The Demand for Gasoline: Evidence from Household Survey Data*

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Forthcoming in: *Journal of Applied Econometrics*

September 28, 2012

*This paper is based on Chapter 2 of Dongfeng Chang's Ph.D. dissertation at the University of Calgary. We would like to thank the following members of Dongfeng's dissertation committee: Herbert Emery, Daniel Gordon, Ron Kneebone, David Walls, and Philip Chang. We also thank Edward Vytlacil and two referees for comments that greatly improved the paper.

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Abstract:

In this paper we investigate the demand for gasoline in Canada using recent annual expenditure data from the Canadian *Survey of Household Spending*, over a 13-year period from 1997 to 2009, on three expenditure categories in the transportation sector: gasoline, local transportation, and inter-city transportation. In doing so, we use three of the most widely used locally flexible functional forms, the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), the quadratic AIDS (QUAIDS) of Banks *et al.* (1997), an extension of the simple AIDS model that can generate quadratic Engel curves, and the Minflex Laurent model of Barnett (1983) that can also generate quadratic Engel curves. We pay explicit attention to economic regularity, argue that unless regularity is attained by luck, flexible functional forms should always be estimated subject to regularity as suggested by Barnett (2002), and impose local curvature to produce inference consistent with neoclassical microeconomic theory. Our findings indicate that the curvature-constrained Minflex Laurent model is the only model that is able to provide theoretically consistent estimates of the Canadian demand for gasoline. Our estimates show that the own-price elasticity for gasoline demand in Canada is between −0.738 and −0.570, less elastic than previously reported in the literature.

JEL classification: C30, D12, Q43.

Keywords: Flexible functional forms; Theoretical regularity; Household data.

1 Introduction

As Hamilton (2011, p. 364) puts is, "I noted in a paper published in the *Journal of Political Economy* in 1983 that at that time, 7 out of the 8 postwar U.S. recessions had been preceded by a sharp increase in the price of crude petroleum [Hamilton (1983)]. Iraq's invasion of Kuwait in August 1990 led to a doubling in the price of oil in the fall of 1990 and was followed by the ninth postwar recession in 1990–1991. The price of oil more than doubled again in 1999–2000, with the tenth postwar recession coming in 2001. Yet another doubling in the price of oil in 2007–2008 accompanied the beginning of recession number 11, the most recent and frightening of the postwar economic downturns. So the count today stands at 10 out of 11, the sole exception being the mild recession of 1960–1961, for which there was no preceding rise in oil prices."

One important parameter for determining the consequences of crude oil price shocks for the macroeconomy is the price elasticity of the demand for gasoline. If the demand for gasoline is very inelastic (so that when the price increases by a certain percentage, the quantity demanded decreases by a much smaller percentage), oil prices changes will have significant effects on the level of economic activity through their effect on consumption. For example, sharp increases in gasoline prices, due to increases in oil prices and supply disruptions (such as, for example, refinery outages and regional supply shortages), will significantly reduce disposable income, causing consumers to curb expenditures across consumption categories. Moreover, higher gasoline prices saddle businesses with higher transportation costs, causing them to pass them along to their customers in the form of higher product prices.

The role of oil prices in the macroeconomy has been the focus of a large number of recent econometric studies — see, for example, Hamilton (2009, 2011), Kilian (2009), Elder and Serletis (2010), and Kilian and Vigfusson (2011), among others. The demand for gasoline has also been investigated extensively in the literature, although the existing major contributions in this area are quite outdated, since they primarily focus on the 1970s and 1980s. See Dahl and Sterner (1991), Espey (1998), and Graham and Glaister (2002) for thorough reviews of hundreds of gasoline demand studies. In recent years, however, a number of significant changes have occurred in the global gasoline market. For example, as can be seen in Figure 1, crude oil and gasoline prices (from the U.S. Energy Information Administration) have become more volatile over the past decade, breaking records in 2007 and 2008, exceeding even levels reached after the oil price shocks in the 1970s. Moreover, according to Hamilton (2009), the earlier oil price shocks were primarily caused by physical disruptions of supply whereas the oil shock of 2007-2008 was caused by strong demand confronting stagnating world production.

The high oil prices and growing concern about global warming have reignited interest in reducing gasoline consumption. Recently, Serletis *et al.* (2010, 2011) use the most recent data (since 1980), published by the International Energy Agency (IEA), for a number of OECD and non-OECD countries (including China and India), to study interfuel substitution elasticities at the sector and aggregate levels. In doing so, they take the approach of using a flexible demand system [namely, the normalized quadratic (NQ), introduced by Diewert and Wales (1988)], following Diewert's (1971) influential paper. This approach, pioneered by Berndt and Wood (1975), Fuss (1977), and Pindyck (1979) in the context of interfactor and interfuel substitution, involves the specification of a differentiable form for the cost function, the application of Shephard's (1953) lemma to derive the cost share equations, and the use of relevant data to estimate the parameters and compute the relevant elasticity measures — the income elasticities and the own- and cross-price elasticities.

With the exception of Serletis *et al.* (2010, 2011), earlier gasoline demand studies that use flexible functional forms have employed the translog functional form, introduced by Christensen *et al.* (1975). Although the translog provides arbitrary elasticity estimates at the point of approximation (that is, locally), there is evidence that this model fails to meet the theoretical regularity conditions of neoclassical microeconomic theory (positivity, monotonicity, and curvature) in large regions. Moreover, in the empirical gasoline demand literature there has been a tendency to ignore theoretical regularity or not to report the results of full regularity checks; for example none of the gasoline demand studies listed in Table 1 reports on theoretical regularity. In this regard, as Barnett (2002, p. 199) put it, "without satisfaction of both curvature and monotonicity, the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid."

In the recent literature there is an attempt to treat theoretical regularity as a maintained hypothesis and built it into the models being estimated, very much like the homogeneity in prices and symmetry properties of neoclassical consumer theory. In particular, Ryan and Wales (1998), drawing on related work by Lau (1978) and Diewert and Wales (1987), suggest a procedure for imposing local curvature conditions and apply their procedure to three locally flexible functional forms — the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), the normalized quadratic (NQ) of Diewert and Wales (1988), and the linear translog. Also, Moschini (1999) suggests a possible reparameterization of the basic translog of Christensen *et al.* (1975) to overcome some problems noted by Ryan and Wales (1998) and imposes curvature conditions locally in the basic translog. More recently, Serletis and Shahmoradi (2007) impose curvature conditions locally on the generalized Leontief model, introduced by Diewert (1974), and Chang and Serletis (2012) on the quadratic AIDS (QUAIDS) of Banks *et al.* (1997), an extension of the simple AIDS model that can generate quadratic Engel curves. Also, Blundell *et al.* (2012) develop a new method for estimating a demand function and apply it to gasoline demand in the United States, using data from the 2001 U.S. National Household Travel Survey. Their method uses Slutsky shape restrictions to improve the precision of a nonparametric estimate of the demand function.

In this paper, we use three flexible functional forms, the AIDS, QUAIDS, and the Minflex Laurent model introduced by Barnett (1983) and Barnett and Lee (1985), to investigate the demand for gasoline in a systems framework, giving the quantity demanded also as a function of the prices of other transportation goods. We pay explicitly attention to theoretical regularity and argue that unless regularity is attained by luck, flexible functional forms should always be estimated subject to regularity, as suggested by Barnett (2002) and more recently by Barnett and Serletis (2008) and Serletis *et al.* (2010, 2011). In doing so, we examine the demand for gasoline in Canada. Canada has only half percent of the world's population, but produces two percent of global greenhouse gas emissions. Moreover, Canada ranks the second-highest among OECD high-income countries in terms of petroleum-based liquid fuel consumption per capita — see Knittel (2011). Also, about 75% of oil-related greenhouse gas emissions in Canada come from vehicle use, not exploration or production, according to Environment Canada (2010). Given the significance of decreasing Canadian gasoline consumption and transportation related greenhouse gas emissions, it is especially important to investigate how Canadians currently respond to changes in gasoline prices.

To obtain our estimates of the demand for gasoline in Canada, we follow Schmalensee and Stoker (1999) and Yatchew and No (2001) and use the recent micro-survey data for Canadian household expenditures. In particular, we use annual expenditure data from the Canadian *Survey of Household Spending*, over a 13-year period from 1997 to 2009, on three expenditure categories in the transportation sector: gasoline, local transportation, and intercity transportation. We ignore demographic or other nonincome consumer characteristics (such as age and household composition), but distinguish between three different household types: single-member households, married couples without children, and married couples with one child. Our findings in terms of regularity violations even when the curvature conditions are imposed are disappointing in the case of the AIDS and QUAIDS models, since the imposition of local curvature conditions on these two models does not eliminate the curvature violations completely for all three household types. Only the curvature-constrained Minflex Laurent model is able to generate results consistent with the theoretical regularity conditions. Based on this model, our estimates of gasoline own-price elasticities for Canada are statistically significant and vary from -0.738 to -0.570, suggesting that the demand for gasoline in Canada is overall less elastic than earlier papers have reported, using methodology different than ours.

