

Department of Mathematics and Statistics  
AMAT 219 - QUIZ 1 - Tuesday, January 24, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators  
To obtain credit you need to show your work. Work should be neat and organized.

Determine each of the following integrals :

$k \in \mathbb{R}$

1.  $\int \arcsin(2x) dx$  (parts)

$$u = 2x \quad \frac{1}{2} du = dx$$

$$\begin{aligned} \frac{1}{2} \int \arcsin(u) du &= \frac{1}{2} \left[ u \arcsin(u) - \int \frac{u}{\sqrt{1-u^2}} du \right] \\ &= \frac{1}{2} \left[ u \arcsin(u) + \sqrt{1-u^2} \right] \end{aligned}$$

$$x \arcsin(2x) +$$

$$\frac{1}{2} \sqrt{1-4x^2} + k$$

2.  $\int \cos(\ln(x)) dx$  (parts twice)

$$\begin{aligned} &\int 1 \cdot \cos(\ln(x)) dx \\ &= x \cos(\ln(x)) + \int x \sin(\ln(x)) \cdot \frac{1}{x} dx \\ &= x \cos(\ln(x)) + \int \sin(\ln(x)) dx \\ &= x \cos(\ln(x)) + x \sin(\ln(x)) - \int \cos(\ln(x)) dx \end{aligned}$$

$$\begin{aligned} &\frac{1}{2} \left[ x \cos(\ln(x)) + x \sin(\ln(x)) \right] \\ &+ k \end{aligned}$$

3.  $\int \frac{x^2}{\sqrt{1-x^2}} dx$  (inverse trig)

$$x = \sin \theta$$

$$\int \frac{\sin^2 \theta \cos \theta d\theta}{\cos \theta} = \int \sin^2 \theta d\theta$$

$$= \frac{1}{2} \left[ -\sin \theta \cos \theta + \theta \right]$$

$$\begin{aligned} &\frac{1}{2} \left[ -x \sqrt{1-x^2} + \arcsin(x) \right] \\ &+ k \end{aligned}$$

4.  $\int \frac{2}{x^2+2x-3} dx$  (partial fraction)

$$\frac{2}{x^2+2x-3} = \frac{2}{(x+3)(x-1)} = \frac{a}{x+3} + \frac{b}{x-1}$$

$$2 = a(x-1) + b(x+3)$$

$$b = \frac{1}{2} = -a$$

$$= -\frac{1}{2} \int \frac{1}{x+3} dx + \frac{1}{2} \int \frac{1}{x-1} dx$$

$$\frac{1}{2} [\ln|x-1| - \ln|x+3|] + k$$

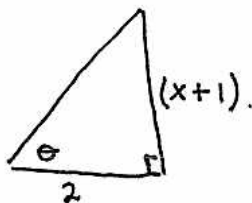
5.  $\int \frac{1}{\sqrt{x^2+2x+5}} dx$  (complete square)

$$= \int \frac{1}{\sqrt{(x+1)^2+4}} dx$$

$$\begin{cases} (x+1) = 2 \tan \theta \\ dx = 2 \sec^2(\theta) d\theta \end{cases}$$

$$= \int \frac{2 \sec^2(\theta) d\theta}{2 \sec(\theta)} = \int \sec(\theta) d\theta$$

$$= \ln |\sec(\theta) + \tan(\theta)|$$



$$\ln \left| \frac{\sqrt{(x+1)^2+4}}{2} + \frac{(x+1)}{2} \right| + k$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes \_\_\_\_\_ or No \_\_\_\_\_