Department of Mathematics and Statistics
AMAT 219 - QUIZ 2 - Tuesday, February 7, 2006

(log = ln)

U of C ID #

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\int \frac{4x+1}{x^3+x^2-2}\,dx$

$\boxed{\log|x-1| - \frac{1}{2}\log((x+1)^2+1) + 2\arctan(x+1) + k}$

$x = 1$ is a root x^3+x^2-2 so

$$\frac{4x+1}{x^3+x^2-2} = \frac{a}{x-1} + \frac{bx+c}{x^2+2x+2}$$

Solve for $a = 1$, $b = -1$, $c = 1$ and

integral $= \int \frac{dx}{x-1} - \int \frac{(x-1)}{(x+1)^2+1}dx = \int \frac{dx}{x-1} - \int \frac{(x+1)\,dx}{(x+1)^2+1} + \int \frac{2}{(x+1)^2+1}dx$

2. Find $\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

$= \lim_{a \nearrow 1} \int_0^a \frac{x}{\sqrt{1-x^2}}dx = \lim_{a\nearrow 1} \left[-(1-x^2)^{\frac{1}{2}}\right|_0^a$

$\boxed{\text{converges to } 1.}$

$= \lim_{a\nearrow 1} \left(-(1-a^2)^{\frac{1}{2}} + 1\right) = 1$

3. Find $\int_3^\infty \frac{1}{9+x^2}\,dx$

$= \lim_{a\to\infty} \int_3^a \frac{1}{9}\left[\frac{1}{1+\left(\frac{x}{3}\right)^2}\right]dx$

$\boxed{\text{converges to } \frac{\pi}{12}.}$

$= \lim_{a\to\infty} \frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big|_3^a$

$= \lim_{a\to\infty} \frac{1}{3}\left[\arctan\left(\frac{a}{3}\right) - \arctan(1)\right] = \frac{1}{3}\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \frac{\pi}{3\cdot 4}$

4. Find $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

$$= \lim_{a\to-\infty}\int_a^0 xe^{-x^2}dx + \lim_{b\to\infty}\int_0^b xe^{-x^2}dx$$

$\boxed{\text{converges to } 0}$

$$= \lim_{a\to-\infty} -\frac{1}{2}e^{-x^2}\Big|_a^0 + \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_0^b$$

$$= -\frac{1}{2}\lim_{a\to-\infty}\left[1+e^{-a^2}\right] - \frac{1}{2}\lim_{b\to\infty}\left[e^{-b^2}-1\right]$$

$$= \frac{1}{2}\left[-1+1\right] = 0$$

5. What is the least value of n that guarantees T_n is within 10^{-3} of $ln(2) = \int_1^2 \frac{1}{x}\,dx$?

Find n so that error estimate for T_n is $\leq 10^{-3}$

$\boxed{n = 13}$

Since $f(x) = \frac{1}{x}$ has $f''(x) = \frac{2}{x^2}$

on $[1,2]$ the error estimate for $T_n = \frac{2\,|2-1|^3}{12\,n^2} = \frac{2}{12n^2}$

need n with $\frac{1}{6n^2} \leq 10^{-3}$, so $\frac{10^3}{6} \leq n^2$;

$166.\overline{6} \leq n^2$, so $n = 13$ is needed.

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes———— or No————