The rest of the paper is organized as follows. Section 2 provides a brief review of different approaches to estimating the demand elasticity of gasoline. Section 3 follows the demand systems approach to gasoline demand and discusses three locally flexible functional forms, paying attention to the imposition of local curvature. Section 4 discusses the flexibility properties of the three functional forms whereas Section 5 focuses on their ability to capture the Engel curve structure of the data. Section 6 discusses the data and related econometric issues and Section 7 presents the empirical results. The final section concludes the paper.

2 Modelling Gasoline Demand

There are different approaches to investigating the demand for gasoline. In this section, we briefly discuss three methods — the time series approach, the single-equation approach, and the demand systems approach based on neoclassical consumer theory.

2.1 The Time Series Approach

Many studies investigate the demand for gasoline using aggregate data and the time series approach — see the Graham and Glaister (2002) survey paper for more details. In these studies, income and the price of gasoline are the most common variables in estimating gasoline demand. The studies employ integration and cointegration techniques to examine the demand for gasoline using a variation of the following model

$$\ln G_t = \alpha_0 + \alpha_1 \ln P_t + \alpha_2 \ln Y_t + \varepsilon_t \tag{1}$$

where G is per capita gasoline consumption, P is the real price of gasoline, Y is real per capita income, and ε is the error term. This model may be expanded to include other variables such as, for example, the stock of vehicles to control for other idiosyncratic aspects of gasoline demand.

This approach requires that we first test for unit roots using alternative unit root testing procedures to deal with anomalies that arise when the data are not very informative about whether or not there is a unit root. If all the variables are integrated of order one [or I(1) in the terminology of Engle and Granger (1987)], then for equation (1) to make any sense the variables must be cointegrated in levels; that is, the equation errors must be stationary. If the errors are nonstationary, then there is no theory linking the left hand side to the right hand side variables in equation (1) or, equivalently, no evidence for the theoretical model in level form. In such cases, some important nonstationary variables might have been omitted. Allowing for first order serial correlation, as is usually done in the literature, is almost the same as taking first differences of the data if the autocorrelation coefficient is close to unity. In that case, the equation errors become stationary, but there is no theory for the model in first differences.

One of the main problems with the time series approach to gasoline demand is that equation (1) is specified in an ad hoc manner. The demand for gasoline is assumed to be a function of the price of gasoline, per capita income, and some other variables, such as vehicle stock and fuel efficiency. There is no underlying economic theory for this specification and the empirical results are sensitive to the selection of sample or time periods. In addition, most studies taking this approach use aggregate data without making a distintion between different sectors of use.

2.2 The Single Equation Approach without Utility Reference

There is an old tradition in applied demand analysis, which specifies the demand function directly without reference to the utility function. Under this approach, the household demand for gasoline is specified as a function of income and the price of gasoline, as in the following log-log demand equation

$$\ln \text{Gallons} = \alpha + \eta_1 \ln \text{Income} + \eta_2 \ln \text{Price} + \epsilon$$

where the coefficient η_1 is the income elasticity of the demand for gasoline and η_2 is the own-price elasticity of gasoline.

One example of a gasoline demand equation with no reference to the utility function is the semiparametric model used by Schmalensee and Stoker (1999)

$$\begin{aligned}\ln \text{Gallons} = G\left(\ln \text{Income},\ \ln \text{Age}\right) + \beta_1 \ln \text{Drvrs} + \beta_2 \ln \text{Size} \\ + \beta_3 \text{Residence} + \beta_4 \text{Region} + \beta_5 \text{Lifecycle} + \epsilon\end{aligned}$$

where Drvrs is the number of drives, Size is the household size, and the remaining qualitative variables refer to location effects, such as urban, rural residence, and regional location. $G(\cdot)$ is estimated nonparametrically and is interpreted as the income-age structure of mean log-gasoline demand, holding all other variables constant. Due to the unavailability of price data at the household level, Schmalensee and Stoker (1999) do not report estimates of price effects.

Similarly, Yatchew and No (2001) estimate a similar model with price and age entering nonparametrically, as follows

$$y = f\left(\text{Price},\ \text{Age}\right) + \boldsymbol{\theta z} + \epsilon$$

where y is the log of total monthly gasoline consumption or the log of total monthly distance traveled, $f(\cdot)$ denotes a generic smooth function, and $\boldsymbol{z}$ includes income, demographic, location, and temporal variables.

2.3 The Neoclassical Consumer Theory Approach

The demand systems approach based on the neoclassical consumer theory allows us to estimate the demand for gasoline in a systems framework, which involves estimating the parameters of an aggregator function — a profit, expenditure, or utility function. Let's assume that the representative consumer has the following utility function

$$u = u(\boldsymbol{c}, l, \boldsymbol{x}) \tag{2}$$

where $\boldsymbol{c}$ is a vector of consumption goods, l is leisure, and $\boldsymbol{x}$ is a vector of transportation goods. The consumer maximizes (2) subject to the budget constraint

$$\boldsymbol{q}'\boldsymbol{c} + wl + \boldsymbol{p}'\boldsymbol{x} = m$$

where $\boldsymbol{q}$ is a vector of prices of the consumption goods, $\boldsymbol{c}$, w is the wage rate, $\boldsymbol{p}$ is a vector of prices of the transportation goods, $\boldsymbol{x}$, and m is total income.

We assume that transportations goods are as a group separable from consumption goods, $\boldsymbol{c}$, and leisure, l. That is, it is possible to write (2) as

$$u = u\left(\boldsymbol{c}, l,\ f\left(\boldsymbol{x}\right)\right) \tag{3}$$

where $f\left(\boldsymbol{x}\right)$ is the aggregator function over transportation goods, $\boldsymbol{x}$. The algebraic requirement of (direct) weak separability in $\boldsymbol{x}$ is that

$$\partial\left(\frac{\partial u/\partial x_i}{\partial u/\partial x_j}\right)/\partial\zeta = 0, \quad x_i, x_j \in \boldsymbol{x} \text{ for } i \neq j; \zeta = \boldsymbol{c}, l.$$

That is, the marginal rate of substitution between any two components of $\boldsymbol{x}$ does not depend upon the values of $\boldsymbol{c}$ and l, meaning that the demand for transportation goods is independent of relative prices outside the transportation sector.

Under the weak separability assumption, we will focus on the details of the demand for three transportation goods, gasoline (x_1), local transportation (x_2), and inter-city transportation (x_3), ignoring other types of goods, in the context of the following problem

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} = y,$$

where $\boldsymbol{x} = (x_1, x_2, x_3)$ is the vector of transportation goods, $\boldsymbol{p} = (p_1, p_2, p_3)$ is the corresponding vector of prices, and y is the total expenditure on gasoline (x_1), local transportation (x_2), and inter-city transportation (x_3). We will estimate the demand system giving the quantity demanded as a function of the prices of all goods and income. For details regarding the theory of multi-stage optimization in the context of consumer theory, see Strotz (1957, 1959), Gorman (1959), and Blackorby *et al.* (1978). See also Barnett and Serletis (2008) for an up-to-date survey of consumer demand analysis.

3 Flexible Demand Systems

Our objective is to estimate a demand system derived from an indirect utility function. The indirect utility function approach is preferred because prices enter as exogenous variables in the estimation process and the demand system is easily derived by applying Roy's identity.

In this section we briefly discuss three flexible functional forms that we use to approximate the unknown underlying indirect utility function of the representative economic agent. They are the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), the quadratic AIDS (QUAIDS) of Banks *et al.* (1997), and the Minflex Laurent (ML) model, introduced by Barnett (1983) and Barnett and Lee (1985). These models are all locally flexible and capable of approximating any unknown function up to the second order (we address the flexibility issue in Section 4).

