Department of Mathematics and Statistics
AMAT 219 - QUIZ 1 - Tuesday, January 24, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

Determine each of the following integrals : $k \in \mathbb{R}$.

1. $\int \arccos(x-5)\,dx$ (parts).

$u = x-5$

$\int \arccos(u)\,du$

$= u\arccos(u) + \int \frac{u\,du}{\sqrt{1-u^2}}$

$= u\arccos(u) - \sqrt{1-u^2}$

$(x-5)\arccos(x-5) - \sqrt{1-(x-5)^2} + k$

2. $\int \sin(\ln(x))dx$ (parts twice)

$= x\sin(\ln(x)) - \int x\cos(\ln(x))\frac{dx}{x}$

$= x\sin(\ln(x)) - \left[x\cos(\ln(x)) + \int x\sin(\ln(x))\frac{dx}{x}\right]$

$= x\sin(\ln(x)) - x\cos(\ln(x)) - \int \sin(\ln(x))\,dx.$

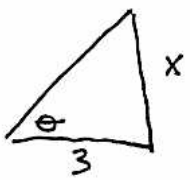
$\frac{x}{2}\left[\sin(\ln(x)) - \cos(\ln(x))\right] + k$

3. $\int \frac{1}{(9+x^2)^{3/2}}\,dx$ (inverse trig).

$\begin{cases} x = 3\tan\theta \\ dx = 3\sec^2(\theta)\,d\theta \end{cases}$

$= \int \frac{3\sec^2(\theta)\,d\theta}{3^3(\sec^2(\theta))^{3/2}}$

$= \frac{1}{9}\int \cos(\theta)\,d\theta = \frac{1}{9}\sin(\theta)$



$= \frac{1}{9}\frac{x}{\sqrt{9+x^2}} + k.$

4. $\int \frac{1}{x^2-x-2}\,dx$ (partial fraction)

$$\frac{1}{x^2-x-2} = \frac{1}{(x-2)(x+1)} = \frac{a}{x-2} + \frac{b}{x+1}$$

$$1 = a(x+1) + b(x-2)$$

$$a = -b = \frac{1}{3}$$

$$\frac{1}{3}\int\left[\frac{1}{x-2} - \frac{1}{x+1}\right]dx$$

$$= \frac{1}{3}\left[\ln|x-2| - \ln|x+1|\right] + k.$$

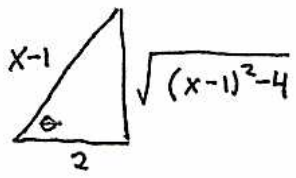
5. $\int \frac{x}{\sqrt{x^2-2x-3}}\,dx$ (complete square)

$$\int \frac{x\,dx}{\sqrt{(x-1)^2-4}}$$

$$\left(\begin{array}{l} x-1 = 2\sec(\theta) \\ dx = 2\sec(\theta)\tan(\theta)\,d\theta \end{array}\right)$$

$$= \int \frac{[2\sec(\theta)+1]\,2\sec(\theta)\tan(\theta)\,d\theta}{2\tan(\theta)}$$

$$= \int (2\sec(\theta)+1)\sec(\theta)\,d\theta = 2\tan\theta + \ln|\sec(\theta)+\tan(\theta)|$$



$$\frac{\sqrt{(x-1)^2-4}}{2} - \ln\left|\frac{x-1}{2} + \frac{\sqrt{(x-1)^2-4}}{2}\right| + k.$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes———— or No————