Investments, 4th ed

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Immunization

In contrast to indexing strategies, many institutions try to insulate their portfolios from interest rate risk altogether. Generally, there are two ways of viewing this risk, depending on the circumstances of the particular investor. Some institutions, such as banks, are concerned with protecting the current net worth or net market value of the firm against interest rate fluctuations. Other investors, such as pension funds, may face an obligation to make payments after a given number of years. These investors are more concerned with protecting the future values of their portfolios.

What is common to the bank and the pension fund, however, is interest rate risk. The net worth of the firm or the ability to meet future obligations fluctuates with interest rates. These institutions presumably might be interested in methods to control that risk. We will see that, by properly adjusting the maturity structure of their portfolios, these institutions can shed their interest rate risk. **Immunization** techniques refer to strategies used by such investors to shield their overall financial status from exposure to interest rate fluctuations.

Net Worth Immunization

Many banks and thrift institutions have a natural mismatch between asset and liability maturity structures. Bank liabilities are primarily the deposits owed to customers, most of which are very short-term in nature and, consequently, of low duration. Bank assets by contrast are composed largely of outstanding commercial and consumer loans or mortgages. These assets are of longer duration than are deposits, and their values are correspondingly more sensitive to interest rate fluctuations. In periods when interest rates increase unexpectedly, banks can suffer serious decreases in net worth—their assets fall in value by more than their liabilities.

The watchword in bank portfolio strategy has become asset and liability management. Techniques called *gap management* were developed to limit the "gap" between asset and liability durations. Adjustable-rate mortgages are one way to reduce the duration of bank asset portfolios. Unlike conventional mortgages, adjustable-rate mortgages do not fall in value when market interest rates rise, because the rates they pay are tied to an index of the current market rate. Even if the indexing is imperfect or entails lags, indexing greatly

diminishes sensitivity to interest rate fluctuations. On the other side of the balance sheet, the introduction of bank certificates of deposit with fixed terms to maturity serves to lengthen the duration of bank liabilities, also reducing the duration gap.

One way to view gap management is that the bank is attempting to equate the durations of assets and liabilities to effectively immunize its overall position from interest rate movements. Because bank assets and liabilities are roughly equal in size, if their durations also are equal, any change in interest rates will affect the values of assets and liabilities equally. Interest rates would have no effect on net worth, in other words. Therefore, net worth immunization requires a portfolio duration of zero. This will result if assets and liabilities are equal in both magnitude and duration.

Concept CHECK

Question 3 • If assets and liabilities are not equal, then immunization requires that $D_A A = D_L L$ where D denotes duration and A and L denote assets and liabilities, respectively. Explain why the simpler condition, $D_A = D_L$, is no longer valid in this case.

Target Date Immunization

In contrast to banks, pension funds think more in terms of future commitments than current net worth. Pension funds have an obligation to provide workers with a flow of income upon their retirement, and they must have sufficient funds available to meet these commitments. As interest rates fluctuate, both the value of the assets held by the fund and the rate at which those assets generate income fluctuate. The pension fund manager, therefore, may want to protect, or "immunize," the future accumulated value of the fund at some target date against interest rate movements.

The nearby box illustrates the dangers that pension funds face when they neglect the interest rate exposure of *both* assets and liabilities. The article points out that when interest rates change, the present value of the fund's liabilities change. For example, although pension funds enjoyed excellent investment returns in 1995, they lost ground because as interest rates fell, the value of their liabilities grew even faster than the value of their assets. The article concludes that funds should match the interest rate exposure of assets and liabilities so that the value of assets will track the value of liabilities whether rates rise or fall.

Pension funds are not alone in this concern. Any institution with a future fixed obligation might consider immunization a reasonable risk management policy. Insurance companies, for example, also pursue immunization strategies. Indeed, the notion of immunization was introduced by F. M. Redington,[6] an actuary for a life insurance company. The idea behind immunization is that duration-matched assets and liabilities let the asset portfolio meet the firm's obligations despite interest rate movements. Consider, for example, an insurance company that issues a guaranteed investment contract, or GIC, for $10,000. (Essentially, GICs are zero-coupon bonds issued by the insurance company to its customers. They are popular products for individuals' retirement-saving accounts.) If the GIC has a five-year maturity and a guaranteed interest rate of 8%, the insurance company is obligated to pay $\$10{,}000 \times (1.08)^5 = \$14{,}693.28$ in five years.

