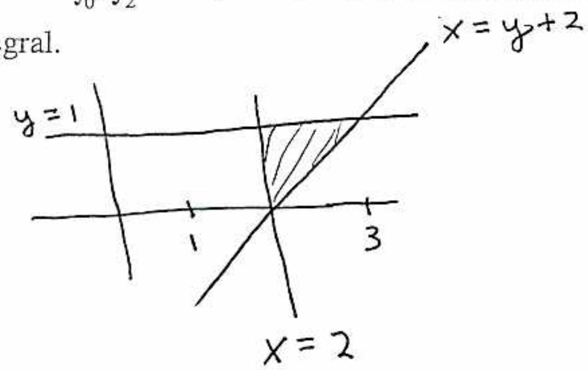
Department of Mathematics and Statistics AMAT 219 - QUIZ 3 - Thursday, March 2, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators

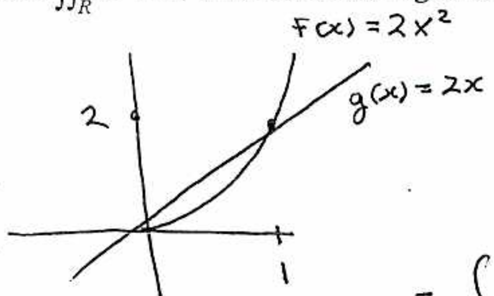
To obtain credit you need to show your work. Work should be neat and organized.

1. Write $\int_0^1 (\int_2^{y+2} \cos(x^2+1)\,dx)\,dy$ as an iterated integral with the order of integration reversed. Do not evaluate the integral.



$$\int_2^3 \left(\int_{x-2}^{1} \cos(x^3+1)\,dy \right) dx$$

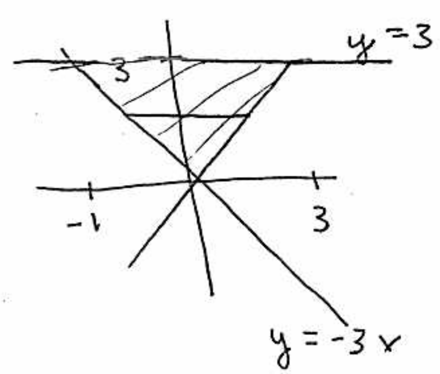
2. Find $\iint_R x^2\,dA$, where R is the region in the $xy-$plane bounded by $f(x) = 2x^2$ and $g(x) = 2x$.



$$\iint_R x^2\,dA = \int_0^1 \int_{2x^2}^{2x} x^2\,dy\,dx \qquad \boxed{1/10}$$

$$= \int_0^1 (2x^3 - 2x^4)\,dx = \frac{1}{2}x^4 - \frac{2}{5}x^5 \Big|_0^1 = \frac{1}{2} - \frac{2}{5}$$

3. Find $\iint_R y\,dA$ by viewing R as an $x-$simple region, where R is the region in the $xy-$plane bounded by $y = -3x$, $y = x$, and $y = 3$.

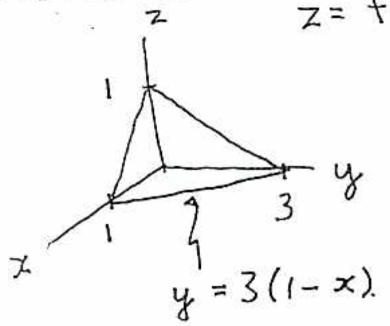


$$\iint_R y\,dA = \int_0^3 \int_{-y/3}^{y} y\,dx\,dy \qquad \boxed{12}$$

$$= \int_0^3 \left(y^2 + y^2/3\right) dy$$

$$= \frac{4}{3}\int_0^3 y^2\,dy = \frac{4}{3\cdot 3} y^3 \Big|_0^3 = 12$$

4. Use double integrals to find the volume of the region in the first octant $(x, y, z \geq 0)$ below the plane $3x + y + 3z = 3$.



$$z = f(x,y) = \frac{1}{3}(3 - 3x - y)$$

1/2

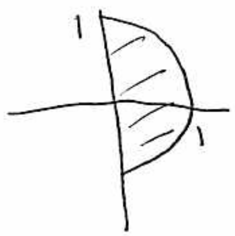
$$\text{Vol} = \int_0^1 \int_0^{3(1-x)} \left(1 - x - \frac{y}{3}\right) dy\, dx$$

$$= \int_0^1 \left[y - xy - \frac{y^2}{6} \Big|_0^{3(1-x)} \right] dx$$

$$= \int_0^1 3(1-x)\left[1 - x - \frac{(1-x)}{2}\right] dx = \int_0^1 3(1-x)\left[\left(\frac{1-x}{2}\right)\right] dx$$

$$= \frac{3}{2}\int_0^1 (1-x)^2 dx = -\frac{3}{2}\,\frac{(1-x)^3}{3}\Big|_0^1 = \frac{3}{2}\cdot\frac{1}{3} = \frac{1}{2}$$

5. Use polar coordinates to find $\iint_R x\, dA$ where R is the region bounded by $x^2 + y^2 = 1$, with $x \geq 0$.



$$\iint_R x\, dA = \int_{-\pi/2}^{\pi/2} \int_0^1 r^2 \cos\theta\, dr\, d\theta$$

2/3

$dA = r\,dr\,d\theta$

$x = r\cos\theta$

$0 \leq r \leq 1$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$$= \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\cos\theta \Big|_0^1 d\theta = \frac{1}{3}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$= \frac{1}{3}\sin\theta \Big|_{-\pi/2}^{\pi/2} = 2/3$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes_____________ or No_____________