3.1 The Almost Ideal Demand System

The AIDS can be derived from the following indirect utility function

$$h(\boldsymbol{p}, y) = [\ln y - \ln a(\boldsymbol{p})]\, \beta_0^{-1} \prod_{i=1}^{n} (p_i)^{-\beta_i} \tag{4}$$

where $i = 1, ..., n$ denotes the number of goods, y is total expenditure, $\boldsymbol{p}$ is a vector of prices,

$$\ln a(\boldsymbol{p}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^* \ln p_i \ln p_j$$

and α, β, and γ are parameters. Solving for $\ln y$ yields the AIDS cost function, as in Deaton and Muellbauer (1980),

$$\ln C(\boldsymbol{p}, u) = (1 - u) \ln a(\boldsymbol{p}) + u \ln b(\boldsymbol{p}) \tag{5}$$

where

$$\ln b(\boldsymbol{p}) = \ln a(\boldsymbol{p}) + \beta_0 \prod_{i=1}^{n} (p_i)^{\beta_i}. \tag{6}$$

Applying Roy's identity to the indirect utility function (4), or Shephard's lemma to the cost function (5), we obtain the AIDS in budget share form

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[\frac{y}{a(\boldsymbol{p})} \right] \tag{7}$$

where $\gamma_{ij} = .5\left(\gamma_{ij}^* + \gamma_{ji}^*\right)$. Economic theory imposes several restrictions on the parameters of the AIDS (7). In particular, symmetry requires $\gamma_{ij} = \gamma_{ji}$ for all i, j and the adding-up and homogeneity conditions require

$$\textstyle\sum_{i=1}^{n} \alpha_i = 1$$

$$\textstyle\sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \gamma_{ij} = \sum_{j=1}^{n} \gamma_{ij} = 0.$$

With n goods, the AIDS share equations (7) contain $(n^2+3n-2)/2$ free parameters (that is, parameters estimated directly).

Local curvature can be imposed using the Ryan and Wales (1998) procedure which applies to those locally flexible demand systems for which, at the point of approximation, the $n \times n$ Slutsky matrix $\boldsymbol{S}$ can be written as

$$\boldsymbol{S} = \boldsymbol{B} + \boldsymbol{C} \tag{8}$$

where $\boldsymbol{B}$ is an $n \times n$ symmetric matrix, containing the same number of independent elements as the Slutsky matrix, and $\boldsymbol{C}$ is an $n \times n$ matrix whose elements are functions of the other parameters of the system. Curvature requires the Slutsky matrix to be negative semidefinite.

In particular, Ryan and Wales (1998) draw on related work by Lau (1978) and Diewert and Wales (1987) and impose curvature by replacing $\boldsymbol{S}$ in (8) with $-\boldsymbol{K}\boldsymbol{K}'$, where $\boldsymbol{K}$ is an $n \times n$ lower triangular matrix so that $-\boldsymbol{K}\boldsymbol{K}'$ is by construction a negative semidefinite matrix. Then solving explicitly for $\boldsymbol{B}$ in terms of $\boldsymbol{K}$ and $\boldsymbol{C}$ yields

$$\boldsymbol{B} = -\boldsymbol{K}\boldsymbol{K}' - \boldsymbol{C}$$

meaning that the model can be reparameterized by estimating the parameters in $\boldsymbol{K}$ and $\boldsymbol{C}$ instead of the parameters in $\boldsymbol{B}$ and $\boldsymbol{C}$. That is, we can replace the elements of $\boldsymbol{B}$ in the estimating equations (7) by the elements of $\boldsymbol{K}$ and the other parameters of the model, thus ensuring that $\boldsymbol{S}$ is negative semidefinite at the point of approximation, which could be any data point.

Applying the Ryan and Wales (1998) procedure, we write the ijth element of the Slutsky matrix associated with the AIDS demand system, at the point $y = p_k = 1$ $(\forall\ k)$, as

$$S_{ij} = \gamma_{ij} - (a_i - b_i a_0)\delta_{ij} + (a_j - b_j a_0)(a_i - b_i a_0) - b_i b_j a_0$$

for $i, j = 1, ..., n$, where $\delta_{ij} = 1$ when $i = j$ and 0 otherwise. Thus, following Ryan and Wales (1998) local curvature can be imposed by replacing the elements of $\boldsymbol{B}$ in the estimating share equations by the elements of $\boldsymbol{K}$ and the other parameters, as follows for the ijth element of $\boldsymbol{B}$

$$\gamma_{ij} = (-KK')_{ij} + (a_i - b_i a_0)\delta_{ij} - (a_j - b_j a_0)(a_i - b_i a_0) + b_i b_j a_0 \tag{9}$$

for $i, j = 1, ..., n$. See Serletis and Shahmoradi (2007) for an example with $n = 3$.

3.2 The Quadratic Almost Ideal Demand System

According to Banks *et al.* (1997), the QUAIDS has the following indirect utility function

$$\ln h(\boldsymbol{p}, y) = \left\{ \left[\frac{\ln y - \ln a(\boldsymbol{p})}{b(\boldsymbol{p})} \right]^{-1} + \lambda(\boldsymbol{p}) \right\}^{-1} \tag{10}$$

where y is total expenditure, $\boldsymbol{p}$ is a vector of prices, $a(\boldsymbol{p})$ is a differentiable, homogeneous function of degree one in prices, and $\lambda(\boldsymbol{p})$ and $b(\boldsymbol{p})$ are differentiable, homogeneous functions of degree zero in prices. In fact, $\lambda(\boldsymbol{p})$ takes the form

$$\lambda(\boldsymbol{p}) = \sum_{i=1}^{n} \lambda_i \ln p_i, \ \text{ where } \sum_{i=1}^{n} \lambda_i = 0$$

and $i = 1, ..., n$ denotes the number of goods.

The specifications of $a(\boldsymbol{p})$ and $b(\boldsymbol{p})$ in (10) are similar to those in the AIDS of Deaton and Muellbauer (1980); they are sufficiently flexible to represent any arbitrary set of first and second derivatives of the cost function, as follows

$$\ln a(\boldsymbol{p}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j$$

$$b(\boldsymbol{p}) = \prod_{i=1}^{n} (p_i)^{\beta_i} .$$

Solving for $\ln y$, yields the QUAIDS cost function

$$\ln C \equiv \ln y = \ln a(\boldsymbol{p}) + \frac{b(\boldsymbol{p}) \ln h(\boldsymbol{p}, y)}{1 - \lambda(\boldsymbol{p}) \ln h(\boldsymbol{p}, y)}. \tag{11}$$

Applying Shephard's lemma to the cost function (11) or Roy's identity to the indirect utility function (10) yields the QUAIDS share equations

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[\frac{y}{a(\boldsymbol{p})} \right] + \frac{\lambda_i}{b(\boldsymbol{p})} \left\{ \ln \left[\frac{y}{a(\boldsymbol{p})} \right] \right\}^2 \tag{12}$$

where s_i is the ith budget share and α, β, γ, and λ are parameters. By setting $\lambda_i = 0$ $(i = 1, ..., n)$, equation (12) reduces to the AIDS share equations.

Economic theory imposes several restrictions on the parameters of the model. In particular, Slutsky symmetry requires

$$\gamma_{ij} = \gamma_{ji}, \ \text{ for all } i, j.$$

Homogeneity of the Marshallian demand functions of degree zero in $(\boldsymbol{p}, y)$ requires

$$\sum_{j=1}^{n} \gamma_{ij} = 0, \ \text{ for all } i.$$

Finally, the adding-up condition requires $\sum_{i=1}^{n} s_i = 1$ or, equivalently,

$$\sum_{i=1}^{n} \alpha_i = 1; \quad \sum_{i=1}^{n} \beta_i = 0; \quad \sum_{i=1}^{n} \lambda_i = 0; \text{ and } \sum_{i=1}^{n} \gamma_{ij} = 0, \text{ for all } j.$$

Following the Ryan and Wales (1998) procedure for imposing local curvature conditions, we can write the ijth element of the QUAIDS Slutsky matrix at the reference point ($p^* = y^* = 1$) as

$$\begin{aligned} S_{ij} = \gamma_{ij} &+ (\alpha_0\beta_i - \alpha_0^2\lambda_i - \alpha_i)\delta_{ij} - \alpha_0\beta_i\beta_j \\ &+ \alpha_0^2\beta_i\lambda_j + \alpha_0^2\beta_j\lambda_i - 2\alpha_0^3\lambda_i\lambda_j + \alpha_i\alpha_j \\ &- \alpha_0\alpha_i\beta_j - \alpha_0\alpha_j\beta_i + \alpha_0^2\alpha_i\lambda_j + \alpha_0^2\alpha_j\lambda_i \\ &+ \alpha_0^2\beta_i\beta_j - \alpha_0^3\beta_i\lambda_j - \alpha_0^3\beta_j\lambda_i + \alpha_0^4\lambda_i\lambda_j \end{aligned}$$

where δ_{ij} is the Kronecker delta, which equals 1 when $i = j$ and 0 otherwise. By replacing $\boldsymbol{S}$ by $-\boldsymbol{K}\boldsymbol{K}'$, the above can be written as

$$\begin{aligned} (-KK')_{ij} = \gamma_{ij} &+ (\alpha_0\beta_i - \alpha_0^2\lambda_i - \alpha_i)\delta_{ij} - \alpha_0\beta_i\beta_j \\ &+ \alpha_0^2\beta_i\lambda_j + \alpha_0^2\beta_j\lambda_i - 2\alpha_0^3\lambda_i\lambda_j + \alpha_i\alpha_j \\ &- \alpha_0\alpha_i\beta_j - \alpha_0\alpha_j\beta_i + \alpha_0^2\alpha_i\lambda_j + \alpha_0^2\alpha_j\lambda_i \\ &+ \alpha_0^2\beta_i\beta_j - \alpha_0^3\beta_i\lambda_j - \alpha_0^3\beta_j\lambda_i + \alpha_0^4\lambda_i\lambda_j. \end{aligned} \tag{13}$$

Solving for the γ_{ij} terms as a function of the $(KK')_{ij}$ terms we can get the restrictions that ensure the negative semidefiniteness of the Slutsky matrix, without destroying the flexibility of (10) as the number of free parameters remains the same. See Chang and Serletis (2012) for an example with $n = 3$.

