

Department of Mathematics and Statistics
 AMAT 219 - QUIZ 2 - Thursday, February 9, 2006

U of C ID #

(log = ln)

45 Minutes, Open Book, NO Calculators

To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\int \frac{-(x+4)}{x^3+x^2-2} dx$

$x=1$ is a root of x^3+x^2-2 so

$\frac{-(x+4)}{x^3+x^2-2} = \frac{a}{x-1} + \frac{bx+c}{x^2+2x+2}$. Solve

$-\log|x-1| + \frac{1}{2}\log|(x+1)^2+1| + \arctan(x+1) + k$

for $a=-1, b=1, c=2$ and integral =

$\int \frac{-1}{x-1} dx + \int \frac{x+2}{x^2+2x+2} dx = -\log|x-1| + \int \frac{x+2}{(x+1)^2+1} dx$

$= -\log|x-1| + \int \frac{(x+1)}{(x+1)^2+1} dx + \int \frac{1}{(x+1)^2+1} dx$

2. Find $\int_0^1 \frac{1}{\sqrt[3]{x-1}} dx$

$= \lim_{a \nearrow 1} \int_0^a \frac{1}{\sqrt[3]{x-1}} dx = \lim_{a \nearrow 1} \left. \frac{3}{2} (x-1)^{2/3} \right|_0^a$

converges to $-3/2$

$= \lim_{a \nearrow 1} \frac{3}{2} [(a-1)^{2/3} - 1] = -3/2$

3. Find $\int_{\sqrt{5}}^{\infty} \frac{1}{5+x^2} dx$

$= \lim_{a \rightarrow \infty} \int_{\sqrt{5}}^a \frac{1}{5 \left(1 + \frac{x^2}{5}\right)} dx$

converges to $\frac{\pi}{4\sqrt{5}}$

$\lim_{a \rightarrow \infty} \frac{1}{\sqrt{5}} \arctan\left(\frac{x}{\sqrt{5}}\right) \Big|_{\sqrt{5}}^a = \lim_{a \rightarrow \infty} \frac{1}{\sqrt{5}} \left[\arctan\left(\frac{a}{\sqrt{5}}\right) - \arctan(1) \right]$

$= \frac{1}{\sqrt{5}} \left[\frac{\pi}{2} - \frac{\pi}{4} \right]$

4. Find $\int_{-\infty}^0 xe^x dx$

$$= \lim_{a \rightarrow -\infty} \int_a^0 xe^x dx$$

$$= \lim_{a \rightarrow -\infty} [xe^x - e^x]_a^0 = \lim_{a \rightarrow -\infty} [-1 + e^a]$$

$$= -1.$$

converges to -1 .

5. What is the least value of n that guarantees M_n is within 10^{-2} of $\ln(2) = \int_1^2 \frac{1}{x} dx$?

$$\text{max value of } |f''(x)| = \frac{2}{x^3} \text{ on } [1, 2]$$

(with $f(x) = \frac{1}{x}$) is 2, so

$$\text{error estimate for } M_n \text{ is } \frac{2|2-1|^3}{24n^2} = \frac{1}{12n^2}.$$

$$\text{need } n \text{ with } \frac{1}{12n^2} < 10^{-2}, \text{ so } \frac{100}{12} \leq n^2; 8.\bar{3} \leq n^2$$

$$n = 3.$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes_____ or No_____