

Department of Mathematics and Statistics
AMAT 219 - QUIZ 1 - Thursday, January 26, 2006

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| U of C ID # |
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45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

Determine each of the following integrals :

$k \in \mathbb{R}$

1. $\int \arctan(x-5) dx$ (parts)

$$u = (x-5), \quad du = dx$$

$$\int \arctan(u) du$$

$$= u \arctan(u) - \int \frac{u}{1+u^2} du$$

$$= u \arctan(u) - \frac{1}{2} \ln(1+u^2)$$

$$(x-5) \arctan(x-5) - \frac{1}{2} \ln(1+(x-5)^2) + k$$

2. $\int e^x \sin(x) dx$ (parts twice)

$$= -e^x \cos(x) + \int e^x \cos(x) dx$$

$$= -e^x \cos(x) + e^x \sin(x) - \int e^x \sin(x) dx$$

$$\frac{1}{2} e^x [\sin(x) - \cos(x)] + k$$

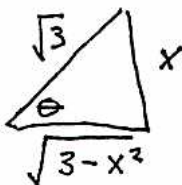
3. $\int \frac{1}{(3-x^2)^{3/2}} dx$ (inverse trig)

$$\begin{cases} x = \sqrt{3} \sin \theta \\ dx = \sqrt{3} \cos \theta d\theta \end{cases}$$

$$\int \frac{\sqrt{3} \cos(\theta) d\theta}{[3 \cos^2(\theta)]^{3/2}} = \frac{1}{3} \int \sec^2(\theta) d\theta$$

$$= \frac{1}{3} \tan(\theta) + k$$

$$\frac{1}{3} \frac{x}{\sqrt{3-x^2}} + k$$



4. $\int \frac{1}{x^3-x} dx$ (partial fraction)

$$\begin{cases} \frac{1}{x^3-x} = \frac{1}{x(x+1)(x-1)} = \frac{a}{x} + \frac{b}{x-1} + \frac{c}{x+1} \\ 1 = a(x-1)(x+1) + b x(x+1) + c x(x-1) \\ a = -1, \quad b = c = \frac{1}{2} \end{cases}$$

$$= \ln|x| + \frac{1}{2} \left[\ln|x-1| + \ln|x+1| \right] + k.$$

5. $\int \frac{x}{x^2-2x+5} dx$ (complete square)

$$\int \frac{x}{(x-1)^2+4} dx = \int \frac{x-1}{(x-1)^2+4} dx + \int \frac{1}{(x-1)^2+4} dx$$

$$= \frac{1}{2} \ln|(x-1)^2+4| + \frac{1}{4} \int \frac{1}{\left(\frac{x-1}{2}\right)^2+1} dx$$

$$= \frac{1}{2} \ln|(x-1)^2+4| + \frac{1}{2} \arctan\left(\frac{x-1}{2}\right) + k$$

| Surname | Given Names | Lab # | Mark (20) |
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I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes _____ or No _____