those on the secondary streets, so to minimize the cost of their daily commute, people maximize their use of the highway. For person $\mathbf{y}$ the journey to work entails traveling south along secondary street y_1 to the highway, then east or west along the highway to secondary street y_2, and then south along y_2 to her place of work; her journey back home is the reverse of her journey to work.

Two firms that retail the convenience good are located on the highway. Their locations are $\mathbf{x} = (x_1, x_2)$, where $0 \leq x_1 \leq x_2 \leq 1$, and their posted prices are $\mathbf{p} = (p_1, p_2)$, where $p_1 \geq 0$ and $p_2 \geq 0$. Firms buy the convenience good at a constant wholesale price c and they incur no other costs.

On their daily journey to and from work, people buy the convenience good from one of the two firms located on the highway. As regards their demand for the convenience good, the transport costs incurred in the journey to and from work are sunk, and therefore have no effect on the full price of the retail good. If on her daily journey a person passes retail firm i, the full price of the convenience good from firm i is just the firm's posted price, p_i. But if the person must depart from her commuting route to get to firm i, then the full price of the convenience good will include the *incremental* transport costs that must be incurred to get to the firm.

We denote the *incremental distance* that a person must travel to get to firm i by d_i. Obviously, d_i is 0 for anyone who passes firm i on her daily commute, and is positive for anyone who does not. For person $\mathbf{y}$, d_i is the following:

$$d_i = |y_1 - x_i| + |y_2 - x_i| - |y_1 - y_2|.$$

To see that this expression accurately represents incremental distance, it is helpful to look at a couple of cases. First suppose that $x_1 < y_1 < y_2$. In this case, $d_i = 2(y_1 - x_i)$, which reflects the fact that to get to firm i she must depart from her usual commute by driving west on the highway from the intersection of the highway with the street where she lives, y_1, to the firm located at x_i, and back to the intersection, a distance of $2(y_1 - x_i)$. Second suppose that $y_1 < x_1 < y_2$. Now $d_i = 0$, because she passes firm i on her usual commute. The full price of the convenience good from firm i is just $p_i + \frac{t}{2} d_i$, where t is the cost per unit of distance of traveling from one point on the highway to another and back again.

Customers buy one unit of the convenience good from the firm offering the lower full price, provided that the lower full price is less than the customers reservation price, $v > 0$. The solution to the customer's utility maximizing problem is, of course, trivial. If both full prices exceed v, she does not buy the convenience good; if one or both of the full prices are less than v, she buys one unit of the good from the firm with the lower full price; if the full prices are equal, she buys 1 unit for a randomly chosen firm.

For some purposes, instead of describing a locational configuration by (x_1, x_2), it is better to describe it by (z, b), where z is the distance between the firms, $z \equiv x_2 - x_1$, and b is what we will call the *balance* of the configuration, defined as $b \equiv 1 - |x_1 - (1 - x_2)|$. Obviously, x_1 is just the distance from firm 1 to the left-hand end of the highway, and $1 - x_2$ is the distance from firm 2 to the right-hand end of the highway. When these distances are equal, the configuration is perfectly balanced and b is equal to 1. The larger is the absolute value of the difference between x_1 and $1 - x_2$, the less balanced in the configuration. In the extreme case where both firms are located at the same end of the highway, b is equal to 0, so $0 \leq b \leq 1$.

It is convenient to choose specific values for some of the model's parameters. With no loss of generality, we can set the wholesale price c equal to 0 and the uniform density of customers d equal to 1. To avoid having to deal with some tedious cases where the entire market is not served we assume that the customer's reservation price v is large in the appropriate sense. Although restrictive, this is almost always done in models of this sort. That leaves the transport cost t. It is convenient to assume that t is equal to one. So, in what follows, we assume that $c = 0, d = 1, t = 1$, and v is large.

3 Analysis of the Model

In this section we derive the demand functions for the CG model, and for purposes of comparison, the demand functions for the Hotelling model, a special case on the CG model that arises when customers make special purpose trips to buy the good. Then we prove a non-existence result for the CG model.

3.1 Demand Functions

With the aid of Figure 1, we can aggregate the choices of customers to get the firms' demand functions in the CG model. The large square area in the figure is the entire set of customers, $\{(y_1, y_2)|0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1\}$. Given $\mathbf{p} = (p_1, p_2)$ and $\mathbf{x} = (x_1, x_2)$, demand aggregation requires that we identify the customers who patronize firm 1, and those who patronize firm 2.

Customers in the square area at the bottom left of the figure, $\{(y_1, y_2)|y_1 \leq x_1, y_2 \leq x_1\}$, are predisposed to buy from firm 1 – these customers pass neither firm on their daily commute, but firm 1 is the closer firm. For any customer

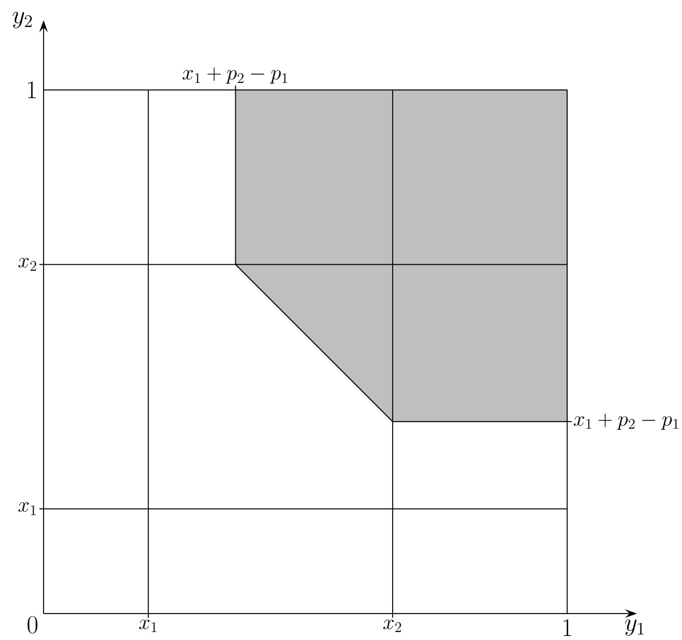

Figure 1: Demand, $p_2 > p_1$. Customers in the grey area patronize firm 2, white area customers patronize firm 1.

in this set, $d_2 - d_1 = x_2 - x_1 = z$ (recall that we defined z to be $x_2 - x_1$). Consequently, if $p_1 < p_2 + z$ all these customers patronize firm 1, if $p_1 > p_2 + z$ they all patronize firm 2, and if $p_1 = p_2 + z$ half patronize firm 1 and half patronize firm 2. So there is a discontinuity in the demand functions at $p_1 = p_2 + z$.