3.3 The Minflex Laurent Model

The Minflex Laurent (ML) model, introduced by Barnett (1983) and Barnett and Lee (1985), is a special case of the Full Laurent model also introduced by Barnett (1983). Following Barnett (1983), the Full Laurent reciprocal indirect utility function is

$$h(\boldsymbol{v}) = a_0 + 2\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} v_i^{1/2} v_j^{1/2} - 2\sum_{i=1}^{n} b_i v_i^{-1/2} - \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} v_i^{-1/2} v_j^{-1/2} \tag{14}$$

where a_0, a_i, a_{ij}, b_i, and b_{ij} are unknown parameters and v_i denotes the income normalized price, p_i/y.

By assuming that $b_i = 0$, $b_{ii} = 0 \; \forall i$, $a_{ij}b_{ij} = 0 \; \forall i, j$, and forcing the off diagonal elements of the symmetric matrices $\boldsymbol{A} \equiv [a_{ij}]$ and $\boldsymbol{B} \equiv [b_{ij}]$ to be nonnegative, (14) reduces to the ML reciprocal indirect utility function

$$h(\boldsymbol{v}) = a_0 + 2\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n} a_{ii} v_i + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} a_{ij}^2 v_i^{1/2} v_j^{1/2} - \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}. \tag{15}$$

Note that the off diagonal elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ are nonnegative as they are raised to the power of two.

By applying Roy's identity to (15), the share equations of the ML demand system are

$$s_i = \frac{a_i v_i^{1/2} + a_{ii} v_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}}{\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n} a_{ii} v_i + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}}. \tag{16}$$

Since the share equations are homogenous of degree zero in the parameters, we follow Barnett and Lee (1985) and impose the following normalization in the estimation of (16)

$$\sum_{i=1}^{n} a_{ii} + 2\sum_{i=1}^{n} a_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij}^2 - \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij}^2 = 1. \tag{17}$$

Hence, there are

$$1 + n + \frac{n(n+1)}{2} + \frac{n(n-1)}{2}$$

parameters in (15), but the $n\,(n-1)\,/2$ equality restrictions, $a_{ij}b_{ij} = 0 \; \forall i, j$, and the normalization (17) reduce the number of parameters in equation (16) to $\left(n^2 + 3n\right)/2$.

As shown by Barnett (1983, Theorem A.3), (15) is globally concave for every $\boldsymbol{v} \geq \boldsymbol{0}$, if all parameters are nonnegative, as in that case (15) would be a sum of concave functions. If the initially estimated parameters of the vector $\boldsymbol{a}$ and matrix $\boldsymbol{A}$ are not nonnegative, curvature can be imposed globally by replacing each unsquared parameter by a squared parameter, as in Barnett (1983).

4 Demand System Approximation Properties

Most of the demand system specifications currently in use (including the three models in this paper) are intended to permit a 'second-order approximation' to an aggregator function.

Also, as noted by Barnett (1983), there are two concepts of second-order approximation in use in the demand literature — those proposed by Diewert (1971) and Christensen *et al.* (1973, 1975).

Regarding Diewert's (1971) definition, consider an n-argument, twice continuously differentiable aggregator function, $h(\boldsymbol{x})$. $h(\boldsymbol{x})$ is a flexible functional form if it contains enough parameters so that it can approximate an arbitrary twice continuously differentiable function h^* to the second order at an arbitrary point $\boldsymbol{x}^*$ in the domain of definition of h and h^*. Thus h must have enough free parameters to satisfy the following $1+n+n^2$ equations

$$h(\boldsymbol{x}^*) = h^*(\boldsymbol{x}^*) \tag{18}$$

$$\nabla h(\boldsymbol{x}^*) = \nabla h^*(\boldsymbol{x}^*) \tag{19}$$

$$\nabla^2 h(\boldsymbol{x}^*) = \nabla^2 h^*(\boldsymbol{x}^*) \tag{20}$$

where $\nabla h(\boldsymbol{x}) = \partial h(\boldsymbol{x})/\partial \boldsymbol{x}$ and $\nabla^2 h(\boldsymbol{x}) = \partial^2 h(\boldsymbol{x})/\partial x_i x_j$ denotes the $n \times n$ symmetric matrix of second-order partial derivatives of $h(\boldsymbol{x})$ evaluated at $\boldsymbol{x}$. The symmetry property follows from the assumption that $h(\boldsymbol{x})$ is twice continuously differentiable.

Barnett (1983) shows that Diewert's (1971) definition of second-order approximation is equivalent to the usual definition in the mathematics of local approximation orders. Regarding that mathematical definition, we follow Barnett (1983) and let h^* be an approximation to the function h and $\|\cdot\|$ to indicate the Euclidian norm. h^* is a second-order local approximation to h at the point $\boldsymbol{x}^*$, if

$$\frac{h^*(\boldsymbol{x}) - h(\boldsymbol{x})}{\|\boldsymbol{x} - \boldsymbol{x}^*\|^2} \to 0 \qquad \text{as} \qquad \boldsymbol{x} \to \boldsymbol{x}^*. \tag{21}$$

If h^* were a series expansion of h, then $h^*(\boldsymbol{x}) - h(\boldsymbol{x})$ would be the remainder term. Equation (21) is commonly interpreted to imply that $h^*(\boldsymbol{x}) - h(\boldsymbol{x})$ converges to zero faster than $\|\boldsymbol{x} - \boldsymbol{x}^*\|^2$. Equation (21) is sometimes equivalently written as $h^*(\boldsymbol{x}) - h(\boldsymbol{x}) = o\left(\|\boldsymbol{x}\text{-}\boldsymbol{x}^*\|^2\right)$, which commonly is read as "$h^*(\boldsymbol{x}) - h(\boldsymbol{x})$ is of little o order $\|\boldsymbol{x} - \boldsymbol{x}^*\|^2$." See Barnett (1983) for more details.

We use Diewert's (1971) definition and examine the flexibility of the AIDS and QUAIDS models based on their cost function, $\ln C(\boldsymbol{p}, u)$, and the flexibility of the Minflex Laurent based on its reciprocal indirect utility function, $h(\boldsymbol{v})$. We find that for the AIDS model, the cost function defined by (5) and 6) has $(n^2+3n)/2$ free parameters, just enough to be a flexible functional form at the point $(\boldsymbol{p}^*, u^*)$ in the class of linearly homogeneous in $\boldsymbol{p}$ cost functions. The cost function of the QUAIDS model in (11) has $(n^2+5n-4)/2$ free parameters, which is $n-1$ more free parameters than the minimum requirement to be a flexible functional form. The Minflex Laurent's reciprocal indirect utility function (15) contains $1+n+n(n+1)/2$ free parameters, which is just enough to be a flexible functional form at the point $\boldsymbol{v}^*$. The flexibility of each model can be achieved by choosing specific parameter values based on conditions (18)-(20).

Hence, the AIDS and Minflex Laurent models have exactly the minimum required number of free parameters and are parsimonious within the class of flexible functional forms, so there is no remaining parametric freedom beyond the ability to satisfy Diewert's (1971) definition of flexibility. The QUAIDS, however, has $n-1$ more free parameters than the minimum requirement to be a locally flexible functional form and is thus more flexible than the AIDS and Minflex Laurent models according to Diewert's (1971) definition.

