Markov Switching Oil Price Uncertainty*

Apostolos Serletis†
Department of Economics
University of Calgary
Calgary, Alberta, T2N 1N4
Canada

and

Libo Xu
Department of Economics
University of San Francisco
San Francisco, California, CA 94117
U.S.A.

Forthcoming in: *Oxford Bulletin of Economics and Statistics*

January 2, 2019

*We would like to thank Jonathan Temple and an anonymous referee for comments that greatly improved the paper.

†Corresponding author. Phone: (403) 220-4092; Fax: (403) 282-5262; E-mail: Serletis@ucalgary.ca; Web: http://econ.ucalgary.ca/serletis.htm.

Abstract:

We investigate whether the United States economy responds negatively to oil price uncertainty and whether oil price shocks exert asymmetric effects on economic activity. In doing so, we relax the assumption in the existing literature that the data are governed by a single process, modifying the Elder and Serletis (2010) bivariate structural GARCH-in-Mean VAR to accommodate Markov regime switching in order to account for changing oil price dynamics over the sample period. We find evidence of asymmetries, against those macroeconomic theories that predict symmetries in the relationship between real aggregate economic activity and the real price of oil.

Word count: 4,525 + 3 Tables and 9 Figures
JEL classification: C32, E32, G31
Keywords: Oil price uncertainty, Real options, GARCH-in-Mean VAR, Markov switching.

1 Introduction

There is an ongoing debate in macroeconomics about how oil price shocks and oil price uncertainty affect the level of economic activity. Those of the view that positive oil price shocks have been the major cause of recessions in the United States (and other oil-importing countries) as, for example, Hamilton (1983, 1996, 2011), Hooker (1996), and Herrera *et al.* (2011), appeal to models that imply asymmetric responses of real output to oil price increases and decreases. These models are able to explain larger economic contractions in response to positive oil price shocks and smaller economic expansions in response to negative ones. On the other hand, those of the view that positive oil price shocks do not cause recessions as, for example, Kilian (2008), Edelstein and Kilian (2009), and Kilian and Vigfusson (2011a,b), appeal to theoretical models of the transmission of exogenous oil price shocks that imply symmetric responses of real output to oil price increases and decreases. These models cannot explain large declines in the level of economic activity in response to positive oil price shocks.

A series of recent papers by Elder and Serletis (2010, 2011), Bredin *et al.* (2011), Rahman and Serletis (2011, 2012), Pinno and Serletis (2013), Jo (2014), Elder (2018), and Serletis and Mehmandosti (2018) also look at the relationship between the price of oil and the level of economic activity, focusing on the role of uncertainty about the price of oil. They appeal to the real options theory, also known as investment under uncertainty, which predicts that firms are likely to delay making irreversible investment decisions in the face of uncertainty about the price of oil, particularly when the cash flow from investments is contingent on the oil price — see, for example, Bernanke (1983), Brennan and Schwartz (1985), Madj and Pindyck (1987), Brennan (1990), Gibson and Schwartz (1990), and Dixit and Pindyck (1994). In doing so, they utilize internally-consistent simultaneous equations empirical models that accommodate an independent role for the effects of oil price uncertainty. They find that oil price uncertainty has had a negative and statistically significant effect on several measures of investment, durables consumption, and aggregate output. They also find that accounting for the effects of oil price uncertainty tends to exacerbate the negative dynamic response of economic activity to a negative oil price shock, while dampening the response to a positive one.

Most of the recent literature on the macroeconomic effects of oil price shocks and oil price uncertainty assumes that the data are governed by a single process. However, as recently noted by Serletis and Mehmandosti (2018, p. 9), "the crude oil market in the United States (and globally) has undergone very significant structural changes over the past 150 years due to technological innovation, revisions in regulatory regimes and market structures, the emergence of new players, such as OPEC, and changes in the sources of supply and demand." Motivated by these considerations, we assume a more complex process to allow for different types of relationship at different times. Two classes of models that allow this to occur have been used so far in the literature — the 'time-varying coefficient model' and the 'Markov switching model.' The former assumes that the relationship changes in every period (which might not be the case in the real world), whereas the latter assumes a switching mechanism from one state to another that is controlled by an unobserved variable governed by a Markov process.

In this paper we take the Markov switching approach, associated with Hamilton (1989, 1990), which has been widely followed in the analysis of economic and financial time series

— see, for example, Garcia and Perron (1989) and Sims and Zha (2006). In doing so, we modify the Elder and Serletis (2010) methodology by assuming Markov regime switching to account for changing oil price dynamics over the sample period. In particular, we use a Markov switching (identified) structural GARCH-in-Mean VAR in real output growth and the change in the real price of oil, associate the oil price change VAR residual with exogenous oil price shocks, use the conditional standard deviation of the forecast error for the change in the real oil price as a measure of uncertainty about the impending real price of oil, and investigate the relationship between the real price of oil and the level of real economic activity in the United States using quarterly data since the early 1970s.

In the context of a mean-square stable structural GARCH-in-Mean VAR with Markov regime switching, we find that oil price uncertainty has a negative and statistically significant effect on the real output growth rate, and that this effect is asymmetric over contractions and expansions in the business cycle, being significantly larger during contractions and periods when large oil price changes occur. We also find that oil price uncertainty tends to amplify the negative dynamic response of the growth rate to positive oil price shocks, and to dampen the positive response to negative oil price shocks during expansions in the business cycle.

The rest of the paper is organized as follows. Section 2 reviews the related empirical literature regarding the macroeconomic effects of oil price uncertainty and the effects of positive and negative oil price shocks on the level of economic activity. In Section 3 we modify the Elder and Serletis (2010) bivariate structural GARCH-in-Mean VAR, by assuming Markov regime switching. We discuss identification and estimation issues in Section 4, present the empirical results in Section 5, and investigate regime stability issues in Section 6. The final section concludes the paper.

2 Literature Review and Motivation

As noted in the Introduction, our paper contributes to an extensive body of literature that investigates the macroeconomic effects of oil price shocks and oil price uncertainty. Much of the literature uses the Elder and Serletis (2010) model. It is a bivariate structural GARCH-in-Mean VAR in real output growth and the change in the real price of oil, as follows

$$\boldsymbol{B}\boldsymbol{z}_t = \boldsymbol{C} + \sum_{i=1}^{k} \boldsymbol{\Gamma}_i \boldsymbol{z}_{t-i} + \boldsymbol{\Psi}\sqrt{\boldsymbol{h}_t} + \boldsymbol{\epsilon}_t \tag{1}$$

$$\boldsymbol{\epsilon}_t | \, \Omega_{t-1} \sim (\boldsymbol{0}, \boldsymbol{H}_t), \quad \boldsymbol{H}_t = \begin{bmatrix} h_{o,t} & 0 \\ 0 & h_{y,t} \end{bmatrix}$$

where $\boldsymbol{z}_t$ is a column vector in the change in the real price of oil, $\Delta \ln o_t$, and the real output growth rate (in the rest of the paper, we will refer to it as the growth rate), $\Delta \ln y_t$, $\boldsymbol{z}_t = [\Delta \ln o_t \;\; \Delta \ln y_t]'$, Ω_{t-1} denotes the information set at time $t-1$, and

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}; \quad \boldsymbol{\Gamma}_i = \begin{bmatrix} \gamma_{i,11} & \gamma_{i,12} \\ \gamma_{i,21} & \gamma_{i,22} \end{bmatrix}; \quad \boldsymbol{\Psi} = \begin{bmatrix} 0 & 0 \\ \psi & 0 \end{bmatrix}; \quad \boldsymbol{h}_t = \begin{bmatrix} h_{o,t} \\ h_{y,t} \end{bmatrix}; \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_{o,t} \\ \epsilon_{y,t} \end{bmatrix}.$$

The system is identified, as in Elder and Serletis (2010), by assuming that the diagonal elements of $\boldsymbol{B}$ are unity, that $\boldsymbol{B}$ is lower triangular, and that the structural shocks, $\boldsymbol{\epsilon}_t$, are uncorrelated.

Elder and Serletis (2010) use a univariate GARCH(1,1) specification to model the conditional variance of the change in the real price of oil, $h_{o,t}$, and the conditional variance of the growth rate, $h_{y,t}$, as follows

$$h_{o,t} = d_{11} + d_{12}\epsilon_{o,t-1} + d_{13}h_{o,t-1} \tag{2}$$

$$h_{y,t} = d_{21} + d_{22}\epsilon_{y,t-1} + d_{23}h_{y,t-1}. \tag{3}$$

They estimate their model by full information maximum likelihood, avoiding Pagan's (1984) generated regressor problems associated with estimating the variance function parameters separately from the conditional mean parameters. The procedure is to maximize the log likelihood with respect to the structural parameters — see Elder and Serletis (2010) for more details.

We estimate the bivariate structural GARCH-in-Mean VAR, consisting of equations (1)-(3), using quarterly data for the United States over the period from 1974:1 to 2016:3. For the real output series, y_t, we use the real GDP series from the Bureau of Economic Analysis. For the oil price series, o_t, we use the composite refiners' acquisition cost (RAC) of crude oil, as compiled by the U.S. Department of Energy, and convert it in real terms by dividing it by the GDP deflator. In Figure 1 we plot the natural log of the real oil price, $\ln o_t$, and its growth rate, $\Delta \ln o_t$, and in Figure 2 the natural log of real output, $\ln y_t$, and its growth rate, $\Delta \ln y_t$.

We follow Elder and Serletis (2010) and estimate the model for $k = 4$ in equation (1), using the logarithmic first differences of the data, $\Delta \ln y_t$ and $\Delta \ln o_t$, over the period from 1974:1 to 2016:3. The estimation results are reported in Table 1. We find that oil price uncertainty has a negative, but statistically insignificant effect on the real output growth rate: $\hat{\psi} = -0.001$ (with a p-value of 0.866). This is in contrast with the evidence reported by Elder and Serletis (2010) who found that oil price uncertainty has a negative and statistically significant effect on $\Delta \ln y_t$; in particular, using quarterly data from 1974:1 to 2008:1, they report that $\hat{\psi} = -0.022$ (with a t-statistic of 2.30). To remove the inconsistency between the results presented in Table 1 and those reported in Elder and Serletis (2010), we re-estimate the model over the Elder and Serletis (2010) sample period, from 1974:1 to 2008:1, and report the results in Table 2. As can be seen in Table 2, oil price uncertainty has a negative and statistically significant effect on the growth rate, $\hat{\psi} = -0.017$ (with a p-value of 0.008), consistent with Elder and Serletis (2010).

