Department of Mathematics and Statistics AMAT 219 - QUIZ 5 - Thursday, April 6, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators

To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\frac{\partial^2 z}{\partial x \partial y}$ for $z = f(4x + 2y)$ given that $f''(t) = \cos(t)$.

Let $t = 4x + 2y$.

$$\frac{\partial z}{\partial x} = \frac{df}{dt} \cdot \frac{\partial t}{\partial x} = 4 \cdot \frac{df}{dt}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 4 \frac{d^2 f}{dt^2} \cdot \frac{\partial t}{\partial y} = 2 \cdot 4 \cdot \frac{d^2 f}{dt^2} = 8 \cos(4x + 2y).$$

2. Find the equation of the plane tangent to the surface $z = 5 + x^2 y + y^2$ at the point $x = 1$, $y = -1$.

At $(1,-1)$ tangent plane has normal vector $\left(\frac{\partial f}{\partial x}(1,-1), \frac{\partial f}{\partial y}(1,-1), -1\right) = (-2, -1)$ since $\frac{\partial f}{\partial x} = 2xy$, $\frac{\partial f}{\partial y} = x^2 + 2y$. A point on the plane is $(1, -1, f(1,-1)) = (1, -1, 5)$, so equation is $-2x - y - z = -6$.

$\boxed{2x + y + z = 6}$

3. Use spherical coordinates to find the mass of the hemisphere $x^2 + y^2 + z^2 \le 1$, $z \ge 0$, with density $\delta(x, y, z) = 5z\sqrt{x^2 + y^2}$ (Hint: $\sqrt{x^2 + y^2} = \rho \sin \phi$ in spherical coordinates).

$\delta = 5 \rho \cos\phi (\rho \sin\phi)$

$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

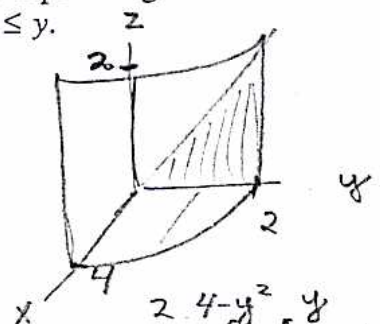
Hemisphere is $\begin{cases} 0 \le \rho \le 1 \\ 0 \le \phi \le \pi/2 \\ 0 \le \theta \le 2\pi \end{cases}$

$$\text{mass} = \iiint_H \delta \, dV = 5 \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho^4 \sin^2\phi \cos\phi \, d\rho \, d\phi \, d\theta$$

$$= 5 \cdot 2\pi \int_0^{\pi/2} \frac{\rho^5}{5}\Big|_0^1 \sin^2\phi \cos\phi \, d\phi = 2\pi \left(\frac{\sin^3\phi}{3} \Big|_0^{\pi/2} \right)$$

$$= \frac{2\pi}{3}$$

4. Use triple integrals to find the volume of the solid enclosed by the region $0 \le x$, $0 \le y$, $x \le 4 - y^2$, and $0 \le z \le y$.

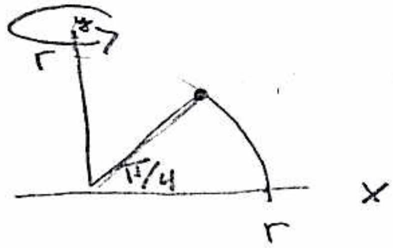


$$Vol = \int_0^2 \int_0^{4-y^2} \int_0^y dz\, dx\, dy = \int_0^2 \int_0^{4-y^2} y\, dx\, dy$$

$$= \int_0^2 y(4-y^2)\, dy = \left[2y^2 - \frac{y^4}{4}\right]_0^2 = 8 - 4 = 4.$$

5. Find the surface area of the surface formed by rotating the planar curve $c(t) = (r\cos(t), r\sin(t))$, $0 \le t \le \pi/4$,

around the y-axis.



$$\text{Surface area} = \int_0^{\pi/4} 2\pi\, x(t) \sqrt{(x')^2 + (y')^2}\, dt$$

$$= \int_0^{\pi/4} 2\pi r \cos(t) \sqrt{r^2 \sin^2 t + r^2 \cos^2 t}\, dt = 2\pi r^2 \int_0^{\pi/4} \cos(t)\, dt$$

$$= \frac{2\pi r^2}{\sqrt{2}} = \sqrt{2}\, \pi r^2.$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes__________ or No__________