Similarly, customers in the square area at the top right of the figure, $\{(y_1, y_2)|y_1 \geq x_2, y_2 \geq x_2\}$, are predisposed to buy from firm 2. If $p_2 < p_1 + z$ all these customers patronize firm 2, if $p_2 > p_1 + z$ they all patronize firm 1, and if $p_2 = p_1 + z$ half patronize firm 1 and half patronize firm 2. So there is a second discontinuity at $p_2 = p_1 + z$.

Customers in the rectangular areas at the top left, $\{(y_1, y_2)|y_1 \leq x_1, y_2 \geq x_2\}$, and bottom right, $\{(y_1, y_2)|y_1 \geq x_2, y_2 \leq x_1\}$, pass both firms on their daily commute. These customers patronize firm 1 if $p_1 < p_2$, firm 2 if $p_2 < p_1$, and half of them patronize firm 1 and half firm 2 if $p_1 = p_2$. So there is a third discontinuity at $p_1 = p_2$.

In Figure 1, we illustrate the demand aggregation for a case in which $p_1 + z > p_2 > p_1$. The convex shaded area at the upper right of the figure is Q_2 and the non-convex non-shaded area is Q_1. Holding p_2 fixed, as we decrease p_1 the shaded area shrinks in a continuous fashion for a time, but as p_1 passes through $p_2 + z$ the customers in the square area at the upper right hand corner jump in mass from firm 2 to firm 1. Conversely, as we increase p_1 the shaded area grows in a continuous fashion for a time, but as p_1 passes through p_2 the customers in rectangular areas at the upper left and lower right of the figure jump in mass from firm 1 to firm 2. In addition, as p_1 passes through p_2, the shapes of the shaded and non-shaded areas flip – the shaded area becoming non-convex and the non-shaded area convex. As we continue to increase p_1, for a time Q_2 increases and Q_1 decreases in a continuous fashion until, as p_1 passes through $p_2 + z$, the customers in the square area at the bottom left of the figure jump in mass from firm 1 to firm 2.

Given the uniform density ($d = 1$), exactly 1 unit is demanded when we aggregate over all customers, so $Q_2(\mathbf{p}, \mathbf{x}) + Q_1(\mathbf{p}, \mathbf{x}) = 1$. Given this, it is enough to write out just one of the demand functions. Firm 1's demand function is presented in Table 1.

Table 1
Firm 1's Demand Function in the CG Model

Quantity Demanded	Price Restriction
0	$p_1 > p_2 + z$
$\frac{x_1^2}{2}$	$p_1 = p_2 + z$
$(x_2 + p_2 - p_1)^2 - \frac{1}{2}(z + p_2 - p_1)^2$	$p_2 + z > p_1 > p_2$
$x_2^2 - \frac{z^2}{2} + x_1(1 - x_2)$	$p_1 = p_2$
$1 - (1 - x_1 + p_1 - p_2)^2 + \frac{1}{2}(z + p_1 - p_2)^2$	$p_2 > p_1 > p_2 - z$
$1 - \frac{(1-x_2)^2}{2}$	$p_1 = p_2 - z$
1	$p_2 - z > p_1$

To facilitate comparison, in Table 2 we have written out firm's 1's demand function for the Hotelling model, where the good is a non-convenience good. Hotelling's model is a special case of ours when customers make a special trip from their home to one of the two firms, and back again, to acquire a non-convenience good. Given this, consumer $\mathbf{y}$ patronizes firm i instead of firm j if $p_i + |y_1 - x_i| < p_j + |y_1 - x_j|$. Building on this choice criterion, we get the demand function for firm 1 in the Hotelling model, presented in Table 2.

Table 2
Firm 1's Demand Function in the Hotelling Model

Quantity Demanded	Price Restriction
0	$p_1 > p_2 + z$
$\frac{x_1}{2}$	$p_1 = p_2 + z$
$\frac{1}{2}(p_2 - p_1 + x_1 + x_2)$	$p_2 + z > p_1 > p_2 - z$
$1 - \frac{1-x_2}{2}$	$p_1 = p_2 - z$
1	$p_2 - z > p_1$

There are discontinuities in the Hotelling demand functions at $p_1 = p_2 + z$ and $p_1 = p_2 - z$ just as there are in the CG model, but there is none at $p_1 = p_2$. Consequently, in the Hotelling model there is no mass of people who always buy from the firm offering the lower price, and no incentive to undercut the other's firm's price to capture them. It is this difference that tends to make the CG model more competitive than the Hotelling model.

In both models, the demand functions of firms 1 and 2 are symmetric in prices when the locational configuration is perfectly balanced (that is, when $b = 1$), and at any common price quantity demanded is 1/2 for both firms. For unbalanced configurations ($b < 1$), at any common price $Q_1 > 1/2 > Q_2 > 0$ if $x_1 > 1 - x_2$, $Q_2 > 1/2 > Q_1 > 0$ if $x_1 < 1 - x_2$, and the smaller is b the larger is the difference in quantities demanded.

3.2 Profit Functions

We can write the profit functions for the CG model as follows:

$$\pi_1(\mathbf{x}, \mathbf{p}) = p_1 Q_1(\mathbf{x}, \mathbf{p}) \tag{1}$$
$$\pi_2(\mathbf{x}, \mathbf{p}) = p_2(1 - Q_1(\mathbf{x}, \mathbf{p})) \tag{2}$$

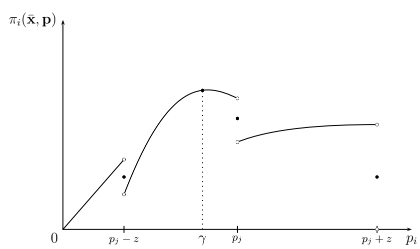

Figure 2: Profit function, firm i, $p_j - z > 0$.