Suppose that the insurance company chooses to fund its obligation with $10,000 of 8% *annual* coupon bonds, selling at par value, with six years to maturity. As long as the market interest rate stays at 8%, the company has fully funded the obligation, as the present value of the obligation exactly equals the value of the bonds.

[6]F. M. Redington, "Review of the Principle of Life-Office Valuations," *Journal of the Institute of Actuaries* 78 (1952).

HOW PENSION FUNDS LOST IN MARKET BOOM

In one of the happiest reports to come out of Detroit lately, General Motors proclaimed Tuesday that its U.S. pension funds are now "fully funded on an economic basis." Less noticed was GM's admission that, in accounting terms, it is still a few cents—well, $3 billion—shy of the mark.

Wait a minute. If GM's pension plans were $9.3 billion in the hole when 1995 began, and if the company, to its credit, shoveled in $10.4 billion more during the year, how come its pension deficit wasn't wiped out in full?

We'll get to that, but the real news here is broader than GM. According to experts, most pension funds actually *lost* ground in 1995, even though, as you may recall, it was a rather good year for stocks and bonds.

True, pension-fund assets did have a banner year. But as is sometimes overlooked, pension funds also have liabilities (their obligations to retirees). And at most funds, liabilities grew at a rate that put asset growth to shame. At the margin, that means more companies' pension plans will be "underfunded." And down the road, assuming no reversal in the trend, more companies will have to pony up more cash.

What's to blame? The sharp decline in interest rates that brought joy to everyone else. As rates fall, pension funds have to set aside more money today to pay off a fixed obligation tomorrow. In accounting-speak, this "discounted present value" of their liabilities rises.

By now, maybe you sense that pension liabilities swing more, in either direction, than assets. How come? In a phrase, most funds are "mismatched," meaning their liabilities are longer-lived than their investments. The longer an obligation, the more its current value reacts to changes in rates. And at a typical pension fund, even though the average obligation is 15 years away, the average duration of its bond portfolio is roughly five years.

If this seems to defy common sense, it does. No sensible family puts its grocery money (a short-term obligation) into common stocks (a long-term asset). And a college sophomore is unlikely to put his retirement savings into two-year bonds. Ordinary Joes and Janes grasp the principle of "matching" without even thinking about it.

But fund managers—the pros—insist on shorter, unmatching bond portfolios for a simple, stupefying reason. They are graded—usually by consultants—according to how they perform against standard (and shorter-term) bond indexes. Thus, rather than invest to keep up with liabilities, managers are investing so as to avoid lagging behind the popular index in any year. A gutsy exception is AMR (average bond duration: 26 years). Its assets will get hammered if rates rise, but they should track liabilities either way.

Source: Roger Lowenstein, "How Pension Funds Lost in Market Boom," *The Wall Street Journal*, February 1, 1996. Excerpted by permission of *The Wall Street Journal*.

Table 16.7A shows that if interest rates remain at 8%, the accumulated funds from the bond will grow to exactly the $14,693.28 obligation. Over the five-year period, the year-end coupon income of $800 is reinvested at the prevailing 8% market interest rate. At the end of the period, the bonds can be sold for $10,000; they still will sell at par value because the coupon rate still equals the market interest rate. Total income after five years from reinvested coupons and the sale of the bond is precisely $14,693.28.

