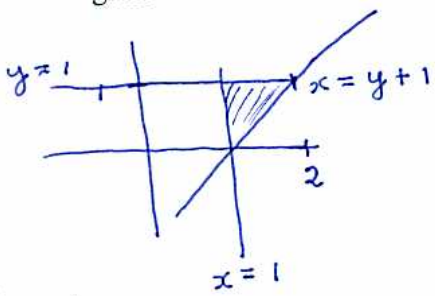
Department of Mathematics and Statistics AMAT 219 - QUIZ 3 - Tuesday, February 28, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators.

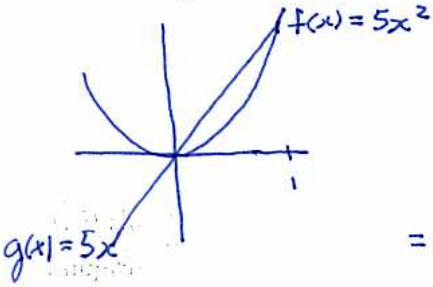
To obtain credit you need to show your work. Work should be neat and organized.

1. Write $\int_0^1 (\int_1^{y+1} \cos(x^2 + 1)\, dx)\, dy$ as an iterated integral with the order of integration reversed. Do not evaluate the integral.



$$\int_1^2 \int_{x-1}^1 \cos(x^2+1)\, dy\, dx$$

2. Find $\iint_R x^2\, dA$, where R is the region in the xy – plane bounded by $f(x) = 5x^2$ and $g(x) = 5x$.

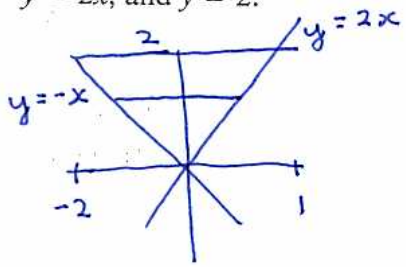


$$\iint_R x^2\, dA = \int_0^1 \int_{5x^2}^{5x} x^2\, dy\, dx$$

$$= \int_0^1 (5x^3 - 5x^4)\, dx = \frac{5}{4}x^4 - x^5 \Big|_0^1 = 1/4$$

1/4

3. Find $\iint_R y\, dA$ by viewing R as an x –simple region, where R is the region in the xy – plane bounded by $y = -x$, $y = 2x$, and $y = 2$.

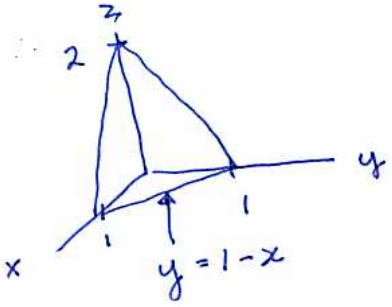


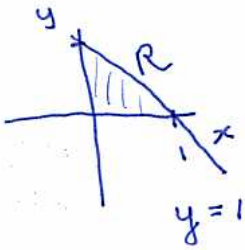
$$\iint_R y\, dA = \int_0^2 \int_{-y}^{y/2} y\, dx\, dy$$

$$= \int_0^2 \left(\frac{y^2}{2} + y^2\right) dy = \frac{y^3}{2} \Big|_0^2 = 4$$

4

4. Use double integrals to find the volume of the region in the first octant $(x, y, z \geq 0)$ below the plane $2x + 2y + z = 2$.





$$z = f(x,y) = 2 - 2x - 2y = 2(1 - x - y).$$

$$\text{Vol} = \iint_R f(x,y)\,dA$$

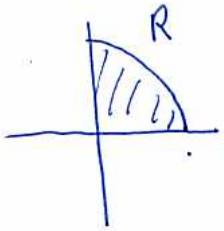
$$= \int_0^1 \int_0^{1-x} (2(1-x) - 2y)\,dy\,dx$$

$$= \int_0^1 \left[2(1-x)^2 - (1-x)^2\right] dx$$

$$= \int_0^1 (1-x)^2\,dx = -\frac{(1-x)^3}{3} = \frac{1}{3}$$

1/3

5. Use polar coordinates to find $\iint_R y\,dA$ where R is the quarter disk in the first quadrant $(x, y \geq 0)$ bounded by $x^2 + y^2 = 1$.



$y = r\sin\theta$

$0 \leq r \leq 1$

$0 \leq \theta \leq \pi/2$

$dA = r\,dr\,d\theta$

$$\iint_R y\,dA = \int_0^{\pi/2} \int_0^1 r^2 \sin\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3}\sin\theta \Big|_{r=0}^{r=1} d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2} \sin\theta\,d\theta = -\frac{1}{3}\cos\theta \Big|_0^{\pi/2} = \frac{1}{3}$$

1/3.

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes_____ or No_____.