Department of Mathematics and Statistics

AMAT 219 - QUIZ 2 - Thursday, February 9, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators

To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\int \frac{x^2-3x-3}{x^3+x^2-2}\,dx$

$\boxed{-\ln|x-1| + \ln((x+1)^2+1) - \arctan(x+1) + k.}$

$x = 1$ is a root of x^3+x^2-2, so

$\frac{x^2-3x-3}{x^3+x^2-2} = \frac{a}{x-1} + \frac{bx+c}{x^2+2x+2}$. Solve

for $a = -1$, $b = 2$, $c = 1$ and integral

$= \int -\frac{1}{x-1}\,dx + \int \frac{2x+1}{x^2+2x+2}\,dx = -\ln|x-1| + \int \frac{2x+2}{(x+1)^2+1}\,dx - \int \frac{1\,dx}{(x+1)^2+}$

2. Find $\int_0^1 \frac{x}{\sqrt[3]{x^2-1}}\,dx$

$= \lim_{a \nearrow 1} \int_0^a \frac{x}{\sqrt[3]{x^2-1}}\,dx = \lim_{a \nearrow 1} \frac{3}{4}(x^2-1)^{2/3}\Big|_0^a$

$= -\frac{3}{4}.$

$\boxed{\text{converges to } -3/4.}$

3. Find $\int_{-1}^{\infty} \frac{1}{1+x^2}\,dx$

$= \lim_{a \to \infty} \int_{-1}^{a} \frac{1}{1+x^2}\,dx$

$\boxed{\text{converges to } \frac{3\pi}{4}.}$

$= \lim_{a \to \infty} \arctan(x)\Big|_{-1}^{a}$

$= \lim_{a \to \infty} \arctan(a) - \arctan(-1) = \pi/2 - (-\pi/4) = \frac{3\pi}{4}$

4. $\int_{-\infty}^{\infty} e^{-|x|}\, dx$

$$= \lim_{a \to -\infty} \int_a^0 e^{-|x|}\,dx + \lim_{b\to\infty} \int_0^b e^{-|x|}\,dx$$

$\boxed{\text{Converges to 2}}$

$$= \lim_{a\to-\infty} \int_a^0 e^{x}\,dx + \lim_{b\to\infty}\int_0^b e^{-x}\,dx$$

$$= \underbrace{\lim_{a\to-\infty} -\int_{-a}^{0} e^{-u}\,du}_{(u = -x \text{ here}).} + \lim_{b\to\infty}\left[-e^{-x}\Big|_0^b\right] = \lim_{a\to\infty}\int_0^a e^{-u}\,du + 1$$

$$= 2.$$

5. What is the least value of n that guarantees M_n is within 10^{-3} of $ln(2) = \int_1^2 \frac{1}{x}\, dx$.

$\boxed{n = 10}$

max value of $|f''(x)| = \frac{2}{x^2}$ on $[1,2]$ is 2 (with $f(x) = \frac{1}{x}$).

Error estimate for M_n is $\frac{2\,|2-1|^3}{24\,n^2} = \frac{1}{12 n^2}$,

need n with $\frac{1}{12n^2} < 10^{-3}$, so $\frac{1000}{12} \le n^2$;

$83.\overline{3} \le n^2$, $n = 10$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes———— or No————