Our evidence, however, suggests that the empirical results in Elder and Serletis (2010) are not robust to including the period after the global financial crisis and Great Recession, potentially because of a change in the economic regime and oil price dynamics. In order to address the issue of changing oil price dynamics and economic regimes over the sample period, in the next section we modify the Elder and Serletis (2010) methodology by assuming Markov regime switching, thus allowing for complicated nonlinear dynamics and changes in the parameters of the structural GARCH-in-Mean VAR. This framework is sufficiently flexible to allow for different types of responses at different times and also utilizes all of the available observations on the series in the estimation of these responses.

3 A Markov Switching Structural VAR

Our empirical framework modifies the Elder and Serletis (2010) mean equation (1) as follows

$$\boldsymbol{B}_{s_t}\boldsymbol{z}_t = \boldsymbol{C}_{s_t} + \sum_{i=1}^{k}\boldsymbol{\Gamma}_{i,s_t}\boldsymbol{z}_{t-i} + \boldsymbol{\Phi}_{s_t}\sqrt{h_{o,t,s_t}} + \boldsymbol{\epsilon}_{t,s_t} \tag{4}$$

$$\boldsymbol{\epsilon}_{t,s_t}|\,\Omega_{t-1} \sim \left(\mathbf{0},\, \boldsymbol{H}_{t,s_t}\right), \quad \boldsymbol{H}_{t,s_t} = \begin{bmatrix} h_{o,t,s_t} & 0 \\ 0 & h_{y,s_t} \end{bmatrix}, \quad \boldsymbol{\Phi}_{s_t} = \begin{bmatrix} 0 \\ \phi_{s_t} \end{bmatrix}$$

Given the scope of the present paper, we keep the Elder and Serletis (2010) setup, and allow Φ_{s_t} to capture the effect of oil price uncertainty on the growth rate. This is the only GARCH-in-Mean term that we consider, and in doing so, we also impose a restriction that oil price uncertainty does not affect the price of oil, as in Elder and Serletis (2010). s_t denotes the unobserved economic regime, and is assumed to follow a first order, homogeneous, two-state Markov chain governed by the transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where $p_{ij} = P\left(s_t = i\,|s_{t-1} = j\right)$, $i, j = 1, 2$ and $p_{11} = 1 - p_{21}$ and $p_{12} = 1 - p_{22}$. All the parameters in the $\boldsymbol{B}_{s_t}$, $\boldsymbol{C}_{s_t}$, $\boldsymbol{\Gamma}_{s_t}$, and $\boldsymbol{\Phi}_{s_t}$ matrices are regime-dependent, taking different values across the two regimes (i and j can only take two values). We assume two regimes in our estimation because of the prior we get from replicating Elder and Serletis (2010). In particular, since including the period of the global financial crisis changes the results significantly, it follows that oil price uncertainty may exert asymmetric effects on output with respect to expansions and contractions in economic activity. Therefore, the two assumed regimes should be able to describe the dynamic interactions between the real price of oil and the real output growth rate across contractionary and expansionary phases of the business cycle. Moreover, as suggested by Hamilton (1988, 1989) the two-regime model is often sufficient.

We also modify the Elder and Serletis (2010) specification for the conditional variances of $\Delta \ln o_t$ and $\Delta \ln y_t$. As in Elder and Serletis (2010) we assume that the conditional variance of $\Delta \ln o_t$, h_{o,t,s_t}, follows a GARCH(1,1) specification

$$h_{o,t,s_t} = d_{1,s_t} + d_{2,s_t}\epsilon_{o,t-1,s_{t-1}} + d_{3,s_t}h_{o,t-1,s_{t-1}} \tag{5}$$

Regarding the variance of the growth rate, h_{y,s_t}, we assume that it takes different values in the two regimes to simplify the estimation of the highly non-linear model, instead of assuming a GARCH(1,1) specification as in Elder and Serletis (2010).

4 Identification, Path Dependence, and Estimation

The lower triangular $\boldsymbol{B}_{s_t}$ matrix implies that the growth rate responds to the contemporaneous change in the oil price. Under such an identifying assumption, oil price shocks could be treated as predetermined, and this assumption has been adopted by Edelstein and Kilian

(2007), Kilian (2009), and Elder and Serletis (2010). In particular, Kilian (2009) states that this assumption is not testable, however, it can be well defended as detailed in Kilian (2009).

The estimation of our framework is also challenging. The GARCH specification (5) for the oil price implies that h_{o,t,s_t} depends on s_t and also indirectly on $\{s_{t-1}, s_{t-2}, \ldots\}$. That is, h_{o,t,s_t} at time t depends on the entire sequence of regimes up to time t. One has to construct the likelihood function by integrating over all possible paths and as it turns out the estimation is not tractable; this is a problem, called path dependence, that typically shows up in the estimation of regime switching GARCH models. To address this problem, we need to use a collapsing procedure that could facilitate evaluation of the likelihood function. In this paper, we follow Gray (1996) and integrate out the regime-dependent error term $\epsilon_{o,t}$ and the regime-dependent variance h_{o,t,s_t} at time $t-1$ by taking the expectation so that the GARCH specification does not require the entire sequence of regimes up to time t. Therefore, we construct the regime-independent error term $\bar{\epsilon}_{o,t}$ and the regime-independent oil price volatility $\bar{h}_{o,t}$ by calculating

$$\bar{\epsilon}_{o,t} = p(s_t = 1|\Omega_{t-1})\epsilon_{o,t,s_t=1} + p(s_t = 2|\Omega_{t-1})\epsilon_{o,t,s_t=2}$$

and

$$\begin{gathered}\bar{h}_{o,t} = p(s_t = 1|\Omega_{t-1})[(\Delta \ln o_t - \epsilon_{o,t,s_t=1})^2 + h_{o,t,s_t=1}] \\ +p(s_t = 2|\Omega_{t-1})[(\Delta \ln o_t - \epsilon_{o,t,s_t=2})^2 + h_{o,t,s_t=2}] \\ -[p(s_t = 1|\Omega_{t-1})(\Delta \ln o_t - \epsilon_{o,t,s_t=1}) + p(s_t = 2|\Omega_{t-1})(\Delta \ln o_t - \epsilon_{o,t,s_t=2})]^2\end{gathered}$$

where $p(s_t = 1|\Omega_{t-1})$ and $p(s_t = 2|\Omega_{t-1})$ are the prediction probabilities from the Hamilton (1989) filter. We then plug $\bar{\epsilon}_{o,t}$ and $\bar{h}_{o,t}$ into the GARCH specification (5) for the oil price so that it becomes

$$h_{o,t,s_t} = d_{1,s_t} + d_{2,s_t}\bar{\epsilon}_{o,t-1} + d_{3,s_t}\bar{h}_{o,t-1}. \tag{6}$$

Thus, h_{o,t,s_t} depends only on the value of s_t and the likelihood function becomes tractable. Under the normality assumption, the density at time t conditional on $s_t = i$ is

$$f(\boldsymbol{z}_t|s_t = i, \Omega_{t-1}) = \frac{1}{2\pi\sqrt{|\boldsymbol{H}_{t,s_t=i}|}} exp\left(-\frac{1}{2}\boldsymbol{\epsilon}^T_{t,s_t=i}\boldsymbol{H}^{-1}_{t,s_t=i}\boldsymbol{\epsilon}_{t,s_t=i}\right)$$

and the density at time t can be found by summing over all possible regimes

$$f(\boldsymbol{z}_t|\Omega_{t-1}) = \sum_{i=1}^{2} f(\boldsymbol{z}_t|s_t = i, \Omega_{t-1})p(s_t = i|\Omega_{t-1}).$$

The log-likelihood is then

$$L = \sum_{t=1}^{T} \ln f(\boldsymbol{z}_t|\Omega_{t-1})$$

with the first element of $vech(\boldsymbol{H}_{t,s_t=i})$, h_{o,t,s_t}, being governed by equation (6). On the other hand, the prediction probabilities are handled by the Hamilton (1989) filter. The estimation is performed in RATS 9.0 using Maximum Likelihood and the BFGS (Broyden, Fletcher, Goldfarb & Shanno) algorithm combined with the derivative-free Simplex pre-estimation method. To implement the estimation, we choose a set of initial values based on the conventional structural GARCH-in-Mean VAR and the pre-estimation of our Markov switching structural GARCH-in-Mean VAR.

To validate our Markov switching model, we estimate it with $k = 4$ in equation (4), and conduct a model comparison using the Bayes factor

$$\text{BF} = \frac{\Pr\left(\text{Data}\,|\text{Unrestricted model}\,\right)}{\Pr\left(\text{Data}\,|\text{Restricted model}\,\right)}$$

where the single regime (or non-Markov switching) structural GARCH-in-Mean VAR is the restricted model. According to Kass and Raftery (1995), the logarithm of the Bayes factor could be approximated using the Bayesian information criterion (BIC), as follows

$$2\ln\text{BF} \approx -\ (\text{BIC of Unrestricted model}\ -\ \text{BIC of Restricted model})$$

with $2\,\text{lnBF} > 2$ providing support for the unrestricted model. In our case, we achieve $2\,\text{lnBF} = 5.862$, which is evidence that the Markov switching structural GARCH-in-Mean VAR should be preferred, compared to its restricted (single regime) version.