5 Engel Curves and Demand System Rank

Household budget data, like the ones used in this paper, give rise to Engel curves (income expansion paths), which are functions describing how a consumer's purchases of some good vary as the consumer's income varies. Engel curves are Marshallian demand functions, with the prices of all goods held constant. Like Marshallian demand functions, Engel curves may also depend on demographic or other nonincome consumer characteristics (such as, for example, age and household composition), which we ignore in this paper.

In this section, we discuss the three demand systems (AIDS, QUAIDS, and Minflex Laurent) in terms of their ability to capture the Engel curve structure of the data and do so in terms of their rank. In particular, Lewbel (1991), extending earlier results by Muellbauer (1975, 1976) and Gorman (1981), defined the rank of any demand system to be the dimension of the space spanned by its Engel curves, holding demographic or other nonincome consumer characteristics fixed. Formally, the rank of a demand system is the smallest value of R such that each budget share can be written as

$$s_i(\boldsymbol{p}, y) = \sum_{r=1}^{R} \phi_{ri}(\boldsymbol{p}) f_r \left[\ln \left(\frac{y}{B(\boldsymbol{p})} \right) \right] \tag{22}$$

where ϕ_{ri} is a function of prices, f_r is a scalar valued function of 'deflated income,' and $B(\boldsymbol{p})$ is a linearly homogeneous function of prices.

More recently, LaFrance and Pope (2006) also use the concept of deflated income, introduced by Lewbel (1989), to determine the rank of demand systems through the indirect utility function, as follows

$$h(\boldsymbol{p}, y) = f \left[\sum_{r=1}^{R} \varphi_r \left(\frac{y}{\pi_r(\boldsymbol{p})} \right) \right] \tag{23}$$

where $\pi_r(\boldsymbol{p})$ is any function that is linearly homogeneous in prices. In the context of (23), the rank is the number of $\varphi_r(y/\pi_r(\boldsymbol{p}))$ functions that maximally span the indirect utility function, $h(\boldsymbol{p}, y)$.

In this section, we follow LaFrance and Pope (2006) and determine the rank of each of the AIDS, QUAIDS, and Minflex Laurent models. In doing so, we start with the indirect utility function, $h(\boldsymbol{p}, y)$, transform it into some function in terms of $\pi_r(\boldsymbol{p})$ and $(y/\pi_r(\boldsymbol{p}))$, and determine the rank by the number of the $\varphi_r(y/\pi_r(\boldsymbol{p}))$ functions that maximally span the indirect utility function.

5.1 The AIDS: A Rank Two Demand System

Let $P_1 = a(\boldsymbol{p})$, where P_1 is a homogeneous function of degree one in prices, and $P_2 = \ln[b(\boldsymbol{p})/a(\boldsymbol{p})]$, where P_2 is a homogeneous function of degree zero in prices. Then the AIDS cost function (5) can be written as

$$\ln C(\boldsymbol{p}, u) = \ln P_1 + uP_2$$

and its corresponding indirect utility function (4) as

$$h(\boldsymbol{p}, y) = \frac{\ln y - \ln P_1}{P_2} = \frac{\ln y - \ln P_1}{y/P_2} \frac{y}{P_2^2}.$$

Taking the log of both sides of the above equation yields

$$\ln h(\boldsymbol{p}, y) = \ln\left[\ln\left(\frac{y}{P_1}\right)\right] - \ln\left(\frac{y}{P_2}\right) + \ln\left(\frac{y}{P_2^2}\right).$$

Defining $\pi_1(\boldsymbol{p}) = P_1$, $\pi_2(\boldsymbol{p}) = P_2$, and $\pi_3(\boldsymbol{p}) = P_2^2$, where $\pi_3(\boldsymbol{p})$ is a homogeneous function of degree zero in prices, the above equation can be written, in terms of the expenditure function, as

$$u = \ln\left[\ln\left(\frac{C(\boldsymbol{p}, u)}{\pi_1(\boldsymbol{p})}\right)\right] - \ln\left(\frac{C(\boldsymbol{p}, u)}{\pi_2(\boldsymbol{p})}\right) + \ln\left[\frac{C(\boldsymbol{p}, u)}{\pi_3(\boldsymbol{p})}\right]$$

with $u = \ln h(\boldsymbol{p}, y)$. Because the last two terms on the right-hand side of this above equation are highly linearly dependent, the model is not a full rank three model, but has rank two — see LaFrance and Pope (2006, Proposition 2).

5.2 The QUAIDS: A Rank Three Demand System

Taking the log of both sides of (10) yields

$$\ln[\ln h(\boldsymbol{p}, y)] = \ln\left[\ln\left(\frac{y}{a(\boldsymbol{p})}\right)\right] - \ln\left[\lambda(\boldsymbol{p})\ln\left(\frac{y}{a(\boldsymbol{p})}\right)\right] - \ln\left[1 + \frac{b(\boldsymbol{p})}{\lambda(\boldsymbol{p})}\ln\left(\frac{y}{a(\boldsymbol{p})}\right)^{-1}\right].$$

Defining $\pi_1(\boldsymbol{p}) = a(\boldsymbol{p})$, $\pi_2(\boldsymbol{p}) = \lambda(\boldsymbol{p})$, and $\pi_3(\boldsymbol{p}) = b(\boldsymbol{p})/\lambda(\boldsymbol{p})$, where $\pi_3(\boldsymbol{p})$ is a homogeneous function of degree zero in prices, then the above yields

$$u = \ln\left[\ln\left(\frac{C(\boldsymbol{p}, u)}{\pi_1(\boldsymbol{p})}\right)\right] - \ln\left[\pi_2(\boldsymbol{p})\ln\left(\frac{C(\boldsymbol{p}, u)}{\pi_1(\boldsymbol{p})}\right)\right] - \ln\left[1 + \pi_3(\boldsymbol{p})\ln\left(\frac{C(\boldsymbol{p}, u)}{\pi_1(\boldsymbol{p})}\right)^{-1}\right]$$

with $u = \ln[\ln h(\boldsymbol{p}, y)]$. This is a full rank three system that does not satisfy proposition 2 in LaFrance and Pope (2006).

5.3 The Minflex Laurent: A Rank Three Demand System

Finally, the ML reciprocal indirect utility function (15) can be written as

$$h(\boldsymbol{p}, y) = \alpha_0 + 2\sum_{i=1}^{n} \alpha_i \sqrt{\frac{p_i}{y}} + \sum_{i=1}^{n} a_{ii} \frac{p_i}{y} + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} a_{ij}^2 \frac{\sqrt{p_i p_j}}{y} - \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} b_{ij}^2 \frac{y}{\sqrt{p_i p_j}}.$$

Defining $\pi_1(\boldsymbol{p}) = \left(2\sum_{i=1}^{n} \alpha_i \sqrt{p_i}\right)^2$, $\pi_2(\boldsymbol{p}) = \sum_{i=1}^{n} a_{ii} p_i$, $\pi_3(\boldsymbol{p}) = \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} a_{ij}^2 \sqrt{p_i p_j}$, and $\pi_4(\boldsymbol{p})^{-1} = \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} b_{ij}^2 / \sqrt{p_i p_j}$, the above can be written as

$$\alpha_0 - \frac{1}{u} = -\left[\frac{C(\boldsymbol{p}, u)}{\pi_1(\boldsymbol{p})}\right]^{-1/2} - \left[\frac{C(\boldsymbol{p}, u)}{\pi_2(\boldsymbol{p})}\right]^{-1} - \left[\frac{C(\boldsymbol{p}, u)}{\pi_3(\boldsymbol{p})}\right]^{-1} + \left[\frac{C(\boldsymbol{p}, u)}{\pi_4(\boldsymbol{p})}\right].$$

As noted by LaFrance and Pope (2006), the Minflex Laurent is not a full rank four system, but is rank three because of the linear dependence of the second and third terms on the right-hand side of the above equation.

6 Data and Econometric Issues

We use annual expenditure data on three expenditure categories in the transportation sector: gasoline (1), local transportation (2), and inter-city transportation (3). The data are from the Canadian *Survey of Household Spending* over a 13-year period, from 1997 to 2009. The associated prices for these expenditure categories are normalized (with 2002 = 100) annual consumer price indices from Statistics Canada, CANSIM II Table 3260021.

Our underlying assumption is that individuals with similar demographic characteristics have similar decision-making behaviors on their expenditures. Thus, in this paper we only consider households aged 25 to 64, living in urban areas with a population of at least 30,000 people who had positive spending on gasoline, local transportation, and inter-city transportation. This excludes Prince Edward Island, the Yukon, the Northwest Territories, and Nunavut. We also focus on residents in English Canada, thereby excluding Quebec from our analysis.