These profit functions are curious characters. Two of the many possibilities are illustrated in Figures 2 and 3. The discontinuities at $p_i = p_j - z$ and $p_i = p_j + z$ in Figure 2 are familiar to students of Hotelling's model. When $p_i < p_j - z$ firm i has captured the entire market, and when $p_i > p_j + z$ it has has conceded the entire market to firm j. The discontinuity at $p_i = p_j$ is the one associated with convenience shopping. Figure 3 illustrates a case where firm i is unable to capture the entire market, because p_j is so low that firm i cannot possibly offer j's captive consumers a price low enough to compensate for the distance they have to travel to get to firm i.

Given p_j, even though the function defined on the interval $(p_j, p_j + z)$ is concave, it is sometimes increasing everywhere, sometimes decreasing everywhere, and sometimes has an interior local maximum. Because this interval is

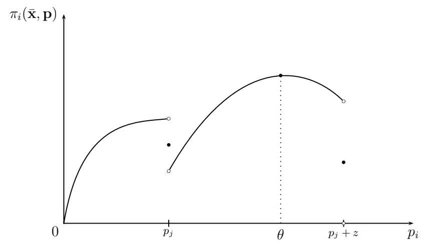

Figure 3: Profit function, firm i, $p_j - z < 0$.

not compact, π_i does not attain a maximum on the interval when π_i is everywhere increasing or decreasing in the interval. The same is true of the interval $(p_j - z, p_j)$. In Figure 2 there is a local maximum in interval $(p_j - z, p_j)$ at $p_i = \gamma$, and in Figure 3 there is a local maximum in interval $(p_j, p_j + z)$ at $p_i = \theta$, and in both cases the local maximum is also a global maximum.

3.3 No Pure Strategy Price Equilibrium

D'Aspremont, Gabszewicz and Thisse (1979) showed that in the original Hotelling model, for certain locational configurations there is no Nash equilibrium in pure price strategies. Roughly, if z is too small, there is no equilibrium in pure price strategies. The equilibrium that Hotelling identified can be thought of as a Nash equilibrium in a restricted strategy space. In effect, in calculating his price equilibrium, Hotelling assumed that neither firm would ever think that it could charge a price that was so low that it captured the entire market without inducing a price response. So he restricted the price strategy space of each firm to prices that were high enough, in relation to the other firm's price, so that the firm could not capture the entire market. Admittedly this is somewhat inelegant, but the reasoning behind it is not obviously wrong, and for that reason Hotelling's equilibrium still has some appeal. Eaton (1972) discusses this restricted equilibrium at some length. Osborne and Pichik (1987) found a Nash equilibrium in mixed strategies of Hotelling's two stage game. They did not, however, find a mixed strategy equilibrium for arbitrary locational configurations.

Because the analysis of the non-existence problem in the CG model is at points a bit messy, it useful to start by summarizing our results.

Proposition 1: Any configuration of locations in the CG model is represented by a point in the large triangular area in Figure 4. For any configuration in the two parts of the figure labeled PSE, there is a Nash equilibrium in pure price strategies, but for configurations in the rest of the figure there is no such equilibrium.

When either $x_1 = 0$ or $x_2 = 1$, the set of customers who always buy from the firm with the lowest price is empty, and the demand discontinuity that distinguishes the GC model is absent. When $x_1 = x_2$, the CG model collapses to the Bertrand model, and there is a pure strategy price equilibrium in which both prices are equal to 0. So we focus on configurations in which $0 < x_1 < x_2 < 1$.

It is clear that there can be no pure strategy equilibrium in which either of the following inequalities holds, $0 < p_2 \leq p_1 - z$ or $0 < p_1 \leq p_2 - z$, because the firm with the higher price, say firm i, could increase its profit by choosing a price just less than $p_j + z$. It also clear that there can be no equilibrium in which one or both prices is equal to 0. If both were equal to 0, either firm could increase its profit by charging a price just less than z, which would allow it to sell to its captive customers (those in the squares at the lower left or the upper right of Figure 1). Further, if one price was positive and one was 0, the firm with price equal to 0 could increase its profit by matching the other firm's price. Finally it is obvious that there can be no equilibrium in which the prices are identical, because either firm could increase its profit by decreasing its price a bit to capture all of the price sensitive customers in the rectangles in the upper left and lower right corners of Figure 1.

So if there is to be a pure strategy equilibrium, one of the prices will have to be strictly less than the other and both will have to be positive. Let us assume that there is such an equilibrium, $\mathbf{p}^* = (p_1^*, p_2^*)$, and for convenience that the firm with the lower price is firm 1, so that $p_2^* > p_1^* > 0$. Firm 1 has to be in the position pictured in Figure 2, pricing at a point where $\partial\pi_1/\partial p_1 = 0$ in interval $(p_2^* - z, p_2^*)$. It is impossible that $p_1^* < p_2^* - z$, because in this interval π_1 is a strictly increasing function on an interval that is open on the right. Similarly, it is impossible that $p_1^* = p_2^* - z$, because π_1 is larger for any price p_1 that is a tiny bit lower than $p_2^* - z$. That leaves only the open interval $(p_2^* - z, p_2^*)$. And because the interval is open, it has to be the case that at p_1^*, $\partial\pi_1/\partial p_1 = 0$. Analogous considerations lead to the conclusion that firm 2 has to be in a situation like that illustrated in Figure 3, pricing at a point where $\partial\pi_2/\partial p_2 = 0$ in the open interval $(p_1^*, p_1^* + z)$.

