AMAT 219 PRACTICE SHEET #11

1. Find first order partial derivatives of:

$(a)\ f(x,y) = y^2 e^{x^2+4y}$ $(b)\ z = \ln(\ \tan(y) + xy^3\ )$ (c)
$g(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

2. Find all partial derivatives of order two for each of the following functions
:

$(a)\ z = \sin(x^2 + 2y^5)$ $(b)\ g(x,y,z) = \ln(x^3y^5z^7)$ (c)
$z = e^{xy}$

3. Find an equation of the plane tangent to the surface $xyz = 8$ at the point $(-1,-2,4)$.

4. Find an equation of the plane tangent to the hyperboloid $2x^2+3y^2-4z^2 = -11$ at the point $(-1,1,2)$.

5. Find vector equation of the line normal to surface $z = \sqrt[3]{x^3+y^2}$ at the point on surface where $x = -2$, $y = 3$.

6. Find an equation of the tangent plane to surface $z = e^{x^2+y^3}$ at the point on the surface where $x = 1$, $y = -1$.

7. Find a unit vector orthogonal (normal) to surface $xyz = -2$ at the point $(1,-2,1)$.

8. Find a unit vector orthogonal to surface $z = \sin(x+2y)$ at the point $(-2,1,0)$.

9. Find the x and $y-$ coordinates of the critical points for the function $f(x,y) = x^4 - 4xy + 2y^2 + 9$.

10. Find the x and $y-$ coordinates of the critical points for the function $f(x,y) = x^2 + xy - y^2 + 3x - 11y + 14$.

11. Find the x and $y-$ coordinates of the critical points for the function $f(x,y) = x^3 + 3xy - 6y - 17$.

12. The relation $x^4 + xy + y^3z + z^4 = 4$ implicitly defines y as a function of x and z, find $\dfrac{\partial y}{\partial z}$.

13. Find $\dfrac{\partial x}{\partial y}$ if $x = x(y,z)$ is defined implicitly by $x^3z + xy^2 + \sin(xyz) = 0$.

14. Given that the relation $3x^5 + 9xy - 2zy^4 + 3z^3 = 11$ implicitly defines x as a function of y and z. Compute $\dfrac{\partial x}{\partial z}$ at the point $(1,1,-1)$.

15. Let $v(x,t) = \sec(\frac{\sqrt{x}}{t})$. Find $2x\frac{\partial v}{\partial x} + t\frac{\partial v}{\partial t}$ and simplify.

16. Let $u(x,t) = \sin(\frac{x^2}{t^3} - 1)$. Find $2t\frac{\partial u}{\partial t} + 3x\frac{\partial u}{\partial x}$ and simplify.

17. Let $z = f(x+2y)$ where $f(t)$ is a function such that $f''(t) = e^{t^3}$. Determine $\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.

18. Let $z = x^4 + 2xy$, where $x = 1 - \sin(2t)$, $y = t\ln(1+t)$. Use the chain rule to find $\frac{dz}{dt}$ at $t = 0$.

19. Suppose $z = f(x,y)$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$. If $\frac{\partial f}{\partial x}(1,1) = 3$, and $\frac{\partial f}{\partial y}(1,1) = -1$, find $\frac{\partial z}{\partial \theta}$ when $r = \sqrt{2}$ and $\theta = \frac{\pi}{4}$.

20. Let $z = \ln(x^3 + 2y)$ where $x = x(r,s)$ and $y = y(r,s)$. Find $\frac{\partial z}{\partial s}$ at $r = 1$, $s = 3$ if $x(1,3) = 0$, $y(1,3) = \frac{1}{2}$, $\frac{\partial x}{\partial s}(1,3) = -1$, and $\frac{\partial y}{\partial s}(1,3) = 2$.

21. Let $z = u(p,q)$ where $p = x^2 + y^2$, $q = x^2 - y^2$, and u is a function such that $\frac{\partial u}{\partial p} = -q$, $\frac{\partial u}{\partial q} = p$.Determine $\frac{\partial z}{\partial x}$.

22. Let $f(t) = (t-2)e^t$ and let $w = f(x+2y)$. Find $\frac{\partial^2 w}{\partial y \partial x}$ and simplify your answer.

23. Show that the function $w = \ln\sqrt{x^2+y^2}$ is Harmonic.

Note : A function is Harmonic if it satisfies Laplace equation $\nabla^2 w = 0$.

24. Let $f(t)$ be a function such that $f''(5) = -2$ and let $W(x,y) = f(x^2 - 4y)$. Determine $\frac{\partial^2 W}{\partial x \partial y}(1,-1)$.

25. Let $z = x^4 + y^3$, where $x = u^2 v$, and $y = uv^2$. Find $\frac{\partial z}{\partial u}$ *at* $u = 1$, $v = -1$.

ANSWERS

3. $4x + 2y - z + 12 = 0$ 4. $2x - 3y + 8z = 11$ 5. $(x,y,z) = (-2,3,1) + t(12,6,-3)$

6. $2x + 3y - z = -2$ 7. $\overrightarrow{n} = \pm\,(-\frac{2}{3}, \frac{1}{3}, -\frac{2}{3})$ 8. $\overrightarrow{n} = \pm\frac{1}{\sqrt{6}}(1,2,-1)$

9. $(x, y) = (0,0), (1,1), (-1,-1)$. 10. $(x, y) = (1,-5)$ 11. $(x, y) = (2, -4)$.

12. $\dfrac{\partial y}{\partial z} = -\dfrac{y^3 + 4z^3}{x + 3y^2 z}$ 13. $\dfrac{\partial x}{\partial y} = -\dfrac{2xy + xz\cos(xyz)}{3x^2 z + y^2 + yz\cos(xyz)}$

14. $-\dfrac{7}{24}$ 15. 0 16. 0

17. $7\ e^{(x+2y)^3}$ 18. -8 19. -4

20. 4 21. $4xy^2$ 22. $2(x+2y)\ e^{x+2y}$

23. Show $w_{xx} + w_{yy} = 0$ 24 16 25. 11