

U of C ID #

45 Minutes, Open Book, NO Calculators

To obtain credit you need to show your work. Work should be neat and organized.

1. Find the parametric equations of the straight line perpendicular to the plane $3x - y + 2z = 5$ and containing the point $(1, 2, 1)$.

$(3, -1, 2)$ is perpendicular vector to plane
so a direction vector for the line.

$$\begin{aligned} x(t) &= 1 + 3t \\ y(t) &= 2 - t \\ z(t) &= 1 + 2t \end{aligned}$$

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2. Find the equation of the plane containing the point $(1, 1, 2)$ and the line $\vec{l}(t) = (2, -1, 1) + t(1, 0, 3)$.

$(1, 1, 2)$ and $(2, -1, 1)$ are points in the plane

so $(2-1, -1-1, 1-2) = (1, -2, -1)$ is a
vector \perp normal of plane.

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Thus $(1, 0, 3) \times (1, -2, -1) = (6, 4, -2)$ is a normal vector
for the plane.

$$6x + 4y - 2z = 6(1) + 4(1) - 2(2)$$

Eq of plane : $6x + 4y - 2z = 6$

3. What is the point of intersection (if it exists) of the lines $\vec{l}_1(t) = (2, -1, 3) + t(3, -4, 1)$ and $\vec{l}_2(s) = (0, 1, 2) + s(-1, 2, 0)$.

Find t, s with $\vec{l}_1(t) = \vec{l}_2(s)$

$$\text{so with } \begin{cases} 2 + 3t = -s \\ -1 - 4t = 1 + 2s \\ 3 + t = 2 \end{cases}$$

Get $t = -1, s = 1$. The point of intersection
is $(-1, 3, 2) = \vec{l}_2(1) = \vec{l}_1(-1)$

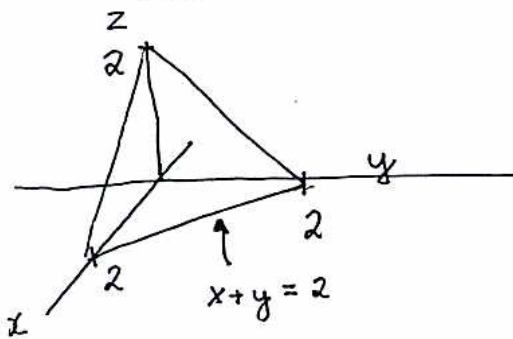
$(-1, 3, 2)$

4. Find the plane containing the point $(0, 1, 1)$ and not meeting the two lines $l_1(t) = (1, 2, 0) + t(0, 1, 2)$ and $l_2(t) = (1, 1, -2) + t(1, -2, 3)$.

For a line not to meet a plane, the direction vector of the line must be perpendicular to the normal vector of the plane. So a normal of the plane is $(0, 1, 2) \times (1, -2, 3) = (7, 2, -1)$.

So equation of plane is $7x + 2y - z = 1$

5. Evaluate $\iiint_E (-x+2) dV$ where E is the solid enclosed by the planes $x = 0, y = 0, z = 0, x+y+z = 2$.



$$= \int_0^2 \int_0^{2-x} \int_0^{2-x-y} (2-x) dz dy dx$$

$$= \int_0^2 \int_0^{2-x} (2-x)(2-x-y) dy dx$$

$$= \int_0^2 \int_0^{2-x} (2-x)(2-x-\frac{y}{2}) y dy dx = \int_0^2 \frac{(2-x)^3}{2} dx = \frac{-(2-x)^4}{4 \cdot 2} \Big|_0^2 = 2$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes _____ or No _____