Finally, data for three household types are extracted from the surveys for our study: single-member households, married couples without children, and married couples with one child. Under these restrictions, we have a total 2,218 observations for the single-member households, 3,326 observations for the married couples, and 6,141 observations for the married couples with one child.

In order to estimate share equation systems such as (7), (12), and (16), a stochastic version must be specified. Also, since only exogenous variables appear on the right-hand side, it seems reasonable to assume that the observed share in the ith equation deviates from the true share by an additive disturbance term u_i. We assume $\boldsymbol{u} \sim N(\mathbf{0}, \boldsymbol{\Omega})$ where $\boldsymbol{u} = (u_1, ..., u_n)'$, $\mathbf{0}$ is a null matrix, and $\boldsymbol{\Omega}$ is the $n \times n$ symmetric positive definite error covariance matrix. Thus, the share equation system for each model for household h can be written in matrix form as

$$\boldsymbol{s}_h = \boldsymbol{g}(\boldsymbol{p}_h, y_h, \vartheta) + \boldsymbol{u}_h, \qquad h = 1, ..., H \tag{24}$$

where $\boldsymbol{s} = (s_1, \cdots, s_n)'$, $\boldsymbol{g}(\boldsymbol{p}_h, y_h, \vartheta) = (\boldsymbol{g}_1(\boldsymbol{p}_h, y_h, \vartheta), \cdots, \boldsymbol{g}_n(\boldsymbol{p}_h, y_h, \vartheta))'$, ϑ is the parameter vector to be estimated, and $\boldsymbol{g}_i(\boldsymbol{p}_h, y_h, \vartheta)$ is given by the right-hand side of each of (7), (12), and (16).

Since the share equation system (24) is estimated as a closed form system with all right hand variables treated as exogenous, there are potential problems of simultaneity bias in the demand elasticity estimates. We acknowledge the fact that income and prices may not be exogenous at the aggregate level in general equilibrium. However, in this study, they are exogenous for households since an error term in an individual consumer's demand equation should not influence the price function that clears the market. It is also to be noted that in this literature, the possibility of endogeneity has been addressed by using iterative three-stage least squares (3SLS), but the results generally have been about the same as those with iterative Zellner estimation — see, for example, Barnett *et al.* (1991).

Finally, since the budget shares sum to 1, the disturbance covariance matrix is singular. To address this issue, Barten (1969) showed that maximum likelihood estimates can be obtained by arbitrarily dropping any equation in the system. We follow Barten (1969) and drop the last equation in each model. All estimations are performed in TSP (version 5.1) using the non-linear full-information maximum likelihood estimation procedure.

7 Empirical Evidence

Tables 2-4 contain a summary of results from the AIDS, QUAIDS, and Minflex Laurent models in terms of parameter estimates (with p-values in parentheses), positivity, monotonicity, and curvature violations, as well as log likelihood values, when the models are estimated without the curvature conditions imposed (unrestricted) and with the curvature conditions imposed for each of the three different types of Canadian households. With the exception of the Minflex Laurent model for married couples with one child, all models satisfy positivity and monotonicity at all sample observations when curvature is not imposed (see the unrestricted column); the unrestricted Minflex Laurent model violates positivity and monotonicity at 1 out of $6,141$ observations in the case of married couples with one child (see Table 4). However, all three unrestricted models violate curvature when curvature is not imposed, except for the AIDS model for single member households which satisfies all three theoretical regularity conditions at all sample observations (see panel A of Table 2).

Because regularity has not been attained (by luck), except for the AIDS when estimated with the data for single members, we follow Barnett (2002) and estimate the models by imposing local curvature, using the methodology discussed in Section 3. The results are disappointing in the case of the AIDS and QUAIDS models, as can be seen under the 'Local curvature imposed' column of Tables 2 and 3; the imposition of local curvature on each of these models reduces the number of curvature violations, but does not completely eliminate them (for all three household types), as is the case with the Minflex Laurent model. We find that the curvature-constrained Minflex Laurent is the only model that is able to provide inferences about the demand for gasoline in Canada that are consistent with neoclassical microeconomic theory (see Table 4). Moreover, the imposition of global curvature on the Minflex Laurent does not have much of an effect on its flexibility, as is indicated by the insignificant differences between the log likelihood values in the unrestricted and restricted versions of the model (see the second last row in Table 4).

In the demand systems approach to estimation of economic relationships, the primary interest, especially in policy analysis, is in how the arguments of the underlying function affect the quantities demanded. This is conventionally completely expressed in terms of income and price elasticities. These elasticities can be calculated from the estimated budget share equations by writing the left-hand side as

$$x_i = \frac{s_i y}{p_i}$$

for $i = 1, ..., n$. In particular, the income elasticities can be calculated by

$$\eta_{iy} = 1 + \frac{y}{s_i}\frac{\partial s_i}{\partial y}$$

for $i = 1, ..., n$, and the Marshallian (or uncompensated) price elasticities can be calculated by

$$\eta_{ij} = \frac{p_j}{s_i}\frac{\partial s_i}{\partial p_j} - \delta_{ij}$$

for $i, j = 1, ..., n$, where δ_{ij} is the Kronecker delta (that is, $\delta_{ij} = 1$ when $i = j$ and 0 otherwise).

We report the income and own- and cross-price elasticities (evaluated at the mean of the data) in Table 5 with p-values in parentheses, using the Delta method to calculate the standard errors. We do so for each of the three transportation categories and for only the demand systems that satisfy the theoretical regularity conditions at all data points of the sample; the AIDS and the curvature-constrained Minflex Laurent models in the case of single member households (in panel A) and the curvature-constrained Minflex Laurent model in the case of married couples (in panel B) and married couples with one child (in panel C).

In Table 5, we show the income elasticities and the own- and cross-price elasticities for each of the three transportation categories. As expected, all the income elasticities, η_1,

η_2, and η_3, are positive, implying that gasoline (1), local transportation (2), and inter-city transportation (3) are all normal goods for all three types of Canadian households. As can be seen, the income elasticity of gasoline demand varies from 0.841 to 0.973 as the family size changes, and is statistically significant in all cases, suggesting that the demand for gasoline in Canada is more elastic to income changes than earlier studies have reported using methodology different than ours. For example, Yatchew and No (2001) estimate a partially linear model with price and age entering nonparametrically using monthly data from October 1994 to September 1996 and find an income elasticity of 0.29. Graham and Glaister (2002) report one of 0.53 for Canada based on Sterner *et al.* (1992), where they applied the lagged endogenous model and annual time series data over the period from 1960 to 1985.

The own-price elasticities (η_{ii}) are all negative (as predicted by the theory), with their absolute values being less than 1, which indicates that the demands for all three types of transportation are inelastic for Canadian families. Our estimates of the gasoline own-price elasticity vary from -0.738 to -0.570 and are statistically significant in all cases, which is overall less elastic than earlier studies have reported; for example, -0.9 in Yatchew and No (2001) and approximately -1.07 in Graham and Glaister (2002). We also test the null hypothesis that they are equal to negative one and reject the null hypothesis at the 5 percent level for all three cases, suggesting that the Canadian demand for gasoline is statistically inelastic. Moreover, since the absolute value of the own-price elasticity of gasoline for married couples with one child is the largest among all three types of households, we conclude that the larger the size of the Canadian family, the more sensitive the family is to gasoline price changes or oil price shocks. Regarding the cross-price elasticities (η_{ij}), we see that most of the off-diagonal terms in panel A of Table 5 are negative and all are negative in panels B and C, indicating that the three transportation categories are gross complements for Canadian households.

Our estimates of the own price elasticity for gasoline demand are different from those reported by Yatchew and No (2001) and Graham and Glaister (2002) for a number of reasons. First, Yatchew and No (2001) use monthly data from October 1994 to September 1996 whereas we use annual household survey data from 1997 to 2009, a much more interesting and longer span of data. We estimate elasticities for each of the three different household types — single-member households, married couples without children, and married couples with one child — while Yatchew and No (2001) provide estimates using data for all Canadian households. Our methodology is also different, as we use the parameters obtained from the curvature-constrained Minflex Laurent model to compute the price elasticities whereas Yatchew and No (2001) use a semiparametric model, which is a partial linear model with price and age entering nonparametrically. Finally, we investigate the demand for gasoline in a systems context together with the demand for local transportation and inter-city transportation; Yatchew and No (2001) investigate the demand for gasoline alone, as a function of its price, household income, and other demographic variables. Finally, the survey paper by

Graham and Glaister (2002) reports a long-run price elasticity for Canada of -1.07, based on Sterner *et al.* (1992) and the use of annual time series data for 21 OECD countries over the period from 1960 to 1985.