5 Empirical Evidence

We estimate the Markov switching structural GARCH-in-Mean VAR described in the previous section with $k = 4$ in equation (4), and report the point estimates of the mean and variance function parameters in Table 3, with p-values in parentheses. According to the $\hat{\phi}$ values reported in Table 3, oil price uncertainty has a negative and statistically significant effect on the growth rate in both regimes. We also find that oil price uncertainty exerts asymmetric effects on real output with respect to economic conditions, as the effect is larger in regime 2, since $\hat{\phi}_{s_t=1} = -0.029$ (with a p-value of 0.037) and $\hat{\phi}_{s_t=2} = -0.605$ (with a p-value of 0.000).

To better understand the two regimes, we calculate the smoothed probabilities

$$p(s_t = i\,|\Omega),\ i \in \{1, 2\}$$

where Ω is the full sample information, and plot the probability of regime 2 in Figure 3, together with Hamilton's (1996, 2003) net oil price increase over the previous four quarters, $\tilde{x}_t$, calculated as a nonlinear function of the growth rate of the real oil price

$$\tilde{x}_t = \max\left[0, \ln o_t - \max\left\{\ln o_{t-1}, \ln o_{t-2}, \ln o_{t-3}, \ln o_{t-4}\right\}\right]$$

in order to filter out increases in the price of oil that represent corrections for recent declines. As can be seen, regime 2 is generally consistent with contractions in the business cycle, includes the recent Great Recession, and corresponds to the times when large oil price changes occur. Moreover, the unconditional variance of the oil price change in regime 2, calculated as $\hat{d}_{21}/(1 - \hat{d}_{22} - \hat{d}_{23})$, is 911.610 and is larger than that in regime 1. We thus conclude that oil price uncertainty is more likely to have a larger negative effect on the growth rate during contractions in the business cycle and at times when large oil price changes occur.

To assess the dynamic response of the growth rate to oil price shocks, we plot the impulse response functions in Figures 4-6. They are obtained as in Elder (2003) and are based on an oil price shock which is the unconditional standard deviation of the change in the real price of oil. In doing so, we report the impulse responses of the growth rate to both positive and negative oil price shocks to address the issue of whether the relationship between real output and the real price of oil is symmetric or asymmetric. Our impulse response functions are regime-dependent, in the sense that they measure the response of the growth rate from time t to time $t + n$ to an oil shock at time t assuming the regime is the same from time t to time $t + n$.

As can be seen in Figure 4, the dynamic response of the real output growth rate to an oil price shock is different in the two regimes. In particular, a positive oil price shock has a large negative and persistent effect on real output in regime 2, but a small negative and nonpersistent effect in regime 1. Similarly, a negative oil price shock has a large positive and persistent effect on real output in regime 2, but almost no effect in regime 1. To assess the statistical significance of the impulse responses reported in Figure 4, we show these responses, together with one-standard error bands, in Figure 5 (in the case of regime 1) and Figure 6 (in the case of regime 2). As can be seen, the negative response of the growth rate to a positive oil price shock is statistically significant in both regimes whereas the positive response to a negative oil price shock is statistically significant only in regime 2. On the basis of these tests, we infer that the responses of the U.S. economy to positive and negative oil price shocks are asymmetric.

Moreover, the impulse response functions across the two regimes are suggesting a new asymmetry. In particular, we find that the effect of an oil price shock on the growth rate, captured by the impulse response function, and the effect of oil price uncertainty on the growth rate, captured by ϕ, are significantly larger in regime 2, suggesting that the U.S. economy tends to be particularly vulnerable to oil price shocks prior to and during recessions. In this regard, Kilian and Vigfusson (2017, p. 1749), in their investigation of the conditional effects of oil price shocks during episodes of net oil price increases, argue that "the magnitude of the conditional response is correlated with a number of indicators of macroeconomic conditions such as consumer confidence, financial stress, the share of oil in GDP, and interest rate expectations that may indicate a heightened vulnerability of the economy to oil price shocks, but most of the time variation appears linked to the prior evolution of the real price of oil." Thus, our empirical results are not only consistent with the literature, but also contribute to the literature, as we find that oil price uncertainty has a larger negative effect on the growth rate at times when the economy is vulnerable to oil price shocks.

To assess how oil price uncertainty contributes to the dynamics of real economic activity, we compare the impulse responses of real output with those impulse responses when ϕ is restricted to be equal to zero. We report these constrained impulse responses, plotted as dashed lines, in Figure 7 (in the case of regime 1) and Figure 8 (in the case of regime 2). In doing so, we also repeat the unconstrained impulse responses (from Figures 5 and 6, respectively), plotted as solid lines, and suppress the error bands for clarity. That is, the solid lines in Figures 7 and 8 are constructed by using the parameter estimates from our bivariate Markov switching GARCH-in-mean VAR with the coefficient on oil price uncertainty in the real output growth equation, ϕ, being unconstrained, and the dashed lines are constructed by using the same parameter estimates but with ϕ constrained to zero.

As can be seen in the first panel of Figure 7, when oil price uncertainty is accounted for, the response of real output growth to a positive oil price shock is negative and more pronounced than when $\phi = 0$. To put it differently, the response of real output growth to a positive oil price shock is amplified when we allow feedback from the conditional standard deviation of oil price changes to output growth. Also, as can be seen in the second panel of Figure 7, when oil price uncertainty is accounted for, the response of real output growth to a negative oil price shock is positive and less pronounced than when $\phi = 0$. That is, the response of real output growth to a negative oil price shock is dampened when we allow feedback from the conditional standard deviation of oil price changes to output growth. These results are consistent with those in Elder and Serletis (2010), and suggest that oil price uncertainty tends to reduce the growth rate during expansions in the business cycle.

Finally, in order to quantify the dynamic response of real output to oil price shocks and oil price uncertainty in regime 2, we plot the unconstrained and constrained impulse responses to positive and negative oil price shocks in Figure 8, in the same fashion as those in Figure 7 for regime 1. We find that the impulse responses of the growth rate to positive and negative oil price shocks are almost the same irrespective of whether ϕ is unconstrained or constrained to zero, suggesting that in regime 2 oil price uncertainty only has a large negative and statistically significant effect on the growth rate on impact (since $\hat{\phi}_{s_t=2} = -0.612$ with a p-value of 0.000 in Table 3). The reason for this result is the absence of the ARCH effect in regime 2, rendering oil price uncertainty nonpersistent.

6 Expected Duration and Stability of the Two Regimes

According to Figure 3, one of the regimes (regime 2) shows up infrequently and is not long-lasting. To quantify the persistence of each regime, we follow Hamilton (1989) and calculate the expected duration of each regime. We find that the expected duration of regime 1 is approximately $1/(1 - \hat{p}_{11}) \approx 14.1$ quarters, and that of regime 2 approximately $1/(1 - \hat{p}_{22}) \approx 1.5$ quarters, suggesting that the economy spends most of the time in regime 1 and that the effect of oil price uncertainty on the growth rate is less severe. However, this does not necessarily imply that the conventional (one-regime) model is enough for us to understand the effect of oil price uncertainty on the growth rate, since the magnitude of that effect is significantly different across the two regimes.

We believe it is important to assess the stability of the two regimes. From an economic point of view, it would be worrying if one or both regimes are unstable, which would make it difficult to interpret our findings. Note that equation (4) with 4 lags could be rewritten as

$$\boldsymbol{z}_t = \boldsymbol{B}_{s_t}^{-1}\boldsymbol{C}_{s_t} + \sum_{i=1}^{k} \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Gamma}_{i,s_t}\boldsymbol{z}_{t-1} + \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Phi}_{s_t}\sqrt{h_{o,t,s_t}} + \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\epsilon}_{t,s_t}$$

and is equivalent to

$$\boldsymbol{Z}_t = \widetilde{\boldsymbol{C}}_{s_t} + \tilde{\boldsymbol{\Gamma}}_{s_t}\boldsymbol{Z}_{t-1} + \tilde{\boldsymbol{\Phi}}_{s_t}\sqrt{h_{o,t,s_t}} + \widetilde{\boldsymbol{\epsilon}}_{t,s_t} \tag{7}$$

where

$$\boldsymbol{Z}_t = \begin{bmatrix} \boldsymbol{z}_t \\ \boldsymbol{z}_{t-1} \\ \boldsymbol{z}_{t-2} \\ \boldsymbol{z}_{t-3} \end{bmatrix}; \quad \widetilde{\boldsymbol{C}}_{s_t} = \begin{bmatrix} \boldsymbol{B}_{s_t}^{-1}\boldsymbol{C}_{s_t} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}; \quad \tilde{\boldsymbol{\Gamma}}_{s_t} = \begin{bmatrix} \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Gamma}_{1,s_t} & \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Gamma}_{2,s_t} & \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Gamma}_{3,s_t} & \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Gamma}_{4,s_t} \\ \boldsymbol{1}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1}_2 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1}_2 & \boldsymbol{0} \end{bmatrix};$$

$$\tilde{\boldsymbol{\Phi}}_{s_t} = \begin{bmatrix} \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\Phi}_{s_t} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}; \quad \widetilde{\boldsymbol{\epsilon}}_{t,s_t} = \begin{bmatrix} \boldsymbol{B}_{s_t}^{-1}\boldsymbol{\epsilon}_{s_t} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}; \quad \boldsymbol{1}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The stability of system (7) is investigated based on the concept called mean-square stability, introduced by Farmer *et al.* (2009) in their study of forward-looking rational expectations Markov switching models. The mean-square stability requires that the first and second moments of the system converge to well-defined limits as the horizon extends to infinity

$$\lim_{t\to\infty} E_0(\boldsymbol{Z}_t) = \boldsymbol{\mu} \tag{8}$$

$$\lim_{t\to\infty} E_0\left(\boldsymbol{Z}_t\boldsymbol{Z}_t^{'}\right) = \boldsymbol{\Theta} \tag{9}$$

where (in our case) μ is an 8-vector and $\boldsymbol{\Theta}$ an 8×8 matrix. Given $\sqrt{h_{o,t,s_t}}$ and $\boldsymbol{\epsilon}_{s_t}$, system (7) will be covariance-stationary in each regime if the eigenvalues of $\tilde{\boldsymbol{\Gamma}}_{s_t}$ in each regime all lie inside the unit circle. Note that system (7) is not a standard VAR, since it includes a GARCH-in-Mean term. However, the GARCH-in-Mean term, $\sqrt{h_{o,t,s_t}}$, is governed by a non-explosive GARCH process, since the sum of the ARCH and GARCH effects in each regime is well below one. Therefore, the GARCH-in-Mean term will not make the $\boldsymbol{Z}_t$ process explosive over time.