The first order condition for firm 1, $\partial\pi_1/\partial p_1 = 0$, dictates that

$$Q_1 + p_1^* \frac{\partial Q_1}{\partial p_1} = 0, \tag{3}$$

and the first order condition for firm 2, $\partial\pi_2/\partial p_2 = 0$, dictates that

$$(1 - Q_1) - p_2^* \frac{\partial Q_1}{\partial p_2} = 0. \tag{4}$$

But $\partial Q_1/\partial p_2 = -\partial Q_1/\partial p_1$, so firm 2's first order condition can be rewritten as

$$(1 - Q_1) + p_2^* \frac{\partial Q_1}{\partial p_1} = 0. \tag{5}$$

Then, solving equations 3 and 4 for $-\partial Q_1/\partial p_1$, we see that

$$\frac{Q_1}{p_1^*} = \frac{1 - Q_1}{p_2^*}, \tag{6}$$

which leads us to the conclusion that

$$\frac{p_2^*}{p_1^*} = \frac{1 - Q_1}{Q_1} \tag{7}$$

Since, by hypothesis $p_1^* < p_2^*$, this condition implies that $Q_1 < 1 - Q_1$, or that $Q_1 < 1/2 < Q_2$. That is, firm 1, the firm with the lower price, has the smaller market. This is not possible, however, if the mass of customers, equal to $2x_1(1 - x_2)$, who always buy from the firm with the lower price is too large. So, if $2x_1(1 - x_2)$ is too large, there is no pure strategy price equilibrium in which $p_1^* < p_2^*$. To sort out the precise meaning of *too large*, first observe that when the firms have identical prices and when we ignore the customers who are indifferent between the firms, firm 1's quantity demanded is $x_2^2 - z^2/2$. So too large means that $x_2^2 - z^2/2 + 2x_1(1 - x_2) > 1/2$. This condition can be rewritten as

$$x_1 > 1 + (1 - x_2) - \sqrt{1 + 2(1 - x_2)^2} \tag{8}$$

The analogous condition for non-existence that arises when we assume that $p_2^* < p_1^*$ is

$$1 - x_2 > 1 + x_1 - \sqrt{1 + 2x_1^2} \tag{9}$$

If both of these conditions hold, there is no pure strategy price equilibrium. In Figure 4, both conditions are satisfied in the area between the curved lines that emanate from the origin, so there is no pure strategy equilibrium in this portion of the figure.

Clearly, this non-existence result is associated with the discontinuity in the demand functions at $p_1 = p_2$. But there are two other discontinuities, at $p_1 = p_2 - z$ and $p_1 = p_2 + z$, and in certain circumstances they too give rise to non-existence. So, let us suppose one of conditions (8) and (9) does not hold; for convenience, that (9) does not hold:

$$1 - x_2 \leq 1 + x_1 - \sqrt{1 + 2x_1^2} \tag{10}$$

Then there is a price pair $(\widehat{p}_1, \widehat{p}_2)$, with $\widehat{p}_1 > \widehat{p}_2$ and $\widehat{Q}_1 > 1/2 > \widehat{Q}_2$, such that there is a local maximum of profit for firm 1 at $\widehat{p}_1$ in interval $(\widehat{p}_2, \widehat{p}_2 + z)$, and a local maximum for firm 2 at $\widehat{p}_2$ in interval $(\widehat{p}_1 - z, \widehat{p}_1)$. However, it is not necessarily the case that $(\widehat{p}_1, \widehat{p}_2)$ is a Nash equilibrium, for it must also be the case that firm 2 cannot increase its profit by choosing a price just below $\widehat{p}_1 - z$, thus capturing the entire market. To say more we need to find the price pair $(\widehat{p}_1, \widehat{p}_2)$. Define Δ as follows: $\Delta = \widehat{p}_1 - \widehat{p}_2$. Then, since $\widehat{p}_2 < \widehat{p}_1 < \widehat{p}_2 + z$, we can write $\widehat{Q}_1$ as

$$\widehat{Q}_1 = (x_2 - \Delta)^2 - \frac{1}{2}(x_2 - x_1 - \Delta)^2 \tag{11}$$

The first order profit maximizing conditions that characterize prices $(\widehat{p}_1, \widehat{p}_2)$ can be written as

$$\widehat{p}_1 = \frac{Q_1}{x_2 - x_1 - \Delta} \tag{12}$$

$$\widehat{p}_2 = \frac{1 - Q_1}{x_2 - x_1 - \Delta} \tag{13}$$

Using the first order conditions and the fact that $\Delta = \widehat{p}_1 - \widehat{p}_2$ we get

$$\Delta = \frac{2\widehat{Q}_1 - 1}{x_2 - x_1 - \Delta} \tag{14}$$

Since $\widehat{Q}_1$ is a function of Δ, equation (14) involves just one endogenous variable, Δ. Further, one can solve the equation to get a closed form expression for Δ.

$$\Delta = \frac{3(x_1 + x_2) - \sqrt{9 + 17x_1^2 + 2x_1 - (1 - x_2)(1 + 2x_1 + x_2)}}{4} \tag{15}$$

Then from equations (11), (12) and (13) we can get closed form solutions for $\widehat{Q}_1, \widehat{p}_1$, and $\widehat{p}_2$. (These expressions are so messy that nothing is gained by writing them out.) Of course, $(\widehat{p}_1, \widehat{p}_2)$ is not a Nash equilibrium if firm 2 has an incentive, given $\widehat{p}_1$, to undercut firm 1's price thereby grabbing the whole market, or if

$$\widehat{p}_1 - z > \widehat{p}_2 \widehat{Q}_1 \tag{16}$$

Intuitively, this inequality will be satisfied for unbalanced configurations in which firm 1's protected market, x_1, is much larger than firm 2's,$1 - x_2$. Consistent with this intuition, it is satisfied in the (roughly) trapezoidal area in the lower right of Figure 4, so there is no pure strategy price equilibrium for configurations in this portion of Figure 4. We used numerical methods to find the boundary on the left side of the trapezoidal area in Figure 4 – the locus of points such that $\widehat{p}_1 - z = \widehat{p}_2 \widehat{Q}_1$. There is, of course, an analogous non-existence condition for the case in which firm 2 has the lower price and smaller market, and this condition is satisfied in the trapezoidal area in the upper right portion of Figure 4.

Regrettably, we are not able to construct a mixed strategy price equilibrium for the CG model. All is not lost, however. There are well know algorithms, and good software, that will find mixed strategy equilibria for games where the strategy space has a finite number of elements. In the next section we use this software to find mixed strategy price equilibria for discretized versions of the CG and Hotelling models.

