AMAT 219 PRACTICE SHEET #5

1. Find $\iint_R x^2 \; dA$ where R is the rectangular region described by $-1 \leq x \leq 1$, $0 \leq y \leq 3$

2. Find $\iint_R xy \; dA$ where R is the rectangular region enclosed by $x = 0, x = 2$, $y = 0$, and $y = 1$.

3. Find $\iint_D \cos(x + 2y) \; dA$ where D is the rectangular region described by $0 \leq x \leq \frac{\pi}{2}$, $0 \leq y \leq \frac{\pi}{4}$.

4. Find $\iint_T y^3 \; dA$ where T is the triangular region enclosed by $y = 2x$, $y = -2x$, and $x = -1$.

5. Let R be the region enclosed by the triangle with vertices at the point $(0, 0)$, $(0, 1)$. and $(1, 1)$.Evaluate $\iint_R 2\sin(\pi y^2) \; dA$

6. Determine $\int_0^1 \left(\int_{\sqrt{x}}^1 \sin(\pi y^3) dy \right) \; dx$.

7. Determine $\iint_R \frac{1}{x^4 + y^2} \; dA$, where R is the region enclosed by $y = x^2$, $y = -x^2$, $x = 1$ and $x = 2$.

8. Use double integrals to find the volume of the region in the first octant $(x, y, z \geq 0)$ below the plane $x + 3y + z = 6$.

9. Find the volume under the surface $f(x, y) = y$ over the region R enclosed by $y = x^2$ and $y = x + 2$ in the $xy-$plane.

10. Determine $\int_0^2 \left(\int_{\frac{y}{2}}^1 e^{x^2} dx \right) \; dy$.

11. Use double integrals to find the volume of the region in the first octant ($x, y, z \geq 0$) bounded by the vertical plane $2x + y = 2$ and the surface $z = x^2$.

12. Find the volume of the solid lying above the triangular region enclosed by $y = x$, $y = 0$ and $x = 1$ and below the surface $z = 2 - x$.

13. Use double integrals to find the volume of the region lying above the planar region bounded by $x = 0$, $y = 0$, and $y = 1 - x$, and below the surface $z = x + 2y$.

14. Let D be the planar region described by $0 \leq y \leq 1 - x^2$, $0 \leq x \leq 1$. Evaluate $\iint_D (1 - x) \; dA$

15. Compute $\iint_R \cos(y^2)\, dA$ where R is the triangular region with vertices at the points $(0,0)$, $(\frac{\pi}{6}, \frac{\pi}{6})$,and $(\frac{\pi}{2}, \frac{\pi}{6})$.

16. Find the volume of the region in the first octant below the plane $2x + 3y + 4z = 12$.

17. Determine $\iint_R \sin(2x - 3y)\ dA$, where R is the region described by $0 \le x \le \frac{\pi}{4}$, $0 \le y \le \pi$.

18. The iterated integral $\int_0^2 \left(\int_{3x^3}^{6x^2} g(x,y)dy \right) dx$ is the double integral of $g(x,y)$ over a planar region R. Express the double integral as an iterated integral with the order of integrals reversed.

19. Use double integrals to find the area enclosed by $y = \sqrt{2 - x^2}$, and $y = x^2$.

20. Let E be a planar region and let $\iint_E f(x,y)\, dA = \int_0^2 \left(\int_{\sqrt{x}}^{\sqrt{6-x^2}} f(x,y)dy \right)$ dx.Sketch the region E.

ANSWERS

(1) 2 (2) 1 (3) 0 (4) 0 (5) $\frac{2}{\pi}$

(6) $\frac{2}{3\pi}$ (7) $\frac{\pi}{4}$ (8) 12 (9) $\frac{36}{5}$ (10) $e - 1$

(11) $\frac{1}{6}$ (12) $\frac{2}{3}$ (13) $\frac{1}{2}$ (14) $\frac{5}{12}$ (15) $\sin(\frac{\pi^2}{36})$

(16) 12 (17) $-\frac{1}{3}$ (18) $\int_0^{24} \left(\int_{\sqrt{\frac{y}{6}}}^{\sqrt[3]{\frac{y}{3}}} g(x,y)dx \right) dy$ (19) $\frac{\pi}{2} + \frac{1}{3}$

(20) R is the planar region in the first quadrant enclosed by $x = 0$, the parabola $y^2 = x$ and the circle $x^2 + y^2 = 6$.

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5. the triangular region is described by : $0 \leq x \leq y$, $0 \leq y \leq 1$.

7. $\int \frac{1}{x^4 + y^2}\, dy = \frac{1}{x^2} \arctan(\frac{y}{x^2})$.

15. The triangular region is enclosed by $y = x$, $y = \frac{x}{3}$, and $y = \frac{\pi}{6}$. Treat region as $x-$simple.