We then study the covariance-stationarity of the system in each regime to verify its mean-square stability. The reason for studying the covariance-stationarity is that covariance-stationarity is strictly stronger than mean-square stability. In other words, covariance-stationarity is a sufficient condition for mean-square stability. Therefore, we will be able to claim that the system is mean-square stable in each regime if the eigenvalues of $\tilde{\boldsymbol{\Gamma}}_{s_t}$ in each regime all lie inside the unit circle. We plot the eigenvalues of $\tilde{\boldsymbol{\Gamma}}_{s_t}$ in the two regimes on the upper-left and upper-right panels of Figure 9. It is clear that the system is covariance-stationary in regime 1, but not in regime 2, since there are three eigenvalues which are outside the unit circle. It follows that the system is mean-square stable in regime 1 and is not surely mean-square stable in regime 2. Considering the explosive impulse response functions of regime 2 in Figure 6, we believe that regime 2 is hardly a mean-square stable regime.

However, one should not take this as evidence that the overall system (7) is not mean-square stable. According to Farmer *et al.* (2009), there are many instances in which a system is unstable in one or more of its regimes. But as long as the explosive regime does not show up at a high frequency, the process of $\boldsymbol{Z}_t$ will converge with finite first and second moments as in (8) and (9). According to Farmer *et al.* (2009), the condition for mean-square stability of VAR-type systems with Markov switching is documented in Costa *et al.* (2004), and requires that all eigenvalues of the matrix

$$\begin{bmatrix} p_{11}\tilde{\mathbf{\Gamma}}_{s_t=1} \otimes \tilde{\mathbf{\Gamma}}_{s_t=1} & p_{21}\tilde{\mathbf{\Gamma}}_{s_t=2} \otimes \tilde{\mathbf{\Gamma}}_{s_t=2} \\ p_{12}\tilde{\mathbf{\Gamma}}_{s_t=1} \otimes \tilde{\mathbf{\Gamma}}_{s_t=1} & p_{22}\tilde{\mathbf{\Gamma}}_{s_t=2} \otimes \tilde{\mathbf{\Gamma}}_{s_t=2} \end{bmatrix}$$

are inside the unit circle. Note that in our case this matrix has 128 eigenvalues, and we report them on the lower-left panel of Figure 9. We find that all of the 128 eigenvalues are inside the unit circle. Therefore, the $\boldsymbol{Z}_t$ process is mean-square stable, since the explosive regime (regime 2) does not occur too frequently. This interpretation is also consistent with the short expected duration of regime 2 that we reported earlier.

7 Conclusion

We investigate whether the United States economy responds negatively to oil price uncertainty and whether oil price shocks exert asymmetric effects on economic activity. We use the methodology recently introduced by Elder and Serletis (2010), but we modify it to accommodate Markov regime switching in order to account for changing oil price dynamics over the sample period. We use quarterly data, over the period from 1974:1 to 2016:3, and present clear evidence of asymmetries, providing evidence against those macroeconomic theories that predict symmetries in the relationship between real aggregate economic activity and the real price of oil.

In the context of a bivariate, mean-square stable, Markov switching structural GARCH-in-Mean VAR, we find that uncertainty about oil prices has a negative and statistically significant effect on the growth rate of real output and that this effect is asymmetric over contractions and expansions in the business cycle, being significantly larger during contractions and periods when large oil price changes occur. We also find that positive oil price shocks have a negative and statistically significant effect on the growth rate and that this effect is also asymmetric over contractions and expansions in economic activity, being larger and more persistent during contractions and periods when large oil price changes occur. Negative oil price shocks also have asymmetric effects, exerting a positive and statistically significant effect on the growth rate only during contractions and periods when large oil price changes occur. Finally, oil price uncertainty tends to amplify the negative dynamic response of the growth rate to positive oil price shocks, and to dampen the positive response to negative oil price shocks during expansions in the business cycle.

References

[1] Bernanke, B.S. (1983). ‘ Irreversibility, uncertainty, and cyclical investment ’, *Quarterly Journal of Economics* Vol. 98, pp. 85-106.

[2] Bredin, D., Elder, L. and Fountas, S. (2011). ‘ Oil Volatility and the option value of waiting: an analysis of the G-7.’ *Journal of Futures Markets* Vol. 31, pp. 679-702.

[3] Brennan, M. (1990). ‘Latent assets’. *Journal of Finance* Vol. 45, pp. 709-730.

[4] Brennan, M. and Schwartz, E. (1985). ‘Evaluating natural resource investment’. *Journal of Business* Vol. 58, pp. 1135-1157.

[5] Costa, O.L.V., Fragoso, M.D. and Marques, R.P. (2004). *Discrete-time markov jump linear systems*, Springer, New York.

[6] Dixit, A. and Pindyck, R.S. (1994). *Investment under uncertainty.* Princeton, NJ: Princeton University Press.

[7] Edelstein, P. and Kilian, L. (2007). ‘The Response of business fixed investment to changes in energy prices: a test of some hypotheses about the transmission of energy price shocks’. *B.E. Journal of Macroeconomics* Vol. 7, pp. 1-41.

[8] Edelstein, P. and Kilian, L. (2009). ‘How sensitive are consumer expenditures to retail energy prices?’. *Journal of Monetary Economics* Vol. 56, pp. 766-779.

[9] Elder, J. (2003). ‘An impulse-response-function for a vector autoregression with multivariate GARCH-in-Mean’. *Economics Letters* Vol. 79, pp. 21-26.

[10] Elder, J. (2018). ‘Oil price volatility: industrial production and special aggregates’. *Macroeconomic Dynamics* Vol. 22, pp. 640-653.

[11] Elder, J. and Serletis, A. (2010). ‘Oil price uncertainty’. *Journal of Money, Credit, and Banking* Vol. 42, pp. 1138-1159.

[12] Elder, J. and Serletis, A. (2011). ‘Volatility in oil prices and manufacturing activity: an investigation of real options’. *Macroeconomic Dynamics* Vol. 15 (Supplement 3), pp. 379–395.

[13] Farmer, R.E.A., Waggoner, D.F. and Zha, T. (2009). ‘Understanding Markov-switching rational expectations models’. *Journal of Economic Theory* Vol. 144, pp. 1849-1867.

[14] Garcia, R. and Perron, P. (1996). ‘An analysis of the real interest rate under regime shifts’. *Review of Economics and Statistics* Vol. 78, pp. 111-125.

[15] Gibson, R. and Schwartz, E.S. (1990). ‘Stochastic convenience yield and the pricing of oil contingent claims’. *Journal of Finance* Vol. 45, pp. 959–976.

[16] Gray, S.F. (1996). ‘Modeling the conditional distribution of interest rates as a regime-switching process’. *Journal of Financial Economics* Vol. 42, pp. 27-62.

[17] Hamilton, J.D. (1983). 'Oil and the macroeconomy since world war II'. *Journal of Political Economy* Vol. 91, pp. 228-248.

[18] Hamilton, J.D. (1988). 'A neoclassical model of unemployment and the business cycle'. *Journal of Political Economy* Vol. 96, pp. 593-617.

[19] Hamilton, J.D. (1989). 'A new approach to the economic analysis of nonstationary time series and the business cycle'. *Econometrica* Vol. 57, pp. 357-384.

[20] Hamilton, J.D. (1990). 'Analysis of time series subject to changes in regime'. *Journal of Econometrics* Vol. 45, pp. 39-70.

[21] Hamilton, J.D. (1996). 'That is what happened to the oil price-macroeconomy relationship'. *Journal of Monetary Economics* Vol. 38, pp. 215-220.

[22] Hamilton, J.D. (2003). "What is an oil shock?" *Journal of Econometrics* Vol. 113, pp. 363-398.

[23] Hamilton, J.D. (2011). 'Nonlinearities and the macroeconomic effects of oil prices'. *Macroeconomic Dynamics* Vol. 15 (Supplement 3), pp. 364-378.

[24] Herrera, A.M., Lagalo, L.G. and Wada, T. (2011). 'Oil price shocks and industrial production: is the relationship linear?'. *Macroeconomic Dynamics* Vol.15 (Supplement 3), pp. 472–497.

[25] Hooker, M.A. (1996). 'What happened to the oil price-macroeconomy relationship?'. *Journal of Monetary Economics* Vol. 38, pp. 195-213.

[26] Jo, S. (2014). 'The effects of oil price uncertainty on global real economic activity'. *Journal of Money, Credit and Banking* Vol 46, pp. 1113-1135.

[27] Kass, R.E. and Raftery, A.E. (1995). 'Bayes factors'. *Journal of the American Statistical Association* Vol. 90, pp. 773-795.

[28] Kilian, L. (2008). 'The Economic effects of oil price shocks'. *Journal of Economic Literature* Vol. 46, pp. 871-909.

[29] Kilian, L. (2009). 'Not all oil price shocks are alike: disentangling demand and supply shocks in the crude oil market'. *American Economic Review* Vol. 99, pp. 1053-1069.

[30] Kilian, L. and Vigfusson, R.J. (2011a). 'Are the responses of the U.S. economy asymmetric in energy price increases and decreases?'. *Quantitative Economics* Vol. 2, pp. 419-453.

[31] Kilian, L. and Vigfusson, R.J. (2011b). 'Nonlinearities in the oil price–output relationship'. *Macroeconomic Dynamics* Vol. 15 (Supplement 3), pp. 337–363.

[32] Kilian, L. and Vigfusson, R.J. (2017). 'The role of oil price shocks in causing U.S. recessions'. *Journal of Money, Credit, and Banking* Vol. 49, pp. 1747-1776.

[33] Majd, S. and Pindyck, R.S. (1987). 'Time to build, option value, and investment decisions'. *Journal of Financial Economics* Vol. 18, pp. 7-27.