Finally, given the nature of our data, it should be noted that the obtained elasticities represent long-run responses rather than short-run behavior. In this regard and in a slightly different context, Houthakker (1965) convincingly argues that individual country (heterogeneous) estimates capture mostly short-run effects, while pooled (homogeneous) estimates reflect mostly long-run adjustments and that the differences between these effects are highly significant both statistically and economically. Our estimates seem to be consistent with an alternative strand of the literature cited in Hamilton (2009) with much smaller numbers for short-run elasticities. In fact, Baltagi and Griffin (1997) find that short-run elasticities are, on average, four times smaller than the corresponding long-run ones.

8 Conclusion

We investigate the demand for gasoline in Canada using recent household survey data, over a 13-year period from 1997 to 2009, on three expenditure categories in the transportation sector: gasoline, local transportation, and inter-city transportation. In doing so, we use three of the most widely used locally flexible functional forms, the AIDS, QUAIDS, and the Minflex Laurent model. As noted in the introduction, so far the literature on the empirical gasoline demand using flexible functional forms did not report the results of full theoretical regularity checks. The present paper represents the first attempt to investigate the demand for gasoline in the context of consistency with neoclassical microeconomic theory. We pay explicit attention to economic regularity, and argue that unless regularity is attained by luck, flexible functional forms should always be estimated subject to regularity as suggested by Barnett (2002). Regularity is important, because it is assumed in the duality theorems (such as Roy's) to derive the demand system from the generating function (usually, the indirect utility function). Without regularity, the resulting demand system is derived from a theorem that is not applicable. For example, if curvature is violated at a point, then at that point Roy's theorem produces local utility minimization, not local utility maximization.

Our findings in terms of regularity violations are disappointing in the case of the AIDS and QUAIDS models, even when the curvature conditions are imposed, since the imposition of local curvature on each of these models, although reduces the number of curvature violations, it does not completely eliminate them, for all three household types. We find that the curvature constrained Minflex Laurent is the only model that is able to provide inferences about the demand for gasoline in Canada that are consistent with neoclassical microeconomic theory. The Minflex Laurent model is a rank three demand system, and more flexible than the QUAIDS model in the Diewert (1971) sense. Based on the Minflex Laurent, our estimates of the own-price elasticity of gasoline demand in Canada are statistically significant and vary from -0.738 to -0.570, suggesting that gasoline demand in Canada is overall less elastic than previously reported.

Regarding the source of the differences between the theoretical regularity properties of the AIDS, QUAIDS, and Minflex Laurent model, they can be seen from inspecting their functional forms. Both the AIDS and QUAIDS are produced from a second-order Taylor series expansion whereas the Minflex Laurent is a special case of a second-order Laurent series expansion in the squared roots of the variables; Barnett (1985) has also produced a minflex Laurent translog flexible functional form by the analogous expansion in the logarithms of the variables. The Laurent series is a generalization of the Taylor series and the Laurent expansion has a better behaved remainder term over the region of the approximation than does the remainder term of the Taylor series expansion. In this regard, Barnett and Lee (1985) compare the global properties of the Minflex Laurent model and two other models based on second-order Taylor series expansions — the generalized Leontief and the translog flexible functional forms — and find that the minflex Laurent model has the largest regular region and that its regular region expands as real income increases. The AIDS and QUAIDs were not considered by Barnett and Lee (1985); in fact the QUAIDS was introduced in 1997. This is the first paper in the literature that compares the regularity properties of the AIDS and QUAIDs to those of the Minflex Laurent.

Based on our evidence, in order to achieve its commitment to reduce greenhouse gas emissions, the Canadian government should impose a larger gasoline tax than previously thought to achieve significant reductions in gasoline consumption. The two instruments that are most often discussed in addressing gasoline consumption in Canada are the federal gasoline tax and the Company Average Fuel Consumption (CAFC) targets. The gasoline tax has the advantages over the CAFC targets, because it encourages improvements in fuel efficiency and reductions in miles driven, while raising tax revenue. In contrast, CAFC targets may encourage more driving, as increases in fuel efficiency reduce the cost of gasoline per mile driven — see West and Williams III (2005) for a discussion of these two instruments in the case of the United States. Moreover, increasing the gasoline tax increases labour supply which produces additional efficiency gains; see West and Williams III (2004a). Therefore, imposing gasoline tax shows a remarkable effect as a emission control policy, which heavily relies on the magnitude of the demand elasticity of gasoline.

As of December 31, 2011, the Canadian federal/excise tax on gasoline was 10 cents per liter, which is the second lowest in the OECD countries — see Natural Resources Canada (2012). In addition, Canada is one of the only two OECD countries whose gasoline tax was less in 2009 than it was in 1998. Moreover, in 2005 Canada's absolute greenhouse gas emissions were 54 percent above 1990 levels, according to the Conference Board of Canada (2011). It is then clear that Canada's tax policy on gasoline has not been as effective in reducing gasoline consumption. If the current tax rate on gasoline consumption is based on elasticity estimates reported in earlier studies as, for example, -0.9 in Yatchew and No (2001) and approximately -1.07 in Graham and Glaister (2002), it is not surprising to see that the tax policy has not been as effective, because Canadian households are in fact less sensitive to changes in gasoline prices than what the earlier literature suggests. According to our estimates, a tax would need to be significantly higher today in order to achieve substantial reductions in gasoline consumption.

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Figure 1. Spot Prices for Crude Oil and Gasoline

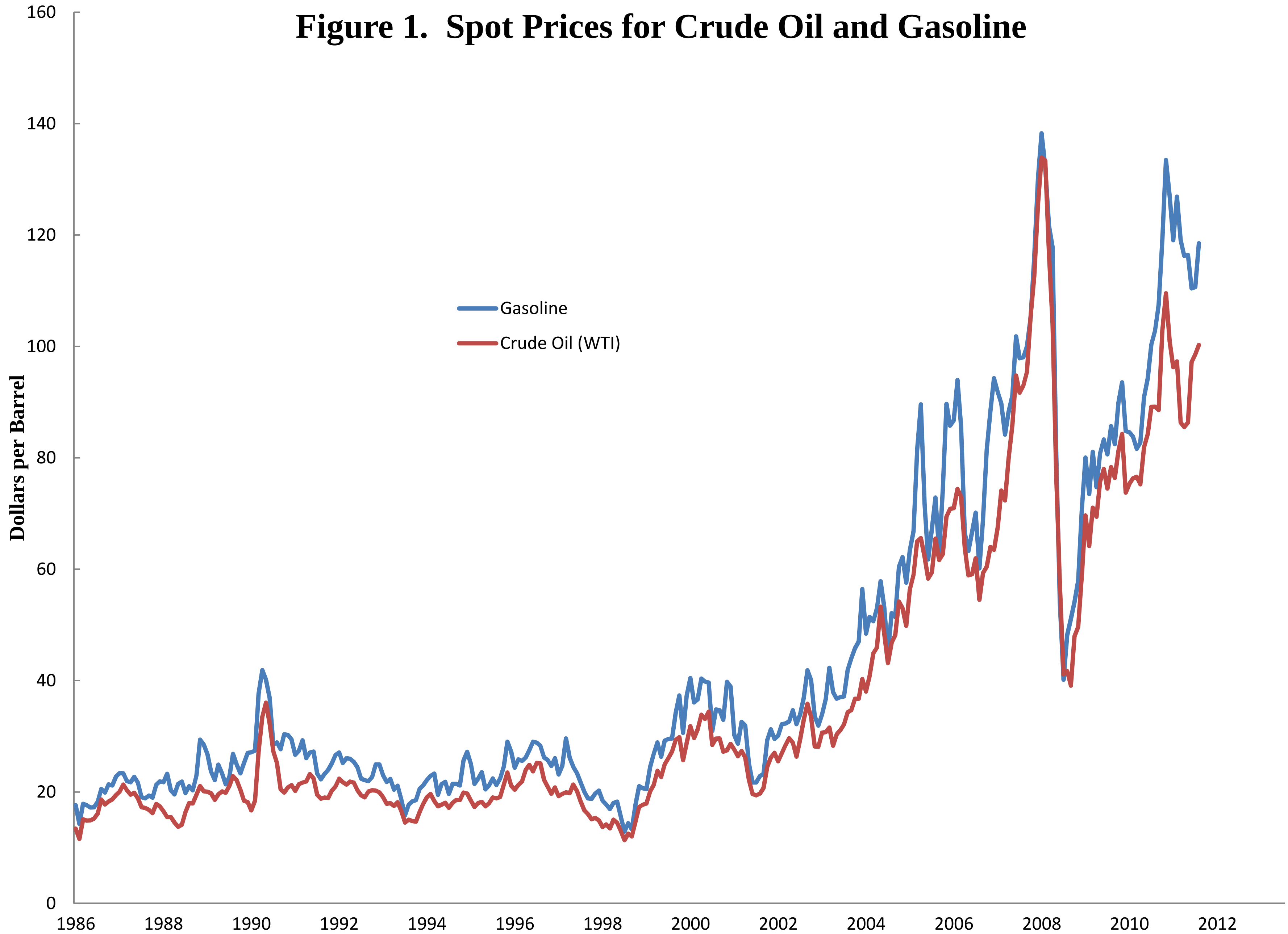

TABLE 1

A SUMMARY OF FLEXIBLE FUNCTIONAL FORMS
ESTIMATION OF GASOLINE DEMAND

Author(s)	Model used	Data used	Curvature imposed
Fuss (1977)	Translog	Canadian	No
Archibald and Gillingham (1980)	Translog	U.S.	No
Considine (1989)	Translog	U.S.	No
Conrad and Schröder (1991)	AIDS	German	No
Nicol (2003)	QUAIDS	U.S. and Canadian	No
West and Williams III (2004b, 2007)	AIDS	U.S.	No
Ghalwash (2008)	QUAIDS	Swedish	No
Wadud *et al.* (2010)	Translog	U.S.	No

