1. For the following statements circle **T** for true or **F** for false. A correct answer is worth 2 marks.

a) For $f(x) = 2x^2 + 5x - x + 7$ and S_4 the Simpson's rule approximation for $\int_0^1 f(x)dx$, then the actual error $\left|S_4 - \int_0^1 f(x)dx\right|$ is zero.

T **F**

b) If $0 \leq f(x) \leq g(x)$ on $[0,\infty)$ and $\int_0^\infty f(x)dx$ converges, then $\int_0^\infty g(x)dx$ converges.

T **F**

c) The integral $\int_0^{\pi/4} (3 + \tan(2t))dt$ is improper.

T **F**

d) The integral $\int_0^1 \frac{x^{1/100}}{x} dx$ diverges.

T **F**

e) The integral $\int_2^\infty \frac{1}{x(\ln(x))} dx$ diverges.

T **F**

f) When $\vec{b} = (1,3,2)$ and $\vec{c} = (2,-1,1)$ then $(\vec{b} \times \vec{c}) \bullet \vec{b} = 0$.

T **F**

2. $\int x\,(\ln(x))^2\,dx$

$$= \frac{x^2}{2}[\ln(x)]^2 - \int \frac{x^2}{2}\cdot 2\frac{\ln(x)}{x}\,dx$$

$$= \frac{x^2}{2}\left[\ln(x)\right]^2 - \int x\ln(x)\,dx$$

$$= \frac{x^2}{2}\left[\ln(x)\right]^2 - \left[\frac{x^2}{2}\ln(x) - \int \frac{x}{2}\,dx\right] = \frac{x^2}{2}\left[[\ln(x)]^2 - \ln(x) + \frac{1}{2}\right] + k.$$

3. $\int x(\arctan(x))\,dx$

$$\boxed{\left(\frac{x^2+1}{2}\right)\arctan(x) - \frac{x}{2} + k}$$

$$= \frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\arctan(x) - \frac{1}{2}\left[\int\left(1 - \frac{1}{1+x^2}\right)dx\right]$$

$$= \frac{x^2}{2}\arctan(x) - \frac{1}{2}\left[x - \arctan(x)\right] + k.$$

4. $\int \frac{1}{x^2 - 4x + 40}\,dx$

$$\boxed{\frac{1}{6}\arctan\left(\frac{x-2}{6}\right) + k}$$

$$= \int \frac{1}{(x-2)^2+36}\,dx$$

$$= \frac{1}{6^2}\int \frac{1}{\left(\frac{x-2}{6}\right)^2+1}\,dx$$

$$= \frac{1}{6}\int \frac{1}{u^2+1}\,du \quad \text{with } u = \frac{x-2}{6}$$

5. $\int \frac{2}{x^2(x-1)}\,dx$

$$\boxed{2\left[\ln|x-1| - \ln|x| + \frac{1}{x}\right] + k}$$

$$\frac{2}{x^2(x-1)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x-1}.$$

$$2 = a\,x(x-1) + b(x-1) + cx^2 = (a+c)x^2 + (b-a)x - b$$

So $b = -2$, $a = -2$, $c = +2$. and integral

$$= \int\left[\frac{-2}{x} - \frac{2}{x^2} + \frac{2}{(x-1)}\right] dx$$

6. $\int \frac{2}{(x-1)(x^2+4)}\,dx = I$

$$\boxed{\frac{2}{5}\left[\ln|x-1| - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\arctan(x/2)\right] + k}$$

$$= \int\left[\frac{a}{x-1} + \frac{bx+c}{x^2+4}\right] dx$$

$$2 = a(x^2+4) + (bx+c)(x-1) = (a+b)x^2 + (c-b)x + [4a-c]$$

So $a = -b = -c$ and $5a = 2$.

$$I = \frac{2}{5}\int\left[\frac{1}{x-1} - \frac{x}{x^2+4} - \frac{1}{x^2+4}\right]dx = \frac{2}{5}\left[\ln|x-1| - \frac{1}{2}\ln(x^2+4) - \frac{1}{4}\int\frac{1}{(\frac{x}{2})^2+1}dx\right]$$

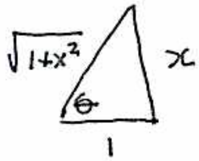
7. $\int \frac{1}{x^2\sqrt{1+x^2}}\,dx = I$

Let $x = \tan(\theta)$. ; $dx = \sec^2(\theta)\,d\theta$

$$I = \int \frac{\sec^2(\theta)\,d\theta}{\tan^2(\theta)\ \sec(\theta)} = \int \frac{\cos(\theta)}{\sin^2(\theta)}\,d\theta = \int \frac{1}{\sin(\theta)}\,\frac{\cos(\theta)}{\sin(\theta)}\,d\theta$$

$$= \int \csc(\theta)\cdot\cot(\theta)\,d\theta = -\csc(\theta) + k. = \frac{-\sqrt{1+x^2}}{x} + k$$

(use ~~tables~~ can tables)



8. $\int \frac{2x^3+2x+1}{x+2}\,dx$

$$= \int \left[2x^2 - 4x + 10 - \frac{19}{x+2}\right] dx$$

$$= \frac{2}{3}x^3 - 2x^2 + 10x - 19\ln|x+2| + k.$$

$$\begin{array}{r} 2x^2 - 4x + 10 \\ x+2 \,\big)\, 2x^3 + 2x + 1 \\ \underline{2x^3 + 4x^2} \\ -4x^2 + 2x \\ \underline{-4x^2 - 8x} \\ 10x + 1 \\ \underline{10x + 20} \\ -19. \end{array}$$