If interest rates change, however, two offsetting influences will affect the ability of the fund to grow to the targeted value of $14,693.28. If interest rates rise, the fund will suffer a capital loss, impairing its ability to satisfy the obligation. The bonds will be worth less in five years than if interest rates had remained at 8%. However, at a higher interest rate, reinvested coupons will grow at a faster rate, offsetting the capital loss. In other words, fixed-income investors face two offsetting types of interest rate risk: *price risk* and *reinvestment rate risk*. Increases in interest rates cause capital losses but at the same time increase the rate at which reinvested income will grow. If the portfolio duration is chosen appropriately, these two effects will cancel out exactly. When the portfolio duration is set equal to the investor's horizon date, the accumulated value of the investment fund at the horizon date will be unaffected by interest rate fluctuations. *For a horizon equal to the portfolio's duration, price risk and reinvestment risk exactly cancel out.*

See Maple Handout

In the example we are discussing, the duration of the six-year maturity bonds used to fund the GIC is five years. You can confirm this using rule 8. Because the fully funded plan has equal duration for its assets and liabilities, the insurance company should be immu-

Table 16.7 Terminal Value of a Bond Portfolio after Five Years (all proceeds reinvested)

Payment Number	Years Remaining until Obligation	Accumulated Value of Invested Payment		
A. Rates remain at 8%				
1	4	$800 \times (1.08)^4$	=	1,088.39
2	3	$800 \times (1.08)^3$	=	1,007.77
3	2	$800 \times (1.08)^2$	=	933.12
4	1	$800 \times (1.08)^1$	=	864.00
5	0	$800 \times (1.08)^0$	=	800.00
Sale of bond	0	10,800/1.08	=	10,000.00
				14,693.28
B. Rates fall to 7%				
1	4	$800 \times (1.07)^4$	=	1,048.64
2	3	$800 \times (1.07)^3$	=	980.03
3	2	$800 \times (1.07)^2$	=	915.92
4	1	$800 \times (1.07)^1$	=	856.00
5	0	$800 \times (1.07)^0$	=	800.00
Sale of bond	0	10,800/1.07	=	10,093.46
				14,694.05
C. Rates increase to 9%				
1	4	$800 \times (1.09)^4$	=	1,129.27
2	3	$800 \times (1.09)^3$	=	1,036.02
3	2	$800 \times (1.09)^2$	=	950.48
4	1	$800 \times (1.09)^1$	=	872.00
5	0	$800 \times (1.09)^0$	=	800.00
Sale of bond	0	10,800/1.09	=	9,908.26
				14,696.02

Note: The sale price of the bond portfolio equals the portfolio's final payment ($10,800) divided by $1 + r$, because the time to maturity of the bonds will be one year at the time of sale.

nized against interest rate fluctuations. To confirm that this is the case, let us now investigate whether the bond can generate enough income to pay off the obligation five years from now regardless of interest rate movements.

Tables 16.7**B** and **C** consider two possible interest rate scenarios: Rates either fall to 7%, or increase to 9%. In both cases, the annual coupon payments from the bond are reinvested at the new interest rate, which is assumed to change before the first coupon payment, and the bond is sold in year 5 to help satisfy the obligation of the GIC.

Table 16.7**B** shows that if interest rates fall to 7%, the total funds will accumulate to $14,694.05, providing a small surplus of $.77. If rates increase to 9% as in Table 16.7**C**, the fund accumulates to $14,696.02, providing a small surplus of $2.74.

Several points are worth highlighting. First, duration matching balances the difference between the accumulated value of the coupon payments (reinvestment rate risk) and the sale value of the bond (price risk). That is, when interest rates fall, the coupons grow less than in the base case, but the gain on the sale of the bond offsets this. When interest rates rise, the resale value of the bond falls, but the coupons more than make up for this loss because they are reinvested at the higher rate. Figure 16.4 illustrates this case. The solid curve traces out the accumulated value of the bonds if interest rates remain at 8%. The dashed curve shows that value if interest rates happen to increase. The initial impact is a capital loss, but this loss eventually is offset by the now-faster growth rate of reinvested funds. At the five-year horizon date, the two effects just cancel, leaving the company able to satisfy its obligation with the accumulated proceeds from the bond.

