AMAT 219 PRACTICE SHEET #3

Determine whether the improper integral converges or diverges using an appropriate limit :

1. $\int_{-2}^{2} \frac{1}{x^6}\, dx$ 2. $\int_{1}^{e} \frac{1}{x\sqrt{\ln(x)}}\, dx$ 3. $\int_{-2}^{\infty} \frac{1}{x^2+4}\, dx$

4. $\int_{-\infty}^{0} 3x^2 e^{x^3}\, dx$ 5. $\int_{0}^{\infty} \frac{e^{-x}}{1+e^{-2x}}\, dx$ 6. $\int_{0}^{2\pi} \sec(\frac{x}{4})\, dx$

7. $\int_{1}^{\infty} \frac{9}{(2-3x)^4}\, dx$ 8. $\int_{0}^{4} \frac{x}{\sqrt{16-x^2}}\, dx$ 9. $\int_{0}^{1} \frac{d}{dx}(\frac{e^x-1}{x})\, dx$

10. $\int_{-3}^{6} (x+2)^{\frac{-2}{3}} dx$ 11. $\int_{0}^{\infty} \frac{1}{(x-1)^2}\, dx$ 12. $\int_{-\infty}^{1} x^2\, e^{-x} dx$

13. $\int_{0}^{1} \ln(x)\, dx$ 14. $\int_{0}^{\frac{\pi}{4}} \tan(2x)\, dx$ 15. $\int_{2}^{\infty} \frac{2}{x^2-1}\, dx$

16. Find the volume of the solid generated by revolving the region enclosed by $y = e^{-x}$
and the $x-axis$, $0 < x < \infty$ about the $y-axis$.

ANSWERS

1. Diverges 2. Converges to 2 3. Converges to $\frac{3\pi}{8}$ 4. Converges to 1

5. Converges to $\frac{\pi}{4}$ 6. Diverges 7. Converges to 1 8. Converges to 4

9. Converges to $e-2$ 10. Converges to 9 11. Diverges 12. Diverges.

13. Converges to -1 14. Diverges 15. Converges to $\ln(3)$ 16. Volume V= 2π

Hints:

Problem # 5: Use the substitution $t = e^{-x}$.

Problem # 6: $\sec(\frac{x}{4})$ has an infinite discontinuity at $x = 2\pi$.

Problem # 9: $\int g'(x)\, dx = g(x) + C$

Problem #13: Use L'Hospital's Rule to show that $\lim_{x \longrightarrow 0^+} x\ln(x) = 0$.

Problem # 16: Using cylindrical shells, the volume V=$2\pi \int_0^\infty xe^{-x}dx$.