

U of C ID #

45 Minutes, Open Book, NO Calculators

To obtain credit you need to show your work. Work should be neat and organized.

1. Find the parametric equations of the straight line perpendicular to the plane  $2x + 3y - z = 9$  and containing the point  $(1, 2, -3)$ .

Direction vector for line is normal to plane, so  $= (2, 3, -1)$

$$x(t) = 1 + 2t$$

$$y(t) = 2 + 3t$$

$$z(t) = -3 - t$$

2. Find the equation of the plane containing the points  $(-1, 1, 2)$ ,  $(2, 2, 1)$ , and  $(1, 0, 3)$ .

Two vectors in plane are

$$(2 - (-1), 2 - 1, 1 - 2) = (3, 1, -1)$$

$$(1 - (-1), 0 - 1, 3 - 2) = (2, -1, 1)$$

so normal to plane is  $(3, 1, -1) \times (2, -1, 1) = (0, -5, -5)$

Eq of plane is  $y + z = 3$

3. What is the line of intersection of the planes  $x - 3y + 2z = 7$  and  $2x + 2y + z = 2$ .

$$\left[ \begin{array}{ccc|c} 1 & -3 & 2 & 7 \\ 2 & 2 & 1 & 2 \end{array} \right] \text{ row reduce}$$

$$\left[ \begin{array}{ccc|c} 1 & -3 & 2 & 7 \\ 0 & 8 & -3 & -12 \end{array} \right]$$

$$\left. \begin{array}{l} x = 3y - 2z + 7 \\ 8y = 3z - 12 \\ z = z \end{array} \right\}$$

$$\begin{cases} x = 3(-3/2 + 3/8t) - 2t + 7 \\ y = (-3/2 + 3/8t) \\ z = t \end{cases}$$

line  $\begin{cases} x = 5/2 - 7/8t \\ y = -3/2 + 3/8t \\ z = t \end{cases}$

$$\propto \left( \frac{5}{2}, -\frac{3}{2}, 0 \right) + t \left( -\frac{7}{8}, \frac{3}{8}, 1 \right)$$

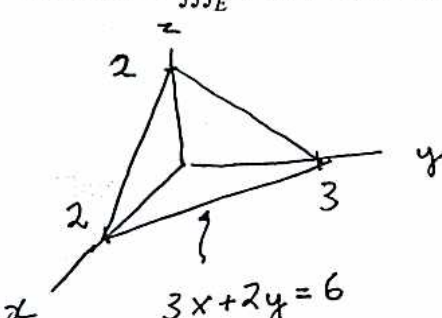
4. Find the plane containing the point  $(0, 1, 1)$  and not meeting the two lines  $l_1(t) = (1, 2, 0) + t(0, 2, 1)$  and  $l_2(t) = (1, 1, -2) + t(2, -1, 1)$ .

For a line and a plane not to meet the direction vector of the line must be perpendicular to the normal vector of the plane. So, a normal of the plane is  $(0, 2, 1) \times (2, -1, 1) = (3, 2, -4)$

and the equation of the plane is

$$3x + 2y - 4z = -2$$

5. Evaluate  $\iiint_E (-3x + 6) dV$  where  $E$  is the solid enclosed by the planes  $x = 0, y = 0, z = 0, 3x + 2y + 3z = 6$ .



$$\begin{aligned} & \int_0^2 \int_0^{\frac{6-3x}{2}} \int_0^{\frac{6-3x-2y}{3}} (6-3x) dz dy dx \\ &= \int_0^2 \int_0^{\frac{6-3x}{2}} (6-3x) \frac{6-3x-2y}{3} dy dx \\ &= \frac{1}{3} \int_0^2 \left[ (6-3x)(6-3x-y)y \right]_0^{\frac{6-3x}{2}} dx = \frac{1}{3} \int_0^2 \frac{(6-3x)^3}{2 \cdot 2} dx \\ &= \frac{-(6-3x)^4}{3 \cdot 3 \cdot 4 \cdot 2 \cdot 2} \Big|_0^2 = \frac{6^4}{3 \cdot 3 \cdot 2^4} = 3^2 = 9. \end{aligned}$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes \_\_\_\_\_ or No \_\_\_\_\_