4 The Discretized Models

We report results for discretized versions of both the Hotelling and CG models. We used a wonderful program called Gambit, developed by McKelvey, McLennan and Turocy (2007), to compute the mixed strategy price equilibria of the

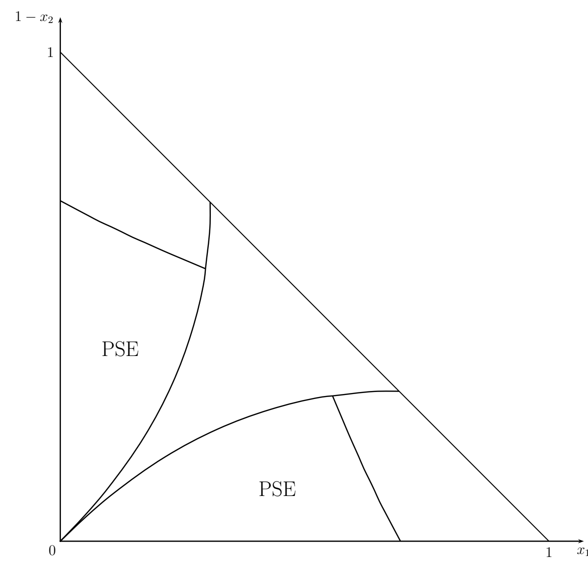

Figure 4: Location configuration of pure strategy equilibria.

discretized models for a number (30 to be precise) of locational configurations of the firms. Gambit offers a number of algorithms, but for our application none of them is universally reliable, so the results that we report were generated by one of three algorithms, *gambit-lcp*, *gambit-enumpoly*, or *gambit-enummixed*.

4.1 Strategy Spaces

We use two sets of prices. In both the smallest price is .001. In one set the *price bandwidth* is .0351 and in the other it is .0451. We use 35 discrete prices with the smaller bandwidth, so permissible prices range from .001 to 1.2295, in steps of .0351, and 30 discrete prices with the larger band with, so with the larger price bandwidth permissible prices range from .001 to 1.3540, in steps of .0451. With the smaller bandwidth, the highest price that received positive probability in any price equilibrium was 1.1242, and for the larger bandwidth it was 1.1285, so the highest permissible prices are irrelevant. This is a desirable because if the highest price did receive positive probability in some equilibrium, that equilibrium would perhaps disappear if higher prices were permitted.

There are 11 permissible locations, .0, .1, .2,..., 1.0. Since the number of permissible locations is 11, there are 121 potential locational configurations. The configurations in which the firms occupy the same location are uninteresting in that there is a pure strategy price equilibrium, analogous to the Bertrand equilibrium, in which both firms choose the smallest permissible price, .001, and in this equilibrium their expected profit is .0005. Obviously, firms will never choose such a configuration. So in the interest of brevity we ignore these 11 configurations. In addition, when two or more configurations are symmetric, there is no need to calculate price equilibria for all of them. Consider, for example, configurations$(.1, .2)$, $(.2, .1)$, $(.9, .8)$ and $(.8, .9)$. In each of these configurations, $(z, b) = (.1, .3)$ – the distance between firms is .1, and the balance of the configuration is .3. Consequently, a price equilibrium for any one of them, is with suitable reinterpretation a price equilibrium for the other three. When the redundant symmetric configurations and the configurations in which both firms occupy the same location are eliminated, we are left with just 30 distinct configurations.

We report results for each of these configurations in a series of seven tables, 3 through 9, that have the same basic format. Along the top of each table are 5 locations, from 0.0 to 0.4, for a firm we call the *column firm*, and *a*long the left edge of the tables are 10 locations, from 0.1 to 1.0, for a firm we call the *row firm*. In the interior cells of the table we report various statistics for the price equilibrium associated with the corresponding locational configuration.

The following observations about the balance (b) and distance between firms (z) for configurations in the these tables will facilitate discussion. Starting at a configuration on the left edge of any of these tables (for example, configuration $(x_r, x_c) = (0.7, 0.0)$), as x_r is reduced and x_c increased by 0.1 we encounter configurations with the same balance b (in the example $b = .7$) and a smaller distance z between firms (in the example, the sequence of z values is .7, 5, .3 and finally .1). Starting again at a configuration on the left edge of a table (for example, configuration $(x_r, x_c) = (0.4, 0.0)$) as both x_r and x_c are increased by 0.1 we encounter more balanced configurations (in the example the sequence of b values is .4, .6, .8 and finally 1) with the same distance between firms (in the example z is .4).

4.2 Comparative Results for Price Equilibria

For many locational configurations there are multiple price equilibria. With the smaller price bandwidth, there are multiple equilibria in the CG model for 12 of 30 configurations (and as many as 7 equilibria), and in the Hotelling model for 11 of 30 configurations (and as many as 5 equilibria). With the larger price bandwidth, in both models there are multiple equilibria for 8 of the 30 configurations – in the Hotelling model in each of these there are 3 equilibria, and in the CG model there are up to 5. Obviously, there is an equilibrium selection issue here, and we address it below. We can, however, say quite a lot about the extent to which prices and profits differ between the models without resolving the issue.

There should be an odd number of equilibria for any configuration and price band width, and *gambit-enummixed* promises to deliver all of them. Nevertheless, there are a small number of cases in the CG model where Gambit returns an even number of equilibria. We spent quite a lot of time trying to solve this problem, with no success.

In Table 3 we report, for every configuration, the ratio of the maximum expected mean price paid by consumers in an equilibrium of the CG model to the minimum expected mean price paid by consumers in an equilibrium of the Hotelling model. Results are for the smaller price bandwidth. Since quantity sold in equilibrium is 1 and marginal cost is 0, in any equilibrium expected mean price in an equilibrium is equal to expected aggregate profit in the equilibrium. If the ratio in Table 3 is less than 1 for some configuration, then regardless of how equilibria are selected expected equilibrium price and aggregate profit are higher in the Hotelling model than they are in the CG model. In just one of the configurations does the ratio exceed 1 (it is equal to 1.01 for configuration $(0.3, 0.1)$), and in 29 it is less than 1. In fact, in most cases it is well below 1 – for 23 of the 30 configurations it is less than .75, and for 16 of 30 it is less than .5. So it is clearly the case that for most locational configurations, the CG model is significantly more price competitive than the Hotelling model.