Table 2. AIDS Parameter Estimates

Goods:
1 = gasoline
2 = local transportation
3 = inter-city transportation

	A. Single members	B. Married couples		C. Married couples with one child	
Parameter	Unrestricted	Unrestricted	Local curvature imposed	Unrestricted	Local curvature imposed
a_0	.047 (.999)	−173.413 (.785)	−227.012 (.720)	−456.471 (.293)	−459.605 (.285)
a_1	.486 (.996)	14.459 (.777)	18.812 (.712)	40.553 (.281)	40.985 (.274)
a_2	.088 (.998)	4.736 (.781)	6.156 (.717)	11.272 (.295)	11.313 (.288)
β_{11}	.091 (.987)	−1.051 (.798)	−1.406 (.732)	−3.503 (.286)	−3.556 (.279)
β_{12}	−.020 (.994)	−.398 (.772)	−.513 (.708)	−1.021 (.085)	−1.027 (.269)
β_{22}	−.052 (.965)	.016 (.973)	−.091 (.845)	−.133 (.631)	−.194 (.478)
b_1	−.064 (.000)	−.080 (.000)	−.081 (.000)	−.088 (.000)	−.088 (.000)
b_2	−.029 (.000)	−.027 (.000)	−.027 (.000)	−.025 (.000)	−.024 (.000)
Positivity violations	0	0	0	0	0
Monotonicity violations	0	0	0	0	0
Curvature violations	0	3326	1397	6141	2894
Log likelihood value	1682.07	2952.04	2950.82	5157.34	5155.72
Number of observations	2218	3326	3326	6141	6141

Notes: Sample period, 1997-2009. Numbers in parentheses are p-values.

Table 3. QUAIDS Parameter Estimates

Goods:

1 = gasoline

2 = local transportation

3 = inter-city transportation

	Single members		Married couples		Married couples with one child	
Parameter	Unrestricted	Local curvature imposed	Unrestricted	Local curvature imposed	Unrestricted	Local curvature imposed
a_0	−7.007 (.006)	5.898 (.013)	5.287 (.001)	−6.764 (.000)	−4.729 (.000)	−4.740 (.000)
a_1	−.579 (.468)	−1.122 (.253)	−1.033 (.172)	−.527 (.409)	.032 (.928)	.029 (.934)
a_2	.381 (.074)	−.018 (.923)	−.191 (.319)	.102 (.600)	.171 (.002)	.167 (.002)
β_{11}	−.438 (.383)	.785 (.198)	.761 (.130)	−.500 (.259)	−.283 (.193)	−.285 (.190)
β_{12}	.046 (.622)	−.002 (.979)	.084 (.401)	−.060 (.468)	−.037 (.120)	−.037 (.119)
β_{22}	−.065 (.403)	−.045 (.545)	.166 (.001)	.067 (.108)	.145 (.000)	.078 (.010)
b_1	.385 (.002)	−.471 (.000)	−.494 (.001)	.404 (.001)	.336 (.001)	.336 (.001)
b_2	−.055 (.333)	−.008 (.885)	−.074 (.183)	.022 (.715)	−.010 (.691)	−.008 (.743)
λ_1	−.033 (.000)	−.033 (.000)	−.038 (.000)	−.037 (.000)	−.047 (.000)	−.047 (.000)
λ_2	.002 (.661)	.002 (.712)	−.004 (.379)	−.004 (.409)	−.002 (.568)	−.002 (.524)
Positivity violations	0	0	0	0	0	0
Monotonicity violations	0	0	0	0	0	0
Curvature violations	1923	59	3326	1651	6141	3373
Log likelihood value	1692.29	1691.89	2967.99	2967.42	5194.76	5192.77
Number of observations	2218	2218	3326	3326	6141	6141

Notes: Sample period, 1997-2009. Numbers in parentheses are p-values.

Table 4. Minflex Laurent Parameter Estimates

Goods:

1 = gasoline

2 = local transportation

3 = inter-city transportation

	Single members		Married couples		Married couples with one child	
Parameter	Unrestricted	Global curvature imposed	Unrestricted	Global curvature imposed	Unrestricted	Global curvature imposed
a_1	−.002 (.593)	.003 (.632)	−.003 (.046)	.002 (.655)	−.004 (.002)	.039 (.590)
a_2	−.003 (.056)	.000 (.999)	−.002 (.001)	.000 (.999)	−.002 (.000)	.011 (.625)
a_3	.013 (.143)	.028 (.086)	.007 (.138)	.025 (.050)	.004 (.069)	.062 (.570)
a_{11}	.369 (.000)	.607 (.000)	.393 (.000)	.761 (.000)	.355 (.000)	.568 (.156)
a_{13}	−.517 (.000)	−.287 (.229)	−.532 (.000)	−.127 (.808)	−.609 (.000)	−.086 (.971)
a_{33}	−.091 (.452)	.000 (.999)	−.108 (.224)	.000 (.999)	−.235 (.001)	.001 (.998)
a_{22}	.169 (.000)	.166 (.490)	.146 (.000)	.153 (.269)	.141 (.000)	.193 (.622)
b_{12}	.000 (.999)	.000 (.999)	.000 (.439)	.000 (.999)	.000 (.999)	.000 (.999)
Positivity violations	0	0	0	0	1	0
Monotonicity violations	0	0	0	0	1	0
Curvature violations	1951	0	2952	0	5402	0
Log likelihood value	1682.83	1677.63	2958.88	2944.63	5181.81	4969.70
Number of observations	2218	2218	3326	3326	6141	6141

Notes: Sample period, 1997-2009. Numbers in parentheses are p-values.

TABLE 5

AIDS AND CURVATURE-RESTRICTED MINFLEX LAURENT INCOME AND PRICE ELASTICITIES

Goods:

1 = gasoline
2 = local transportation
3 = inter-city transportation

Good		Elasticities			
		Income	Own- and cross-price		
(i)	Model	η_i	η_{i1}	η_{i2}	η_{i3}
		A. Single members (2218 observations)			
(1)	AIDS	.869 (.000)	−.750 (.000)	−.029 (.552)	−.090 (.000)
	Minflex	.862 (.000)	−.593 (.000)	−.099 (.000)	−.170 (.029)
(2)	AIDS	.678 (.450)	−.064 (.811)	−1.548 (.067)	.934 (.000)
	Minflex	.799 (.000)	−.481 (.000)	−.099 (.890)	−.219 (.759)
(3)	AIDS	1.221 (.975)	−.277 (.997)	.150 (.998)	−1.095 (.970)
	Minflex	1.234 (.000)	−.415 (.000)	−.099 (.587)	−.719 (.001)
		B. Married couples (3326 observations)			
(1)	Minflex	.841 (.000)	−.570 (.000)	−.087 (.000)	−.183 (.003)
(2)	Minflex	.790 (.000)	−.511 (.000)	−.087 (.844)	−.192 (.665)
(3)	Minflex	1.278 (.000)	−.499 (.000)	−.087 (.396)	−.692 (.000)
		C. Married couples with one child (6141 observations)			
(1)	Minflex	.973 (.000)	−.738 (.000)	−.067 (.000)	−.169 (.000)
(2)	Minflex	.887 (.000)	−.342 (.000)	−.375 (.209)	−.170 (.585)
(3)	Minflex	1.077 (.000)	−.340 (.000)	−.067 (.449)	−.670 (.000)

Note: Numbers in parentheses are p-values.