[34] Pagan, A. (1984). 'Econometric issues in the analysis of regressions with generated regressors'. *International Economic Review* Vol. 25, pp. 221–247.

[35] Pinno, K. and Serletis, A. (2013). 'Oil price uncertainty and industrial production'. *The Energy Journal* Vol. 34, pp. 191-216.

[36] Rahman, S. and Serletis, A. (2011). 'The Asymmetric effects of oil price shocks'. *Macroeconomic Dynamics* Vol. 15 (Supplement 3), pp. 437–471.

[37] Rahman, S. and Serletis, A. (2012). 'Oil price uncertainty and the Canadian economy: evidence from a VARMA, GARCH-in-Mean, Asymmetric BEKK model'. *Energy Economics* Vol. 34, pp. 603-610.

[38] Serletis, A. and Mehmandosti, E.A. (2018). '150 years of the oil price-macroeconomy relationship'. *Macroeconomic Dynamics* (forthcoming).

[39] Sims, C.A. and Zha, T. (2006). 'Were there regime switches in U.S. monetary policy?'. *American Economic Review* Vol. 96, pp. 54-81.

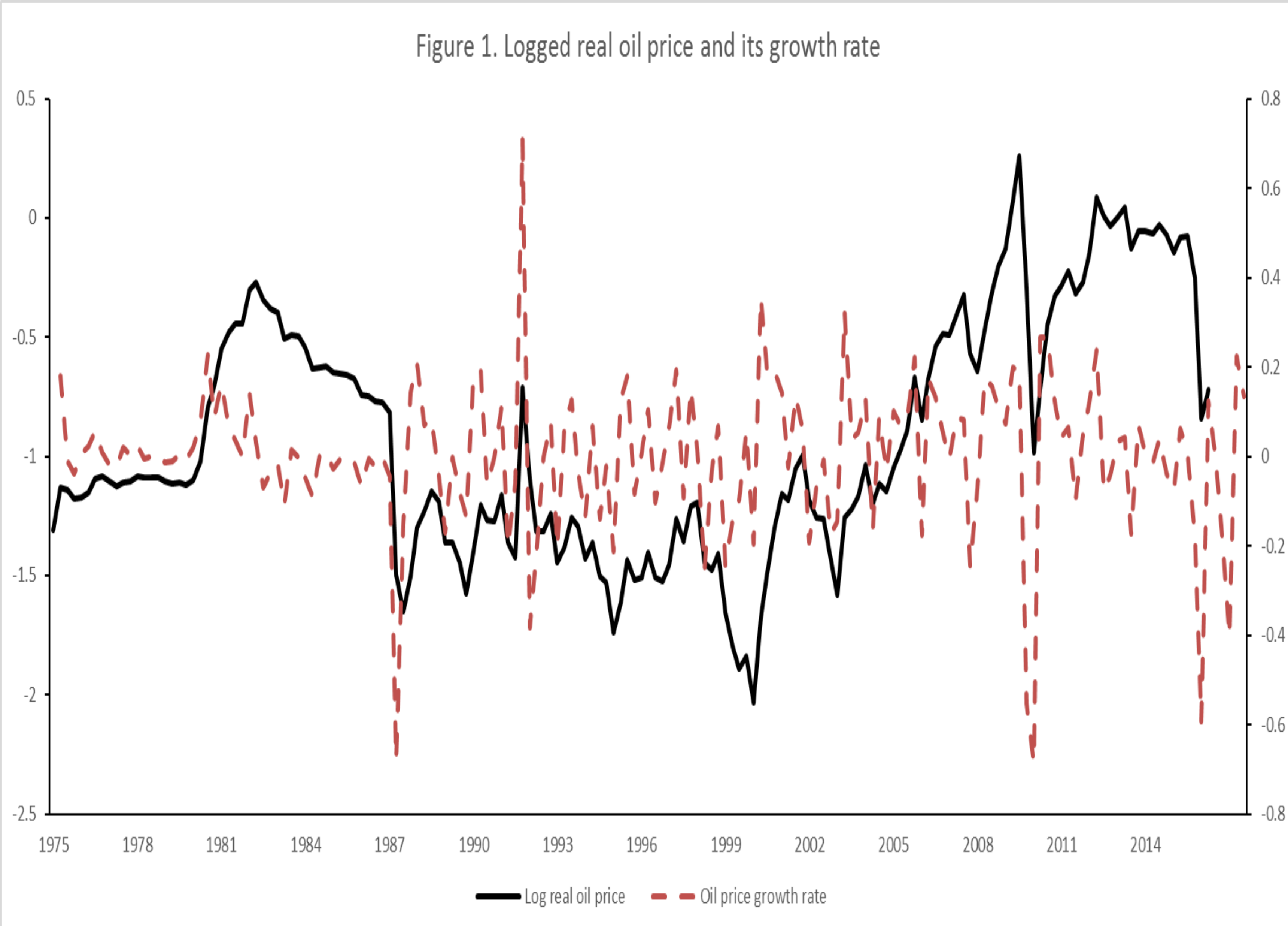

Figure 1. Logged real oil price and its growth rate
0.5
0
-0.5
-1
-1.5
-2
-2.5
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Log real oil price
Oil price growth rate

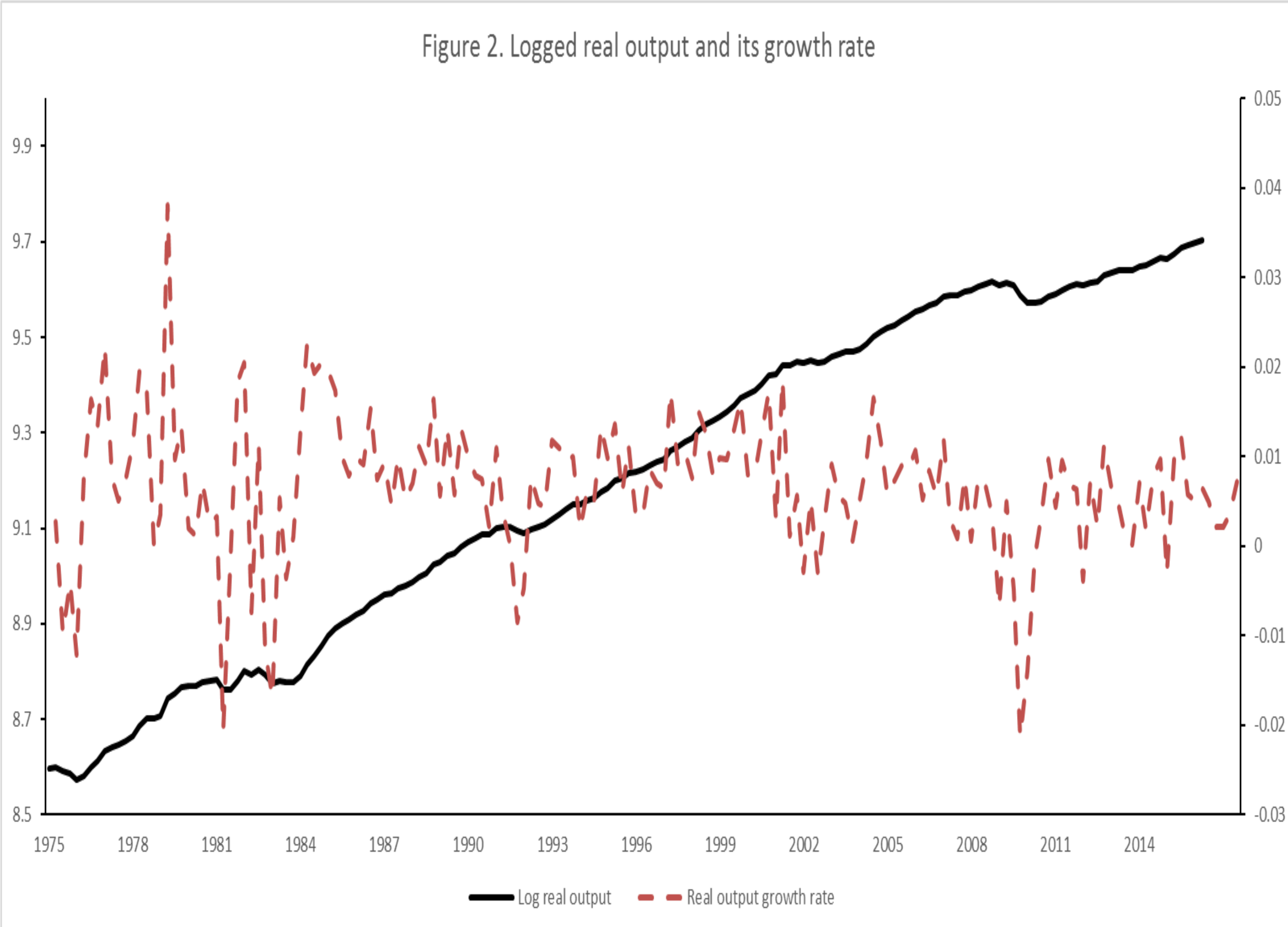
Figure 2. Logged real output and its growth rate
9.9
9.7
9.5
9.3
9.1
8.9
8.7
8.5
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Log real output
Real output growth rate

Table 1. Parameter estimates of the structural GARCH-in-Mean VAR: 1974:Q1 - 2016:Q3

A. Conditional mean equation

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ -0.002\ (0.293) & 1 \end{bmatrix}; \boldsymbol{\Gamma_1} = \begin{bmatrix} -0.029\ (0.751) & -0.714\ (0.635) \\ -0.003\ (0.376) & 0.327\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma_2} = \begin{bmatrix} -0.105\ (0.234) & 2.655\ (0.081) \\ 0.002\ (0.474) & 0.171\ (0.024) \end{bmatrix};$$

$$\boldsymbol{\Gamma_3} = \begin{bmatrix} 0.012\ (0.864) & 2.655\ (0.081) \\ 0.002\ (0.474) & 0.171\ (0.024) \end{bmatrix}; \boldsymbol{\Gamma_4} = \begin{bmatrix} -0.063\ (0.404) & 0.827\ (0.553) \\ 0.001\ (0.813) & 0.069\ (0.345) \end{bmatrix}; \boldsymbol{C} = \begin{bmatrix} 2.195\ (0.211) \\ 0.368\ (0.013) \end{bmatrix}; \boldsymbol{\Psi} = \begin{bmatrix} 0 & 0 \\ -0.001\ (0.866) & 0 \end{bmatrix}$$

B. Conditional variance equation

$d_{11} = 153.589\ (0.000)$; $d_{12} = 0.574\ (0.031)$; $d_{13} = 0.000$;

$d_{21} = 0.023\ (0.107)$; $d_{22} = 0.220\ (0.000)$; $d_{23} = 0.757\ (0.000)$.

