Department of Mathematics and Statistics AMAT 219 - QUIZ 5 - Tuesday, April 4, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators

To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\frac{\partial^2 z}{\partial x \partial y}$ for $z = f(2x + 3y)$ given that $f''(t) = \log(t)$.

Let $t = 2x + 3y$

$$\frac{\partial z}{\partial y} = \frac{df}{dt}\frac{\partial t}{\partial y} = 3 \cdot \frac{df}{dt}$$

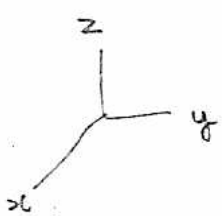
$$\frac{\partial^2 z}{\partial x \partial y} = 3\frac{d^2 f}{dt^2}\frac{dt}{dx} = 2 \cdot 3 \cdot \frac{d^2 f}{dt}(t) = 6 \cdot \log(2x+3y)$$

2. Find the equation of the plane tangent to the surface $z = 3 + x^2 + y^3$ when $x = 1, y = -1$.

at $(1,-1)$ tangent plane has normal vector $\left(\frac{\partial f}{\partial x}(1,-1), \frac{\partial f}{\partial y}(1,-1), -1\right) = (2, 3, -1)$ for $f(x,y) = 3 + x^2 + y^3$. a point on the plane is $(1, -1, f(1,-1)) = (1, -1, 3)$.

Equation is $2x + 3y - z = -4$

3. Use spherical coordinates to find the mass of the hemisphere $x^2 + y^2 + z^2 \leq 1$, $x \geq 0$, with density $\delta(x, y, z) = 5z^2$



$z = \rho \cos\phi$ so

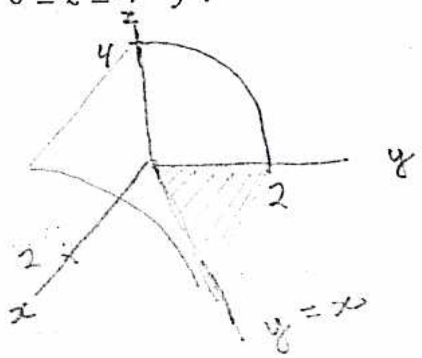
$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

Hemisphere is $\begin{cases} 0 \leq \rho \leq 1 \\ -\pi/2 \leq \theta \leq \pi/2 \\ 0 \leq \phi \leq \pi \end{cases}$

$$\text{Mass} = \iiint_H 5z^2 \, dV = 5\int_{-\pi/2}^{\pi/2}\int_0^{\pi}\int_0^1 \rho^4 \cos^2\phi \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= 5 \cdot \pi \int_0^{\pi} \frac{\rho^5}{5}\Big|_0^1 \cos^2\phi \sin\phi \, d\phi = -\pi \cdot \frac{\cos^3\phi}{3}\Big|_0^{\pi} = \frac{2\pi}{3}$$

4. Use triple integrals to find the volume of the solid enclosed by the region, $0 \le y$, $0 \le x \le y$, and $0 \le z \le 4 - y^2$.



$$\text{Vol} = \int_0^2 \int_0^y \int_0^{4-y^2} dz\, dx\, dy$$

$$= \int_0^2 (4-y^2)\, y\, dy$$

$$= \left[2y^2 - \frac{y^4}{4}\right]\Bigg|_0^2 = 8 - 4 = 4.$$

5. Find the arclength of the planar parametric curve $c(t) = (t^2, \frac{t^3}{3} + 2)$, $0 \le t \le 1$.

$$\int_0^1 \sqrt{(x')^2 + (y')^2}\, dt$$

$$= \int_0^1 \sqrt{(4t^2 + t^4)}\, dt$$

$$= \int_0^1 t\sqrt{4 + t^2}\, dt = \frac{1}{3}(4+t^2)^{3/2}\Big|_0^1 = \frac{5^{3/2}}{3}$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes__________ or No__________