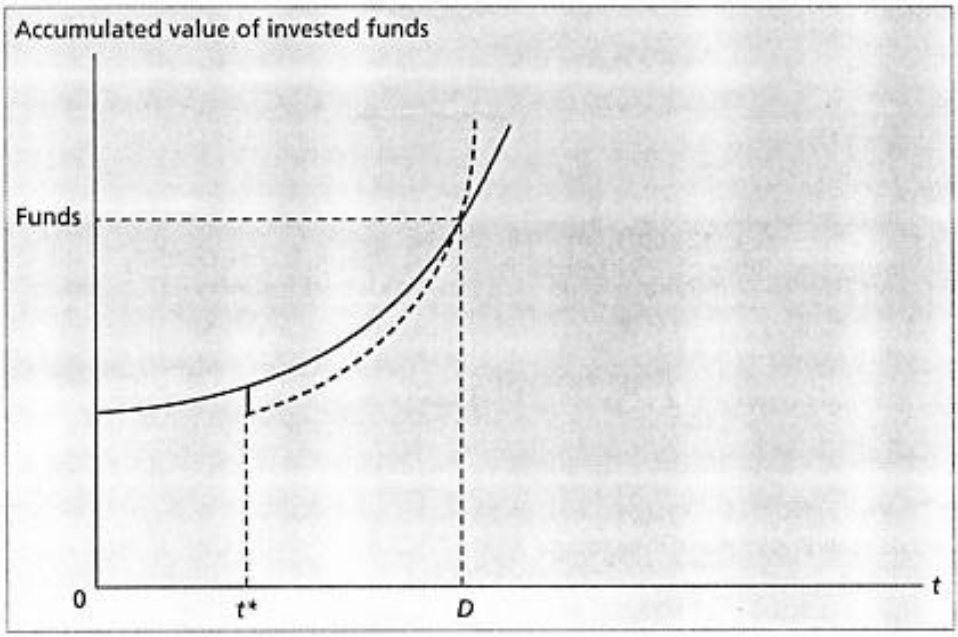


Figure 16.4 Growth of invested funds. The solid colored curve represents the growth of portfolio value at the original interest rate. If interest rates increase at time t^*, the portfolio value initially falls but increases thereafter at the faster rate represented by the broken curve. At time D (duration) the curves cross.

Table 16.8 Market Value Balance Sheet

Assets		Liabilities	
A. Interest rate = 8%			
Bonds	$10,000	Obligation	$10,000
B. Interest rate = 7%			
Bonds	$10,476.65	Obligation	$10,476.11
C. Interest rate = 9%			
Bonds	$ 9,551.41	Obligation	$ 9,549.62

Notes:

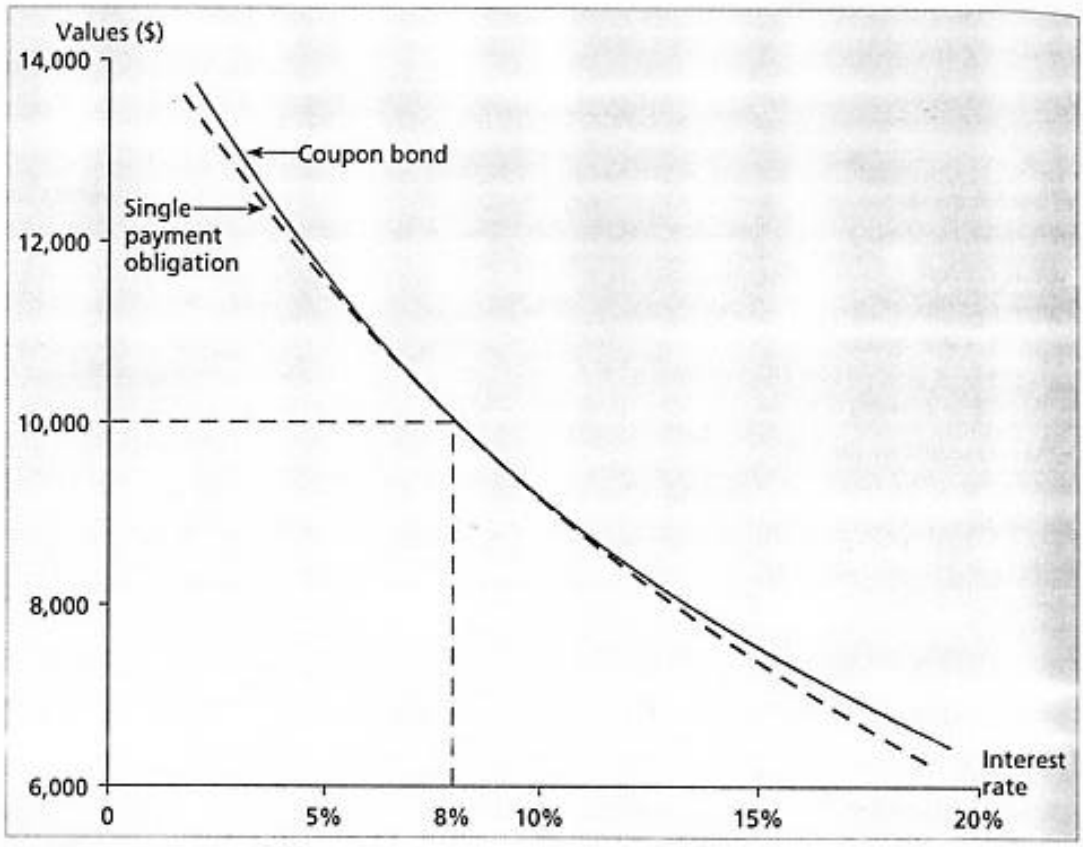
$$\text{Value of bonds} = 800 \times \text{Annuity factor}(r, 6) + 10{,}000 \times \text{PV factor}(r, 6)$$