Notes: Sample period, quarterly data, 1974:Q1-2016:Q3. A coefficient of 0.000 indicates that the nonnegativity constraint is binding. Numbers in parentheses are p-values.

Table 2. Parameter estimates of the structural GARCH-in-Mean VAR: 1974:Q1 - 2008:Q1

A. Conditional mean equation

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0.003\ (0.268) & 1 \end{bmatrix};\ \boldsymbol{\Gamma_1} = \begin{bmatrix} 0.053\ (0.575) & 0.005\ (0.998) \\ -0.003\ (0.320) & 0.267\ (0.001) \end{bmatrix};\ \boldsymbol{\Gamma_2} = \begin{bmatrix} -0.126\ (0.144) & 1.960\ (0.257) \\ 0.002\ (0.590) & 0.224\ (0.011) \end{bmatrix};$$

$$\boldsymbol{\Gamma_3} = \begin{bmatrix} 0.108\ (0.179) & -2.552\ (0.112) \\ -0.005\ (0.089) & -0.095\ (0.307) \end{bmatrix};\ \boldsymbol{\Gamma_4} = \begin{bmatrix} -0.055\ (0.541) & 0.272\ (0.861) \\ -0.000\ (0.964) & 0.057\ (0.473) \end{bmatrix};\ \boldsymbol{C} = \begin{bmatrix} 1.318\ (0.467) \\ 0.707\ (0.000) \end{bmatrix};\ \boldsymbol{\Psi} = \begin{bmatrix} 0 & 0 \\ -0.017\ (0.008) & 0 \end{bmatrix}$$

B. Conditional variance equation

$d_{11} = 191.145\ (0.000)$; $d_{12} = 0.070\ (0.313)$; $d_{13} = 0.000$;

$d_{21} = 0.018\ (0.005)$; $d_{22} = 0.205\ (0.000)$; $d_{23} = 0.778\ (0.000)$.

Notes: Sample period, quarterly data, 1974:Q1-2008:Q1.A coefficient of 0.000 indicates that the nonnegativity constraint is binding. Numbers in parentheses are p-values.

Table 3. Parameter estimates of the Markov switching structural GARCH-in-Mean VAR: 1974:Q1 - 2016:Q3

A. Conditional mean equation

$$\boldsymbol{B}_{s_t=1} = \begin{bmatrix} 1 & 0 \\ -0.003\ (0.484) & 1 \end{bmatrix}; \boldsymbol{\Gamma}_{1,s_t=1} = \begin{bmatrix} 0.183\ (0.008) & -0.064\ (0.946) \\ -0.004\ (0.148) & 0.207\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma}_{2,s_t=1} = \begin{bmatrix} -0.049\ (0.537) & 1.295\ (0.123) \\ 0.001\ (0.815) & 0.040\ (0.609) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_{3,s_t=1} = \begin{bmatrix} 0.082\ (0.158) & -3.049\ (0.005) \\ -0.004\ (0.127) & -0.010\ (0.848) \end{bmatrix}; \boldsymbol{\Gamma}_{4,s_t=1} = \begin{bmatrix} -0.017\ (0.785) & 0.794\ (0.336) \\ -0.000\ (0.906) & 0.045\ (0.578) \end{bmatrix}; \boldsymbol{C}_{s_t=1} = \begin{bmatrix} 1.041\ (0.508) \\ 0.922\ (0.000) \end{bmatrix}; \boldsymbol{\Phi}_{s_t=1} = \begin{bmatrix} 0 \\ -0.029\ (0.037) \end{bmatrix};$$

$$\boldsymbol{B}_{s_t=2} = \begin{bmatrix} 1 & 0 \\ 0.004\ (0.000) & 1 \end{bmatrix}; \boldsymbol{\Gamma}_{1,s_t=2} = \begin{bmatrix} -0.496\ (0.079) & -2.604\ (0.759) \\ -0.020\ (0.000) & 0.730\ (0.000) \end{bmatrix}; \boldsymbol{\Gamma}_{2,s_t=2} = \begin{bmatrix} -0.918\ (0.047) & -1.338\ (0.914) \\ -0.025\ (0.000) & 0.351\ (0.000) \end{bmatrix};$$

$$\boldsymbol{\Gamma}_{3,s_t=2} = \begin{bmatrix} -0.033\ (0.956) & 1.287\ (0.918) \\ -0.043\ (0.000) & -0.019\ (0.084) \end{bmatrix}; \boldsymbol{\Gamma}_{4,s_t=2} = \begin{bmatrix} -0.306\ (0.320) & -3.029\ (0.717) \\ -0.013\ (0.000) & -0.216\ (0.000) \end{bmatrix}; \boldsymbol{C}_{s_t=2} = \begin{bmatrix} -6.698\ (0.344) \\ 17.490\ (0.000) \end{bmatrix}; \boldsymbol{\Phi}_{s_t=2} = \begin{bmatrix} 0 \\ -0.605\ (0.000) \end{bmatrix}.$$

B. Conditional variance equation

$d_{1,s_t=1} = 93.181\ (0.000)$; $d_{2,s_t=1} = 0.204\ (0.016)$; $d_{3,s_t=1} = 0.000\ (0.999)$; $h_{\Delta y,s_t=1} = 0.370\ (0.000)$;

$d_{1,s_t=2} = 908.875\ (0.000)$; $d_{2,s_t=2} = 0.000\ (0.999)$; $d_{3,s_t=2} = 0.003\ (0.627)$; $h_{\Delta y,s_t=2} = 0.001\ (0.000)$.

C. Transition matrix

$$\boldsymbol{P} = \begin{bmatrix} 0.929\ (0.000) & 0.666\ (0.000) \\ 0.071\ (0.001) & 0.334\ (0.010) \end{bmatrix}$$

Notes: Sample period, quarterly data, 1974:Q1-2016:Q3. A coefficient of 0.000 indicates that the nonnegativity constraint is binding. Numbers in parentheses are p-values.

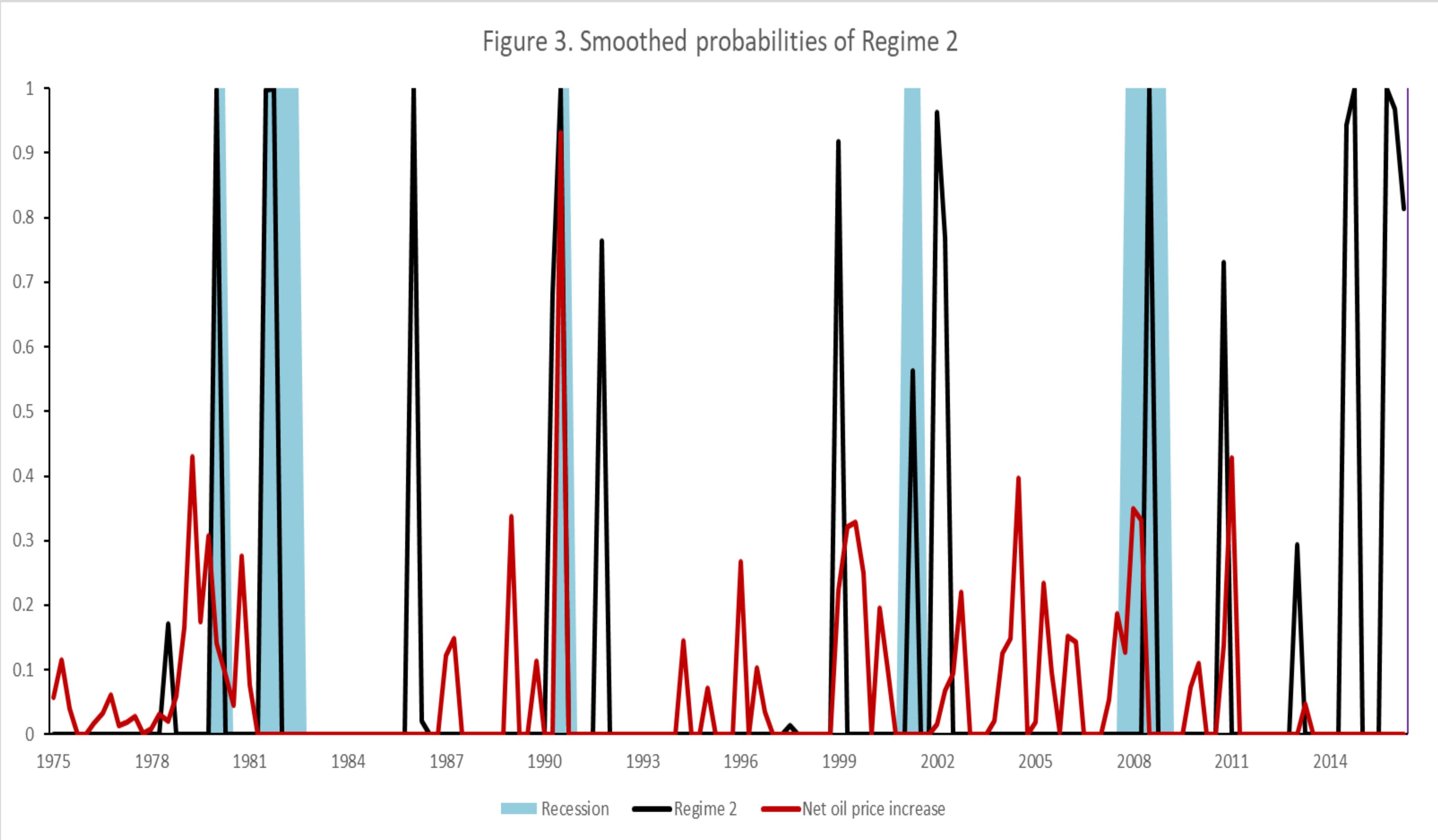
Figure 3. Smoothed probabilities of Regime 2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Recession
Regime 2
Net oil price increase

