AMAT 219 PRACTICE SHEET #7

1. Use double integrals to find the coordinates of the centre of mass of the planar region enclosed by $y = x$, $y = -x$, and $x = 1$ with density $\delta(x, y) = y^2$.

2. Use double integrals to find the $x-$ coordinate of the centroid of the planar region R enclosed by $y = 4x^2$, $y = -3x$, and $x = 1$.

3. Refer to problem 2 above.

Use Pappus's Theorem to find the volume of the solid generated by rotating the region R about the line $x = \dfrac{46}{17}$.

4. Use double integrals to find the coordinates of the centre of mass of the planar region enclosed by $y = -2x$, $y = x$, and $y = 2$ if the density function is given by $\delta(x, y) = y$.

5. A lamina has the shape of the planar region enclosed by $y = 1 - x^2$, and $y = 0$.The density at the point (x, y) is given by the function $\delta(x, y) = 1 + x$.

Determine:
(a) The mass of the lamina
(b) The moment of the lamina about the $x-$ axis.
(c) The moment of the lamina about the $y-$ axis
(d) Coordinates of the centre of mass of the lamina.

6. Use double integrals to find the coordinates of the centroid of the region in the plane bounded by the upper semi-circle centered at the origin and with radius b.

7. Determine $\iint_R xy^2 \, dA$ where R is a planar region with mass equal to 3, centre of mass at $(\overline{x}, \overline{y}) = (1, 4)$ and density function $\delta(x, y) = xy$.

8. Use double integrals to find the coordinates of the centre of mass of the region given by $x^2 + y^2 \leq 4$, $x \geq 0$, $y \geq 0$.

9. The coordinates of the centroid of the triangular region T with vertices at the points $(0, 0)$, $(0, 6)$, and $(3, 0)$ is the point $(1, 2)$. Use Pappus's Theorem to find the volume of the solid generated by rotating the region T about the line $x = 3$.

10. Use double integrals to find the coordinates of the centre of mass of the planar region bounded by $y = 0$ and $y = \sqrt{25 - x^2}$ if the density function is given by $\delta(x, y) = x^2 + y^2$.

11. Use double integrals to find the coordinates of the centroid of the region D bounded by $y = \sqrt{x}$, $x = 0$, and $y = 1$ and hence apply Pappus's Theorem to find the volume of the solid generated by revolving region D about the line $y = -1$.

12. Use double integrals to find the mass of the region given by $0 \le y \le \sin(x)$, $0 \le x \le \pi$ where density $\delta(x, y) = 4xy$.

ANSWERS

1. $(\overline{x}, \overline{y}) = (\frac{4}{5}, 0)$ 2. $\overline{x} = \frac{12}{17}$ 3. Volume $V = \frac{34}{3}\pi$ 4. $(\overline{x}, \overline{y}) = (\frac{3}{8}, \frac{3}{2})$

5. (a) $\frac{4}{3}$ (b) $\frac{8}{15}$ (c) $\frac{4}{15}$ (d) $(\overline{x}, \overline{y}) = (\frac{1}{5}, \frac{2}{5})$.

6. $(\overline{x}, \overline{y}) = (0, \frac{4b}{3\pi})$ 7. 12 8. $(\overline{x}, \overline{y}) = (\frac{8}{3\pi}, \frac{8}{3\pi})$ 9. 36π

10. $(\overline{x}, \overline{y}) = (0, \frac{8}{\pi})$ 11. $(\overline{x}, \overline{y}) = (\frac{3}{10}, \frac{3}{4})$; Volume $V = \frac{7\pi}{6}$ 12. $\frac{\pi^2}{2}$