$$\text{Value of obligation} = \frac{14{,}693.28}{(1+r)^5} = 14{,}693.28 \times \text{PV factor}(r, 5)$$

We can also analyze immunization in terms of present as opposed to future values. Table 16.8A shows the initial balance sheet for the insurance company's GIC account. Both assets and the obligation have market values of $10,000, so that the plan is just fully funded. Tables 16.8B and C show that whether the interest rate increases or decreases, the value of the bonds funding the GIC and the present value of the company's obligation change by virtually identical amounts. Regardless of the interest rate change, the plan remains fully funded, with the surplus in Table 16.8B and C just about zero. The duration-matching strategy has ensured that both assets and liabilities react equally to interest rate fluctuations.

Figure 16.5 is a graph of the present values of the bond and the single-payment obligation as a function of the interest rate. At the current rate of 8%, the values are equal, and the obligation is fully funded by the bond. Moreover, the two present value curves are tangent at $y = 8\%$. As interest rates change, the change in value of both the asset and the obligation is equal, so the obligation remains fully funded. For greater changes in the interest rate,

Figure 16.5
Immunization.



however, the present value curves diverge. This reflects the fact that the fund actually shows a small surplus at market interest rates other than 8%.

Why is there any surplus in the fund? After all, we claimed that a duration-matched asset and liability mix would result in indifference to interest rate shifts. Actually, such a claim is valid only for *small* changes in the interest rate, because as bond yields change, so too does duration. (Recall rule 4 for duration and footnote 4.) In our example, although the duration of the bond is indeed equal to 5 years at a yield to maturity of 8%, it rises to 5.02 years when its yield falls to 7% and drops to 4.97 years at $y = 9\%$; that is, the bond and the obligation were not duration-matched *across* the interest rate shift, so that the position was not fully immunized.

This example highlights the importance of **rebalancing** immunized portfolios. As interest rates and asset durations change, a manager must rebalance the portfolio of fixed-income assets continually to realign its duration with the duration of the obligation. Moreover, even if interest rates do not change, asset durations *will* change solely because of the passage of time. Recall from Figure 16.2 that duration generally decreases less rapidly than does maturity. Thus, even if an obligation is immunized at the outset, as time passes the durations of the asset and liability will fall at different rates. Without portfolio rebalancing, durations will become unmatched and the goals of immunization will not be realized. Obviously, immunization is a passive strategy only in the sense that it does not involve attempts to identify undervalued securities. Immunization managers still actively update and monitor their positions.