Figure 4. Impulse response functions for the Markov switching structural GARCH-in-Mean VAR

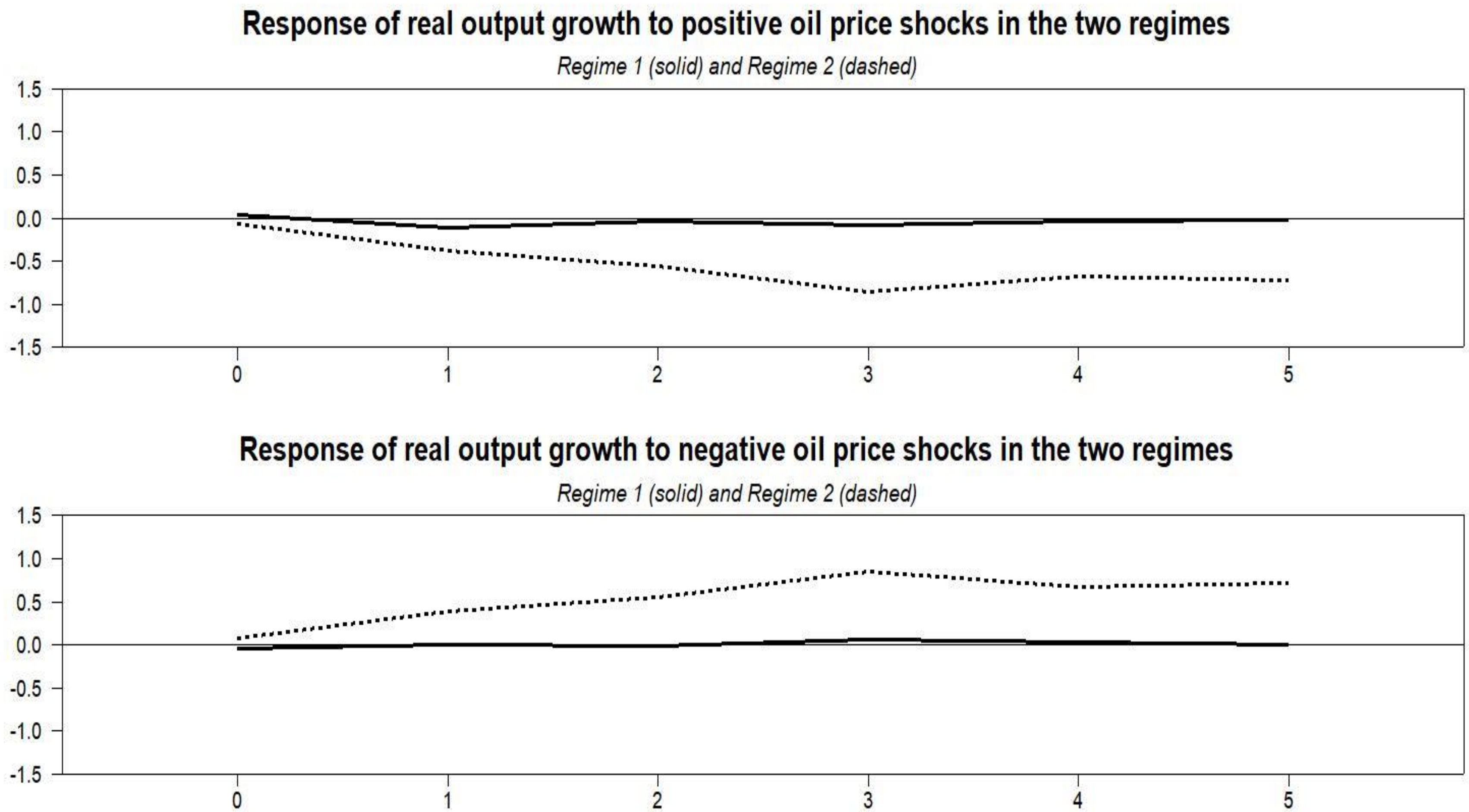

Figure 5. Impulse response functions for the Markov switching structural GARCH-in-Mean VAR in regime 1

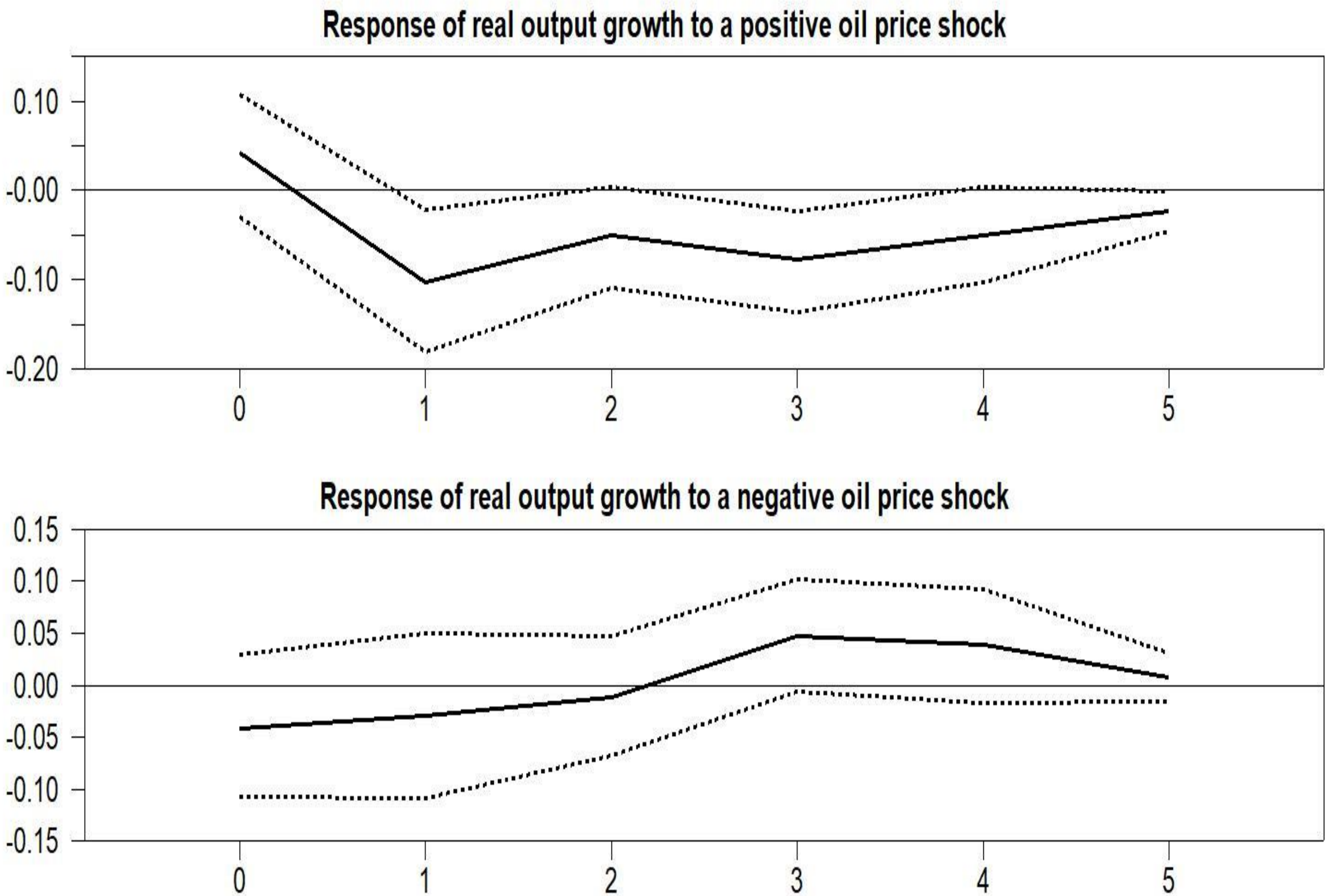

Figure 6. Impulse response functions for the Markov switching structural GARCH-in-Mean VAR in regime 2