The smallest ratios in Table 3 are for relatively balanced configurations (specifically, those for which $b \geq .8$) in which the distance between the firms is not too large ($z \leq .6$). The intuition for this result is the following. In these configurations, a significant number of customers pass both firms on their daily journeys to and from work, so the price battle for these customers in the CG model is fierce. On the other hand, the distance between the firms is large enough so that in the Hotelling model one firm must post a price that is significantly lower than the other's posted price if it is to capture the entire market, which tends to limit price competition in this model.

For many of the configurations for which there are multiple price equilibria, there is a strict Pareto-dominant equilibrium – profits of both firms are higher in a strict Pareto-dominant equilibrium than they are in any of the other equilibria. In the Hotelling model, there is always such an equilibrium, and in the CG model there is one for four fifths of the configurations where there are multiple price equilibria. In Tables 4 through 9 we report results for selected equilibria. Our primary selection criterion is Pareto-dominance – if there is a Pareto-dominant equilibrium we select it. Our secondary criterion is aggregate profit – if there is no Pareto-dominant equilibrium, we select the equilibrium with the highest aggregate profit. Admittedly, our secondary criterion is somewhat arbitrary, but there does not appear to be a better or more defensible way of choosing among equilibria in the CG model when there is no Pareto-dominant equilibrium.[4]

In Tables 4 and 5 we report for all configurations expected equilibrium prices

[4]For completeness, we note that when the firms occupy the same location, there is a second pure strategy equilibrium in which both prices are the second lowest permissible price, .0361 with the smaller band width and .0461 with the larger price bandwidth, and this is the Pareto-dominant equilibrium. In this equilibrium, expected price and expected aggregate profit are either .0361 or .0461, somewhat smaller than all of the expected prices and aggregate profits in Tables 4 and 5.

in the selected price equilibrium for both price bandwidths and both models – the CG model in Table 4 and the Hotelling model in Table 5. The top and bottom entries in each cell are for the smaller and larger price bandwidths respectively. The first thing to note is that price bandwidth has no dramatic impact on expected equilibrium prices – the pattern of results across cells is the same for both bandwidths, and within cells the results are quite similar. The second thing to note is that expected prices in the selected equilibria tend to be quite a bit higher in the Hotelling model. In fact, in just one of 60 comparisons is expected equilibrium price higher in the CG model, for configuration $(0.3, 0.1)$ with the smaller price bandwidth, and in more than half of the configurations prices for both bandwidths are more than twice as high in the Hotelling model

In the following proposition we summarize results with respect to relative prices and aggregate profits in the two models.

Proposition 2: For a large majority of the configurations considered, expected price is higher in the Hotelling model than it is the CG model, and for half of them it is more than twice as high. In none of the configurations considered is expected price significantly larger in the CG model than in the Hotelling model. The difference in expected price is largest for configurations that are highly balanced ($b \geq .8$) and the distance between firms is not too large ($z \leq .6$).

One can also get some interesting insights regarding the comparative statics of expected prices in these models from Tables 4 and 5. In both models, if we fix b (balance) and vary z (distance between firms), we see that expected prices rise as z rises. In the Hotelling model, if we fix z and vary b, we see that expected prices rise as b rises. The results of the corresponding exercise for the CG model are quite different. First of all, when z is constant and less than .4, there is no monotonic relationship between expected prices and b – initially expected prices increase as b increases, but then they fall. Secondly, over the entire range of possibilities the tendency for expected prices to fall dominates, so as gross generalization it is fair to say that as configurations become more balanced, holding z fixed, expected prices fall in the GC model.

In Tables 6 and 7 we report, for the smaller price bandwidth, the expected equilibrium prices of both firms. In each cell the top entry is for the row firm and the bottom is for the column firm. Table 4 pertains to the CG model and Table 5 to the Hotelling model. In both models, the expected prices of the two firms are identical for perfectly balanced configurations (where $b = 1$). For configurations that are not perfectly balanced ($b < 1$) the column firm's expected price is lower than the row firm's, because the row firm has the larger protected market ($1 - x_r > x_c$). Comparing results across the two tables, we see that for 28 of 30 configurations, both prices are higher in the Hotelling model than in the CG model, and further that prices of both firms in the Hotelling model are more than twice as high as those in the CG model for 17 of 30 configurations.[5] This is consistent with Proposition 2.

[5]Results for configuration $(.3, .1)$ in Tables 4 and 5 are apparently at odds with the result in Table 3 for this configuration. In both models the price equilibrium is unique, so the

These tables also suggest that the ratio of the expected price of the column firm to that of the row firm is smaller in CG model than it is in the Hotelling model. Table 8 confirms this fact. In this table we report the ratio for both models. In each cell the top entry pertains to the CG model and the bottom entry to the Hotelling model. The following proposition captures the results reported in Table 8.

Proposition 3: For configurations that are not perfectly balanced, the normalized variation in price across firms is larger in the CG model than in the Hotelling model.

4.3 The Location Game

Although our primary interest is in the stage 2 price equilibrium, for completeness we briefly consider the stage 1 game in which firms choose locations. In Table 9 we report for both models the best response functions of the row firm in the stage 1 location game. A CG in a cell indicates a best response in the CG model, an H a best response in the Hotelling model, and a B a best response in both models. Results are for the larger price band width.

The best response functions tell us that, in both models, there is no tendency toward minimum differentiation. As the row firm approaches the column firm two forces are in play, one that attracts the row firm to the column firm and one that repels the row firm. The attractive force is driven by the fact that as the row firm gets closer to the column firm, the row firm's protected market gets bigger – consequently, holding prices fixed, quantity demanded from the row firm, and hence its profit, increases. The repulsive force is driven by the fact that as the row firm gets closer to the column firm, equilibrium prices fall – consequently, holding quantities fixed, profit of the row firm diminishes. In both models the forces come into balance when the distance between firms is still fairly large (when $z \geq .4$), so there is no tendency toward minium differentiation in either model.

