Department of Mathematics and Statistics AMAT 219 - QUIZ 5 - Thursday, April 6, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\frac{\partial^2 z}{\partial x \partial y}$ for $z = f(2x - 4y)$ given that $f''(t) = \cosh(t)$.

Let $t = 2x - 4y$

$$\frac{\partial z}{\partial x} = \frac{df}{dt}\frac{\partial t}{\partial x} = 2\frac{df}{dt}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 2\frac{d^2 f}{dt^2}\frac{\partial t}{\partial y} = -8\frac{d^2 f}{dt^2} = -8\cosh(2x-4y).$$

2. Find the equation of the plane tangent to the surface $z = x^2 + xy - y^2$ at the point $x = 0,\ y = -1$.

$\boxed{x - 2y + z = 1}$

at $(0,-1)$ tangent plane has normal vector $\left(\frac{\partial f}{\partial x}(0,-1), \frac{\partial f}{\partial y}(0,-1), -1\right)$ with $f = x^2 + xy - y^2$.

$\frac{\partial f}{\partial x} = 2x + y$, $\frac{\partial f}{\partial y} = x - 2y$ so normal vector is $(-1, 2, -1)$. Point on plane is $(0, -1, f(0,-1)) = (0,-1,-1)$

so equation is $-x + 2y - z = -1$

3. Use spherical coordinates to find the mass of the hemisphere $x^2 + y^2 + z^2 \le 1$, $y \ge 0$ z, with density $\delta(x,y,z) = 6z(x^2+y^2)$ (Hint: $\sqrt{x^2+y^2} = \rho\sin\phi$ in spherical coordinates).

$\delta = 6\rho\cos\phi(\rho\sin\phi)^2$

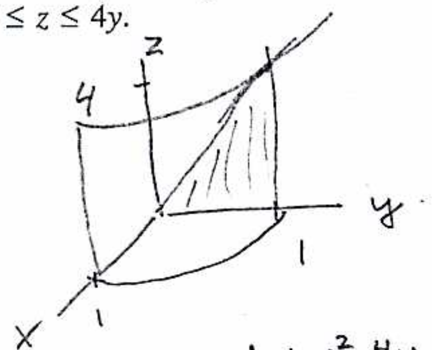
$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

Hemisphere is $\begin{cases} 0 \le \rho \le 1 \\ 0 \le \theta \le 2\pi \\ 0 \le \phi \le \pi/2 \end{cases}$

$$\text{mass} = \iiint_H \delta \, dV = 6\int_0^{\pi}\int_0^{\pi/2}\int_0^1 \rho^5 \sin^3\phi\cos\phi \, d\rho\, d\phi\, d\theta$$

$$= \pi\, 6\int_0^{\pi/2}\left[\frac{\rho^6}{6}\Big|_0^1\right]\sin^3\phi\cos\phi\, d\phi = \pi\left[\frac{\sin^4\phi}{4}\right]_0^{\pi/2} = \frac{\pi}{4}.$$

4. Use triple integrals to find the volume of the solid enclosed by the region $0 \le x$, $0 \le y$, $x \le 1 - y^2$, and $0 \le z \le 4y$.

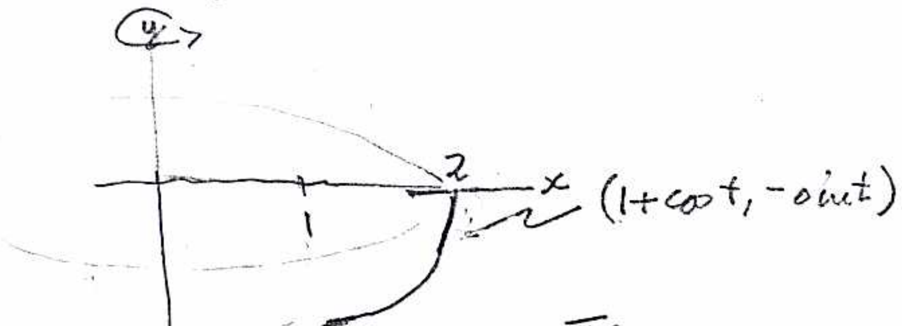


$$\text{Vol} = \int_0^1 \int_0^{1-y^2} \int_0^{4y} dz\, dx\, dy = \int_0^1 \int_0^{1-y^2} 4y\, dx\, dy$$

$$= \int_0^1 4y(1-y^2)\, dy = \left[2y^2 - y^4\right]_0^1 = 2 - 1 = 1.$$

5. Find the surface area of the surface formed by rotating the planar curve $c(t) = (1 + \cos(t), -\sin(t))$, $0 \le t \le \pi/2$,

around the y-axis.



$$\text{Surface area} = \int_0^{\pi/2} 2\pi\,(1+\cos(t))\sqrt{(x')^2 + (y')^2}\, dt$$

$$= \int_0^{\pi/2} 2\pi\,(1+\cos(t))\sqrt{\sin^2(t) + \cos^2(t)}\, dt = 2\pi \int_0^{\pi/2} (1 + \cos t)\, dt = 2\pi\left(\tfrac{\pi}{2} + \sin\tfrac{\pi}{2}\right)$$

$$= 2\pi\left(\tfrac{\pi}{2} + 1\right)$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes__________ or No__________