As another example of the need for rebalancing, consider a portfolio manager facing an obligation of $19,487 in seven years, which, at a current market interest rate of 10%, has a present value of $10,000. Right now, suppose that the manager wishes to immunize the obligation by holding only three-year zero-coupon bonds and perpetuities paying annual

coupons. (Our focus on zeros and perpetuities helps keep the algebra simple.) At current interest rates, the perpetuities have a duration of 1.10/.10 = 11 years. The duration of the zero is simply three years.

For assets with equal yields, the duration of a portfolio is the weighted average of the durations of the assets comprising the portfolio. To achieve the desired portfolio duration of seven years, the manager would have to choose appropriate values for the weights of the zero and the perpetuity in the overall portfolio. Call w the zero's weight and $(1 - w)$ the perpetuity's weight. Then w must be chosen to satisfy the equation

$$w \times 3 \text{ years} + (1 - w) \times 11 \text{ years} = 7 \text{ years}$$

which implies that $w = ½$. The manager invests $5,000 in the zero-coupon bond and $5,000 in the perpetuity, providing annual coupon payments of $500 per year indefinitely. The portfolio duration is then seven years, and the position is immunized.

Next year, even if interest rates do not change, rebalancing will be necessary. The present value of the obligation has grown to $11,000, because it is one year closer to maturity. The manager's funds also have grown to $11,000: The zero-coupon bonds have increased in value from $5,000 to $5,500 with the passage of time, while the perpetuity has paid its annual $500 coupon and still is worth $5,000. However, the portfolio weights must be changed. The zero-coupon bond now will have duration of 2 years, while the perpetuity remains at 11 years. The obligation is now due in 6 years. The weights must now satisfy the equation

$$w \times 2 + (1 - w) \times 11 = 6$$

which implies that $w = ⅚$. Now, the manager must invest a total of $11,000 × ⅚ = $6,111.11 in the zero. This requires that the entire $500 coupon payment be invested in the zero and that an additional $111.11 of the perpetuity be sold and invested in the zero in order to maintain an immunized position.

Of course, rebalancing of the portfolio entails transaction costs as assets are bought or sold, so one cannot rebalance continuously. In practice, an appropriate compromise must be established between the desire for perfect immunization, which requires continual rebalancing, and the need to control trading costs, which dictates less frequent rebalancing.

Concept CHECK

Question 4 • What would be the immunizing weights in the second year if the interest rate had fallen to 8%?

Cash Flow Matching and Dedication

The problems associated with immunization seem to have a simple solution. Why not simply buy a zero-coupon bond that provides a payment in an amount exactly sufficient to cover the projected cash outlay? If we follow the principle of **cash flow matching** we automatically immunize the portfolio from interest rate movement because the cash flow from the bond and the obligation exactly offset each other.

Cash flow matching on a multiperiod basis is referred to as a **dedication strategy**. In this case, the manager selects either zero-coupon or coupon bonds that provide total cash flows in each period that match a series of obligations. The advantage of dedication is that it is a once-and-for-all approach to eliminating interest rate risk. Once the cash flows are matched, there is no need for rebalancing. The dedicated portfolio provides the cash necessary to pay the firm's liabilities regardless of the eventual path of interest rates.

Cash flow matching is not more widely pursued probably because of the constraints that it imposes on bond selection. Immunization-dedication strategies are appealing to firms

that do not wish to bet on general movements in interest rates, but these firms may want to immunize using bonds that they perceive are undervalued. Cash flow matching, however, places so many more constraints on the bond selection process that it can be impossible to pursue a dedication strategy using only "underpriced" bonds. Firms looking for underpriced bonds give up exact and easy dedication for the possibility of achieving superior returns from the bond portfolio.

Sometimes, cash flow matching is not possible. To cash-flow-match for a pension fund that is obligated to pay out a perpetual flow of income to current and future retirees, the pension fund would need to purchase fixed-income securities with maturities ranging up to hundreds of years. Such securities do not exist, making exact dedication infeasible.

Concept CHECK

Question 5 • How would an increase in trading costs affect the attractiveness of dedication versus immunization?