Using the best response functions in Table 9 and the fact that the column firm's best response is symmetric to the row firm's, we can readily find the equilibria of the location game. In the Hotelling model, there are two equilibria – in the first, one firm is located at .7 and the other at .2; in the second one firm is located at .3 and the other at .8. For both equilibrium configurations, $z = .5$ and $b = .9$. In the CG model there are also two equilibria: in the first, one firm is located at .6 and the other at .1; in the second one firm is located at .9 and the other at .4. For both of these configurations, $z = .5$ and $b = .7$. So, while the distance between firms in the equilibrium configurations is .5 in both models, the equilibrium configurations in the CG model are less balanced than those in the Hotelling model.

ratio reported in Table 3 is based on the equilibria reported in Tables 4 and 5. There is no contradiction here. Even though the expected prices of both firms are higher in the Hotelling model than in the convenience model, aggregate profits are slightly higher in the convenience model owing to market share considerations.

5 Conclusions

The computational approach that we use in this paper is unusual, so it makes sense to say something about it. As a matter of fact, we never see prices like $\$\sqrt{2}$, and the computers that merchants use could not in fact handle such prices. As far as prices are concerned, the world of commerce is digital. In Canada, for example, prices are denominated in dollars and cents, and the price for any particular item offered by any retailer is invariably an integer multiple of one cent. Economists do not usually model prices in this discrete way, because it is more elegant, and often a lot simpler, to develop and examine theories using the set of non-negative real numbers as the price strategy space. Presumably, when we choose to do this, the choice reflects a view that the elegance and simplicity outweigh the loss of reality that is involved. In other words, a continuous strategy space is often a convenient simplifying assumption. But, in the CG model, a continuum of prices frustrates the effort to understand the implications of convenience shopping, because there is no price equilibrium in pure strategies and no handy way of finding the mixed strategy equilibrium. That being the case, there is a lot to be said in favor of positing a finite set of discrete prices and using numerical techniques to explore the equilibrium properties of the model. That is what we did in section 4 of this paper. Of course the inability to find a mixed strategy equilibrium in a continuous price strategy space is not peculiar to the CG model. This suggests that the computational approach used in this paper may have many fruitful applications.

The results reported in this paper suggest a general hypothesis, namely that convenience shopping is pro competitive. We can be somewhat more specific. In typical models of spatial competition, by assumption, convenience shopping is not possible. People are simply not allowed to buy goods as they move about their environment. Instead they are forced to make special purpose trips to buy them. The more specific hypothesis is this: if this arbitrary proscription of convenience shopping is eliminated in the context of a specific model of spatial competition, equilibrium prices in the model with convenience shopping will be lower than those in the same model without convenience shopping. And our results suggest that they might be a lot lower.

The Hotelling model of spatial competition spawned a large and still growing literature on horizontal product differentiation. In this literature, models of spatial differentiation like Hotelling's have alternative interpretations as models of horizontal differentiation in which goods are described by their location in an abstract product characteristics space, as opposed to a geographic landscape. The fact that models of spatial competition map to models of differentiation in characteristics space suggests that there may be a pro competitive phenomenon in these characteristics models analogous to convenience shopping. With this in mind, we reinterpret the CG model as a model of horizontal differentiation with customers who are *not too picky* about their preferred good. We call it the NTP model of horizontal differentiation.

The good offered by firm i is described by (x_i, p_i). The good's characteristic is $x_i, 0 \leq x_i \leq 1$, and its price is p_i. Instead of being a location on a highway, x_i

is a point in an abstract product characteristic space. A customer is described by two parameters, $(z_1, z_2), 0 \leq z_1 \leq z_2 \leq 1$, that define a *preferred set* of product characteristics, namely the set $\{x|z_1 < x < z_2\}$. The customer is indifferent to any two products offered at identical prices if both are in her preferred set. In the usual address model the preferred set of every customer is just a point in the characteristic space – every customer has exactly one preferred product. In the NTP model we are sketching here, customers are not so picky about what they buy – their preferred sets are *fat*. The larger is the customer's preferred set, the less picky is the customer. Preferences in the NTP model are described by the following indirect utility function. Given the opportunity to buy a product with characteristic x at price p, the customer's utility is

$$\begin{aligned} V &= 0 \text{ if } q \leq 0 \\ V &= v - p \text{ if } q \geq 1 \text{ and } z_1 < x < z_2 \\ V &= v - p - t(\ z_1 - x) \text{ if } q \geq 1 \text{ and } x < z_1 \\ V &= v - p - t(\ x - z_2) \text{ if } q \geq 1 \text{ and } z_2 < x \end{aligned}$$

If the consumer does not buy at least one unit of the good (if $q \leq 0$), her utility is 0. If she buys at least one unit of the good (if $q \geq 1$), her utility is $v - p$ if the good's characteristic is in her preferred set, and it is $v - p - tr$ if the good is a distance r from the nearest boundary of her preferred set. Given the choice between two products, (x_1, p_1) and (x_2, p_2), that yield indirect utilities V_1 and V_2, the customer buys neither good if both V_1 and V_2 are less than 0, and otherwise she buys one unit of the good with the larger indirect utility.

To complete the homeomorphism between the NTP model and the CG model we need to do a bit more. First we need to define the parameters that describe an individual in the NTP model, (z_1, z_2), in terms of the parameters we used to describe an individual in the CG model, (y_1, y_2). The following does the trick.

$$z_1 = \min(y_1, y_2) \text{ and } z_2 = \max(y_1, y_2)$$

Then we need to construct the implied density function for (z_1, z_2) in the NTP model. In the CG model, we assumed that there was a uniform density, equal to 1, of (y_1, y_2) on the unit square pictured in Figure 1. For the NTP model, this density assumption yields a uniform density of (z_1, z_2), equal to 2, on the triangle in (z_1, z_2) space with corners (0,0), (0,1) and (1,0). This completes the homeomorphism.

One might, of course, very well prefer some other density function for the NTP model. However, as long as the preferred product sets are fat (not just points), or as long as customers are *not too picky*, the demand functions for the NTP model will exhibit the same pattern of discontinuities seen in the CG model, and these discontinuities will have pro competitive effects similar to the pro competitive effects we saw in the CG model.

It is, of course, clear to anyone who is familiar with models in the Hotelling tradition that any model of spatial competition is homeomorphic to some address model of horizontal differentiation. There is nothing new in this. For those of us who study product differentiation, what might be worth thinking about is the following. If a significant portion of customers do in fact have fat preferred product sets, then there will be some customers who will see products that are differentiated in characteristics as perfect substitutes. The battle to attract these not very picky customers will make these models somewhat more competitive than the related model where all preferred product sets are just points in characteristics space.

