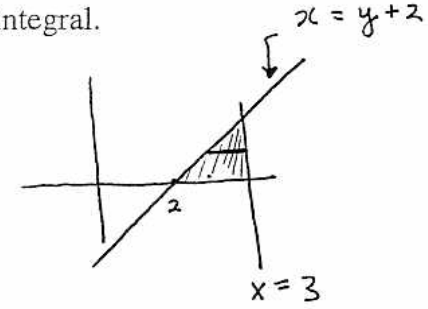
Department of Mathematics and Statistics AMAT 219 - QUIZ 3 - Thursday, March 2, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators

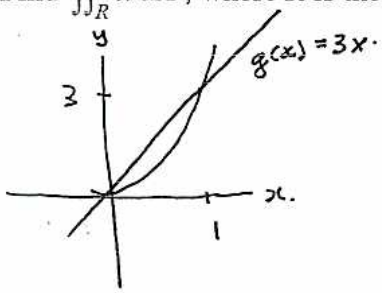
To obtain credit you need to show your work. Work should be neat and organized.

1. Write $\int_0^1 \left(\int_{y+2}^3 \cos(x^2+1)\, dx \right) dy$ as an iterated integral with the order of integration reversed. Do not evaluate the integral.



$$\int_2^3 \int_0^{x-2} \cos(x^2+1)\, dy\, dx$$

2. Find $\iint_R x\, dA$, where R is the region in the xy – plane bounded by $f(x) = 3x^2$ and $g(x) = 3x$.

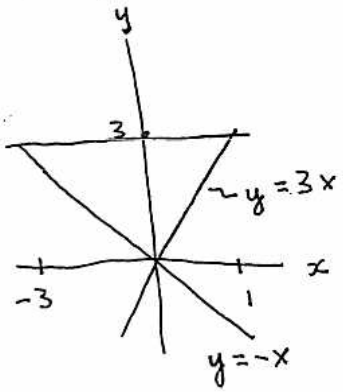


$$\iint_R x\, dA = \int_0^1 \int_{3x^2}^{3x} x\, dy\, dx$$

$$= \int_0^1 (3x^2 - 3x^3)\, dx = x^3 - \tfrac{3}{4}x^4 \Big|_0^1 = \tfrac{1}{4}.$$

1/4

3. Find $\iint_R y\, dA$ by viewing R as an x –simple region, where R is the region in the xy – plane bounded by $y = -x$, $y = 3x$, and $y = 3$.



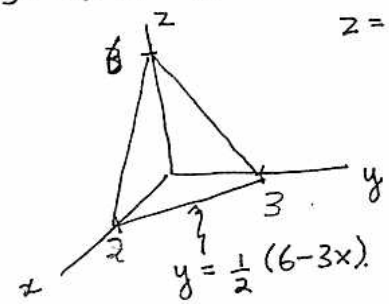
$$\iint_R y\, dA = \int_0^3 \int_{-y}^{y/3} y\, dx\, dy$$

$$= \int_0^3 \left(\frac{y^2}{3} + y^2 \right) dy$$

$$= \frac{4}{3} \int_0^3 y^2\, dy = \frac{4}{3 \cdot 3} y^3 \Big|_0^3 = 4 \cdot 3$$

12

4. Use double integrals to find the volume of the region in the first octant ($x, y, z \geq 0$) below the plane $2x + 2y + z = 6$.

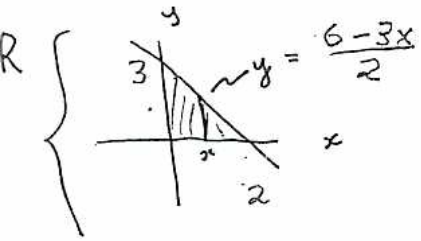


$z = f(x,y) = 6 - 3x - 2y$

$$Vol = \iint_R f(x,y)\,dA = \int_0^2 \int_0^{\frac{(6-3x)}{2}} (6 - 3x - 2y)\,dy\,dx$$

6

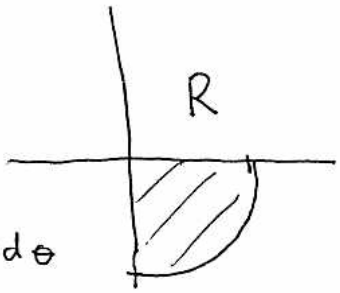
$$= \int_0^2 \left[6y - 3xy - y^2\right]_0^{\frac{6-3x}{2}} dx = \int_0^2 \left[6 - 3x - \left(\frac{6-3x}{2}\right)\right]\left(\frac{6-3x}{2}\right) dx$$



$$= \int_0^2 \left(\frac{6-3x}{2}\right)^2 dx = -\frac{2}{3 \cdot 3}\left(\frac{6-3x}{2}\right)^3 \Bigg|_{x=0}^{x=2}$$

$$= +\frac{2}{3 \cdot 3}(3)^3 = 6$$

5. Use polar coordinates to find $\iint_R x\,dA$ where R is the quarter disk in the fourth quadrant ($x \geq 0$, $y \leq 0$) bounded by $x^2 + y^2 = 1$.



$dA = r\,dr\,d\theta$

$x = r\cos\theta$

$0 \leq r \leq 1$

$-\frac{\pi}{2} \leq \theta \leq 0$

$\left(\text{or } \frac{3\pi}{2} \leq \theta \leq 2\pi\right)$

$$\iint_R x\,dA = \int_{-\pi/2}^{0} \int_0^1 r^2 \cos\theta\,dr\,d\theta$$

1/3

$$= \int_{-\pi/2}^{0} \frac{r^3}{3}\cos\theta \Big|_0^1 d\theta = \frac{1}{3}\int_{-\pi/2}^{0} \cos\theta\,d\theta$$

$$= \frac{1}{3}\sin\theta \Big|_{-\pi/2}^{0} = 1/3$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes_____ or No_____