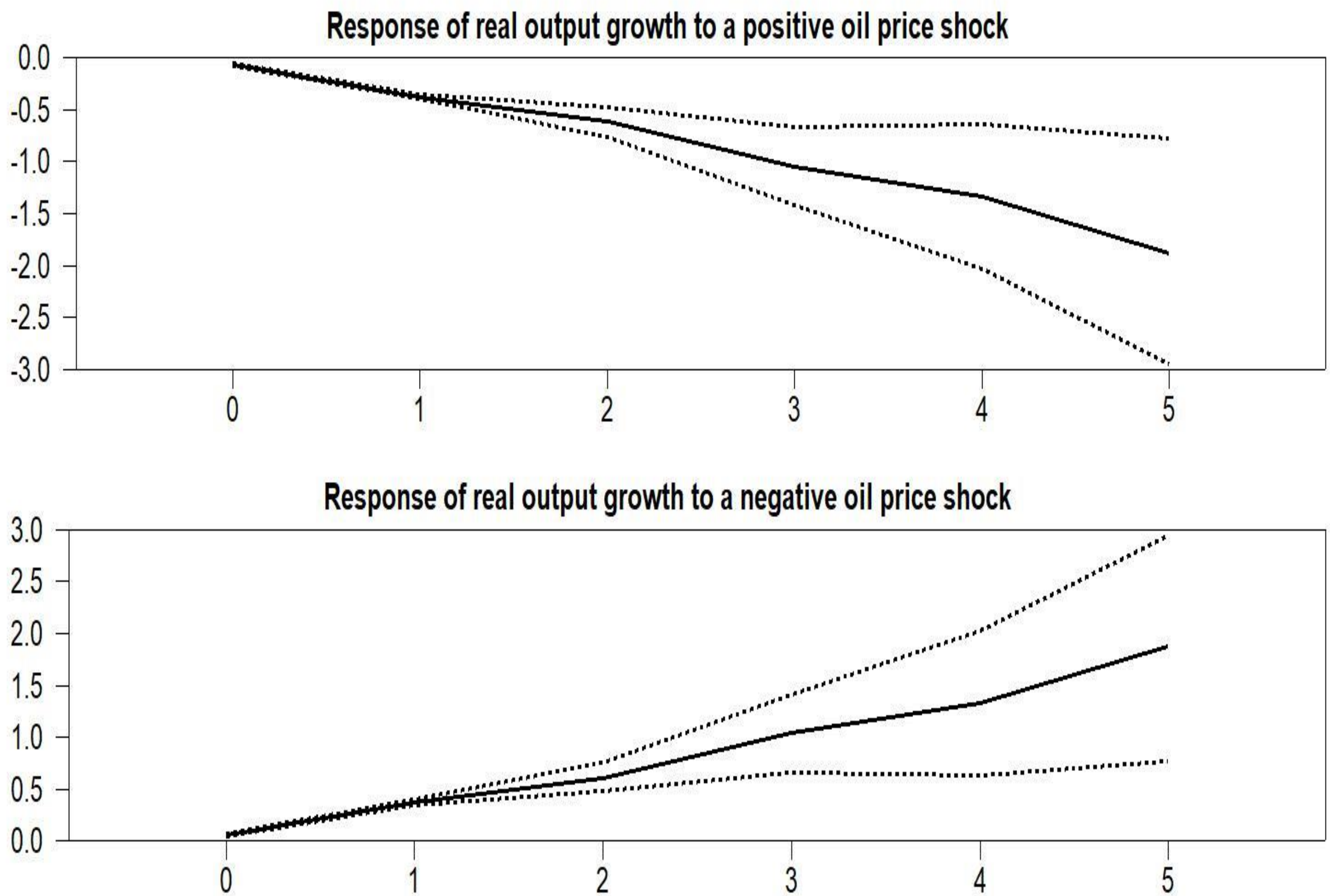

Figure 7. Impulse response functions for the Markov switching structural GARCH-in-Mean VAR in regime 1

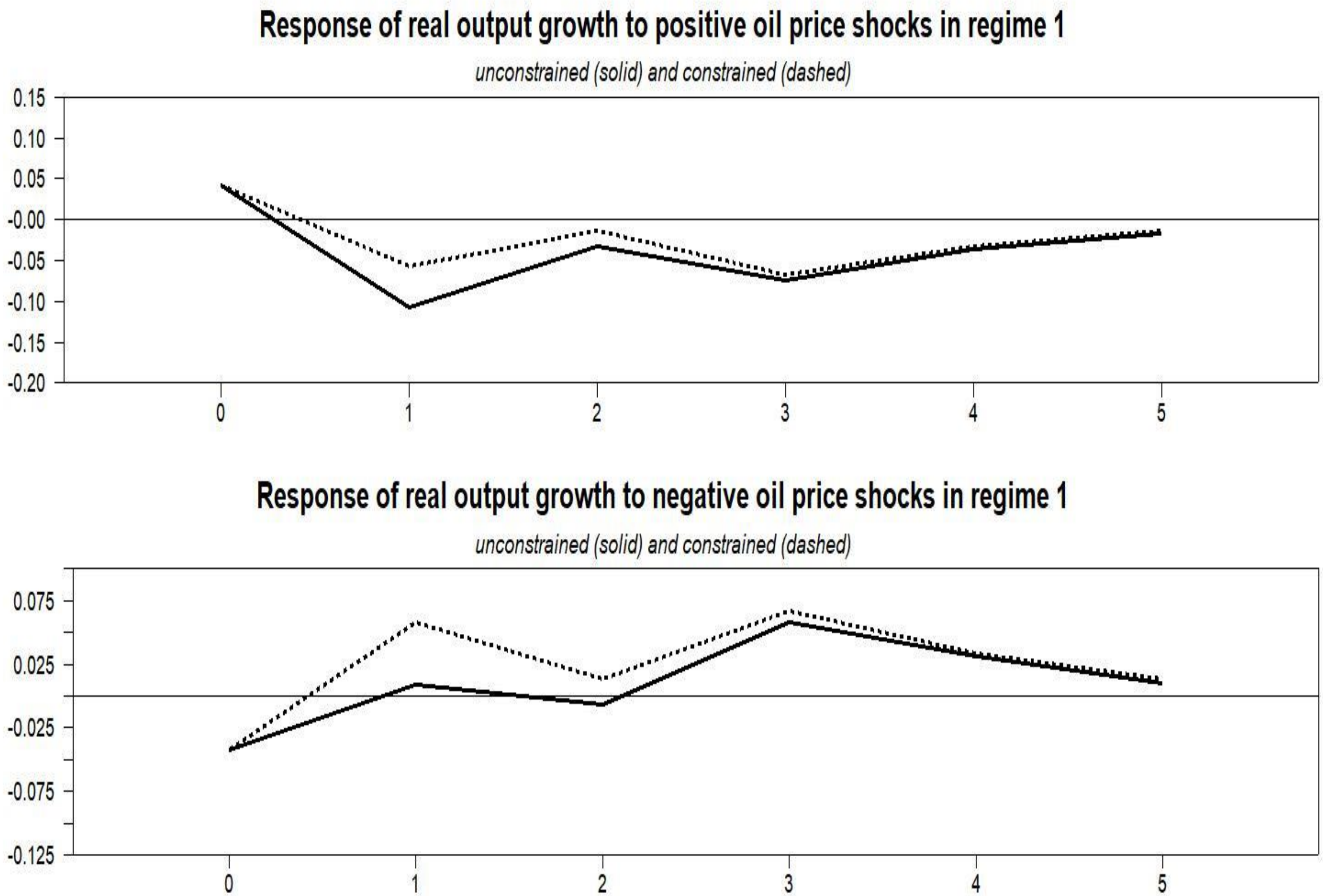

Figure 8. Impulse response functions for the Markov switching structural GARCH-in-Mean VAR in regime 2

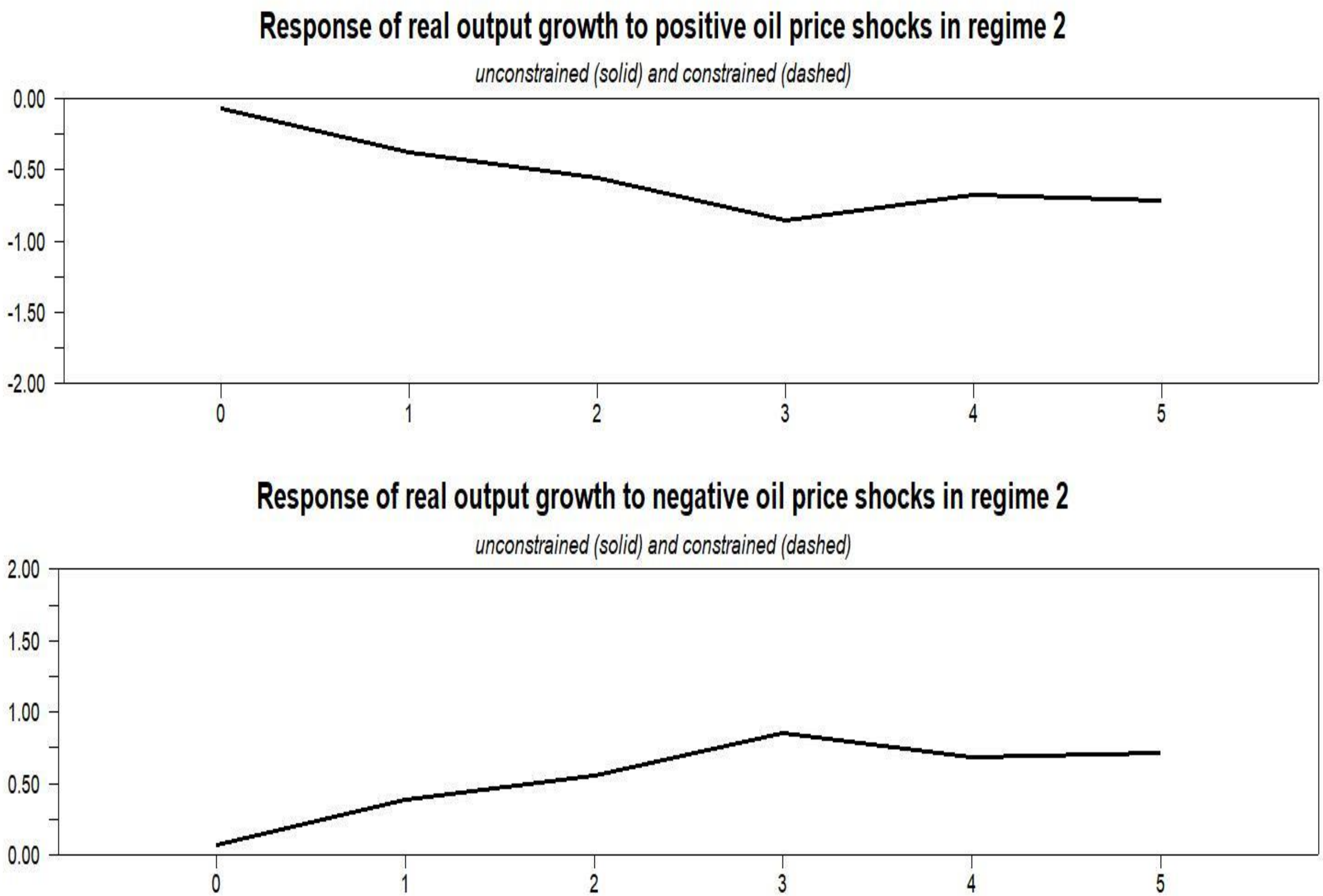

Figure 9. Eigenvalues for the stability of the Markov switching structural GARCH-in-Mean VAR