Table 3

Relative Prices (and Aggregate Profits)

$$\frac{\textbf{Maximum Expected Equilibrium Price in Convenience Model}}{\textbf{Minimum Expeted Equilibrium Price in Hotelling Model}}$$

Price Band Width: .0351

	0.0	**0.1**	**0.2**	**0.3**	**0.4**
0.1	.97	—	—	—	—
0.2	.96	.86	—	—	—
0.3	.95	1.01	.81	—	—
0.4	.71	.82	.68	.53	—
0.5	.58	.62	.45	.37	.36
0.6	.45	.45	.34	.25	.28
0.7	.47	.42	.28	.22	—
0.8	.49	.40	.31	—	—
0.9	.50	.43	—	—	—
1.0	.54	—	—	—	—

Table 4

Expected Customer Prices, Convenience Model

Price Band Widths: .0351 (top) and .0451 (bottom)

	0.0	**0.1**	**0.2**	**0.3**	**0.4**
0.1	.10	—	—	—	—
	.13	—	—	—	—
0.2	.20	.10	—	—	—
	.21	.12	—	—	—
0.3	.33	.26	.13	—	—
	.33	.27	.14	—	—
0.4	.37	.36	.24	.10	—
	.39	.35	.25	.13	—
0.5	.43	.40	.26	.16	.07
	.43	.39	.25	.16	.09
0.6	.42	.40	.27	.18	.12
	.42	.40	.27	.18	.14
0.7	.47	.41	.28	.20	—
	.44	.40	.28	.20	—
0.8	.48	.40	.30	—	—
	.46	.41	.30	—	—
0.9	.49	.42	—	—	—
	.48	.45	—	—	—
1.0	.53	—	—	—	—
	.54	—	—	—	—

Table 5

Expected Customer Prices, Hotelling Model

Price Band Widths: .0351 (top) and .0451 (bottom)

	0.0	**0.1**	**0.2**	**0.3**	**0.4**
0.1	.11	—	—	—	—
	.13	—	—	—	—
0.2	.21	.11	—	—	—
	.22	.14	—	—	—
0.3	.35	.26	.16	—	—
	.36	.28	.20	—	—
0.4	.52	.44	.35	.19	—
	.50	.43	.36	.24	—
0.5	.73	.66	.58	.43	.21
	.76	.61	.55	.45	.28
0.6	.92	.88	.81	.70	.47
	.93	.90	.78	.69	.51
0.7	1.03	1.02	1.01	.93	—
	1.00	1.02	.99	.90	—
0.8	1.02	1.02	1.02	—	—
	1.02	.99	1.04	—	—
0.9	1.02	1.02	—	—	—
	.99	1.04	—	—	—
1.0	1.02	—	—	—	—
	1.04	—	—	—	—

Table 6

Expected Prices of Firms r and c, Convenience Model

Price Band Width: .0351

	0.0	**0.1**	**0.2**	**0.3**	**0.4**
0.1	.11	—	—	—	—
	.06	—	—	—	—
0.2	.24	.12	—	—	—
	.13	.06	—	—	—
0.3	.40	.30	.15	—	—
	.21	.21	.10	—	—
0.4	.46	.42	.29	.11	—
	.25	.28	.21	.09	—
0.5	.49	.46	.30	.19	.09
	.32	.34	.23	.15	.07
0.6	.49)	.44	.31	.20	.13
	.32	.35	.25	.17	.13
0.7	.53	.46	.30	.20	—
	.39	.37	.27	.20	—
0.8	.53	.42	.31	—	—
	.42	.37	.31	—	—
0.9	.53	.42	—	—	—
	.46	.42	—	—	—
1.0	.53	—	—	—	—
	.53	—	—	—	—

Table 7

Expected Prices of Firms r and c, Hotelling Model

Price Band Width: .0351

	0.0	**0.1**	**0.2**	**0.3**	**0.4**
0.1	.11	—	—	—	—
	.08	—	—	—	—
0.2	.24	.13	—	—	—
	.16	.12	—	—	—
0.3	.42	.31	.18	—	—
	.28	.26	.16	—	—
0.4	.61	.50	.39	.20	—
	.43	.41	.38	.20	—
0.5	.83	.73	.62	.45	.22
	.63	.61	.59	.45	.23
0.6	1.03	.96	.86)	.73	.49
	.80	.81	.80	.72	.49
0.7	1.12	1.09	1.04	.94	—
	.91	.95	.98	.94	—
0.8	1.09	1.05	1.02	—	—
	.95	.98	1.02	—	—
0.9	1.05	1.02	—	—	—
	.98	1.02	—	—	—
1.0	1.02	—	—	—	—
	1.02	—	—	—	—

Table 8

Relative Prices of Firms, Both Models

$$\frac{\text{Lower Expected Price}}{\text{Higher Expected Price}}$$

Price Band Width: .0351

	0.0	**0.1**	**0.2**	**0.3**	**0.4**
0.1	.55	—	—	—	—
	.73	—	—	—	—
0.2	.54	.50	—	—	—
	.67	.92	—	—	—
0.3	.53	.70	.67	—	—
	.67	.84	.89	—	—
0.4	.54	.67	.72	.82	—
	.70	.82	.97	1.00	—
0.5	.65	.74	.77	.79	.78
	.76	.84	.95	1.00	1.05
0.6	.65	.80	.81	.85	1.00
	.78	.84	.93	.99	1.00
0.7	.74	.80	.90	1.00	—
	.81	.87	.94	1.00	—
0.8	.79	.88	1.00	—	—
	.87	.93	1.00	—	—
0.9	.87	1.00	—	—	—
	.93	1.00	—	—	—
1.0	1.00	—	—	—	—
	1.00	—	—	—	—

Table 9

Row Firm's Best Responses, Both Models

Price Band Width: .0451

	0.0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0.0						H					
0.1						CG	B	CG			
0.2								H			
0.3									H	H	H
0.4									CG	CG	
0.5	CG										CG
0.6		CG	CG								
0.7	H	H	H								
0.8				H							
0.9				CG	B	CG					
1.0						H					

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