Other Problems with Conventional Immunization

If you look back at the definition of duration in equation 16.1, you note that it uses the bond's yield to maturity to calculate the weight applied to the time until each coupon payment. Given this definition and limitations on the proper use of yield to maturity, it is perhaps not surprising that this notion of duration is strictly valid only for a flat yield curve for which all payments are discounted at a common interest rate.

If the yield curve is not flat, then the definition of duration must be modified and $CF_t/(1 + y)^t$ replaced with the present value of CF_t, where the present value of each cash flow is calculated by discounting with the appropriate interest rate from the yield curve corresponding to the date of the *particular* cash flow, instead of by discounting with the *bond's* yield to maturity. Moreover, even with this modification, duration matching will immunize portfolios only for parallel shifts in the yield curve. Clearly, this sort of restriction is unrealistic. As a result, much work has been devoted to generalizing the notion of duration. Multifactor duration models have been developed to allow for tilts and other distortions in the shape of the yield curve, in addition to shifts in its level. (We refer to some of this work in the suggested readings at the end of this chapter.) However, it does not appear that the added complexity of such models pays off in terms of substantially greater effectiveness.[7]

Finally, immunization can be an inappropriate goal in an inflationary environment. Immunization is essentially a nominal notion and makes sense only for nominal liabilities. It makes no sense to immunize a projected obligation that will grow with the price level using nominal assets such as bonds. For example, if your child will attend college in 15 years and if the annual cost of tuition is expected to be $15,000 at that time, immunizing your portfolio at a locked-in terminal value of $15,000 is not necessarily a risk-reducing strategy. The tuition obligation will vary with the realized inflation rate, whereas the asset portfolio's final value will not. In the end, the tuition obligation will not necessarily be matched by the value of the portfolio.

On this note, it is worth pointing out that immunization is a goal that may well be inappropriate for many investors who would find a zero-risk portfolio strategy unduly conservative. Full immunization is a fairly extreme position for a portfolio manager to pursue.

[7]G. O. Bierwag, G. C. Kaufman, and A. Toevs, eds., *Innovations in Bond Portfolio Management: Duration Analysis and Immunization* (Greenwich, CT: JAI Press, 1983).

Derive the formula for the modified duration of a coupon bond, where j is the coupon rate per period, i is the yield rate per period, and n is the number of payment periods. Evaluate it when n = 5, and j = i = 0.08.

```
> a := j * ( 1 - 1 / ( 1 + i ) ** n ) / i + 1 / ( 1 + i ) ** n;
```

$$a := \frac{j\left(1 - \frac{1}{(1+i)^n}\right)}{i} + \frac{1}{(1+i)^n}$$

```
> b := diff( j * ( 1 - 1 / ( 1 + i ) ** n ) / i + 1 / ( 1 + i ) **
  n, i);
```

$$b := \frac{j\,n}{(1+i)^n\,(1+i)\,i} - \frac{j\left(1 - \frac{1}{(1+i)^n}\right)}{i^2} - \frac{n}{(1+i)^n\,(1+i)}$$

```
> -b/a;
```

$$-\frac{\frac{j\,n}{(1+i)^n\,(1+i)\,i} - \frac{j\left(1 - \frac{1}{(1+i)^n}\right)}{i^2} - \frac{n}{(1+i)^n\,(1+i)}}{\frac{j\left(1 - \frac{1}{(1+i)^n}\right)}{i} + \frac{1}{(1+i)^n}}$$

```
> moddur := simplify( -b/a );
```

$$moddur := -\frac{j\,n\,i - j\,(1+i)^n - j\,(1+i)^n\,i + j + j\,i - n\,i^2}{i\,(j\,(1+i)^n - j + i)\,(1+i)}$$

```
> macdur := moddur * ( 1 + i );
```

$$macdur := -\frac{j\,n\,i - j\,(1+i)^n - j\,(1+i)^n\,i + j + j\,i - n\,i^2}{i\,(j\,(1+i)^n - j + i)}$$

```
> n := 6; i := 0.08; j := 0.08;
```

$$n := 6$$

$$i := .08$$

$$j := .08$$

```
> moddur;
```

$$4.622879662$$

```
> macdur;
```

$$4.992710035$$

```
>
```