9. $\int \sec^3(x)dx = \int \sec^2(x)\sec(x)\,dx$

$$= \tan(x)\cdot\sec(x) - \int \tan^2(x)\sec(x)dx$$

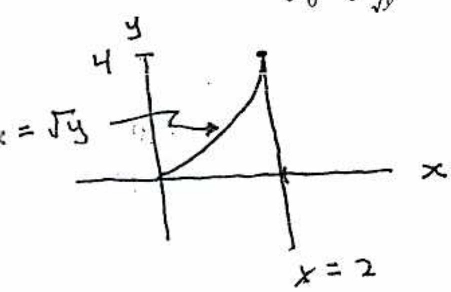
$$= \tan(x)\cdot\sec(x) - \int (\sec^2(x)-1)\sec(x)dx$$

$$= \tan(x)\sec(x) - \int \sec^3(x)\,dx + \int \sec(x)dx$$

So $\int \sec^3(x)dx = \frac{1}{2}\left[\tan(x)\sec(x) + \ln|\sec(x)+\tan(x)|\right] + k.$

10. Evaluate $\int_0^4 \left(\int_{\sqrt{y}}^2 \cos(x^3)\,dx\right) dy$ by first reversing the order of integration.

$$\frac{\sin(8)}{3}$$



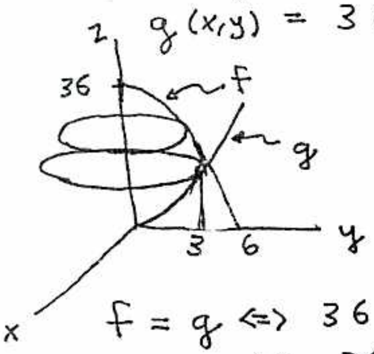
$$= \int_0^2 \int_0^{x^2} \cos(x^3)\,dy\,dx = \int_0^2 x^2\cos(x^3)\,dx = \frac{1}{3}\sin(x^3)\Big|_{x=0}^{x=2}$$

11. Use double integrals to find the volume enclosed by the two surfaces $z = 36 - (x^2 + y^2)$, and $z = 3(x^2 + y^2)$.

Let $f(x,y) = 36 - (x^2+y^2) = 36 - r^2$ where (r, θ) are polar coordinates.

$g(x,y) = 3(x^2+y^2) = 3r^2$

$\boxed{162\pi}$



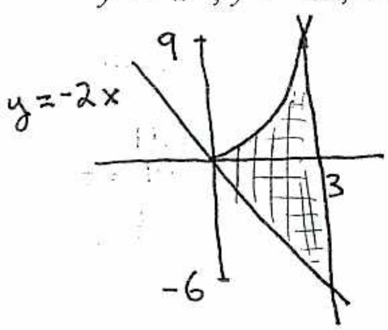
$$\text{Volume} = \iint_R (f-g)(x,y)\,da$$ where R = disk about origin of radius 3.

$$= \int_0^{2\pi}\int_0^3 (36 - 4r^2)\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^3 36r - 4r^3\,dr$$

$f = g \iff 36 - r^2 = 3r^2$
$\iff 36 = 4r^2 \iff 9 = r^2$

$$= 2\pi\left[18r^2 - r^4\right]_0^3 = 2\pi\left[3^4\right] = 2\cdot 81$$

12. Use double integrals to find the area of the lamina which occupies the planar region enclosed by $y = x^2$, $y = -2x$, $0 \le x \le 3$

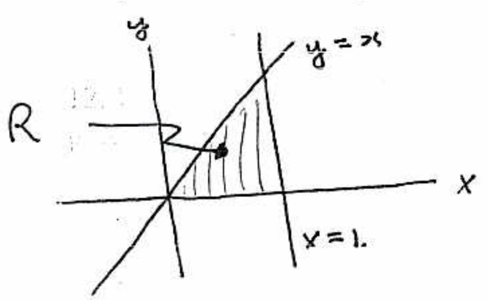


$$\text{area} = \int_0^3\int_{-2x}^{x^2} dy\,dx$$

$\boxed{18}$

$$= \int_0^3 (x^2 + 2x)\,dx = \left[\frac{x^3}{3} + x^2\right]\Big|_0^3 = 9 + 9 = 18$$

13. Find the x- coordinate of the center of mass of the triangular planar region D enclosed by $y = 0$, $x = 1$, and $y = x$ if the density function is given by $\delta(x,y) = x$.



x - coordinate of center of mass $\boxed{3/4}$

$$= \frac{\iint_R x\,\delta(x,y)\,da}{\text{Mass}} = \frac{1/4}{1/3} = 3/4$$

$$\text{Mass} = \iint_R \delta(x,y)\,da = \int_0^1\int_0^x x\,dy\,dx = \int_0^1 x^2\,dx = \frac{1}{3}$$

$$\iint_R x\,\delta(x,y)\,da = \int_0^1\int_0^x x^2\,dy\,dx = \int_0^1 x^3\,dx = \frac{1}{4}$$