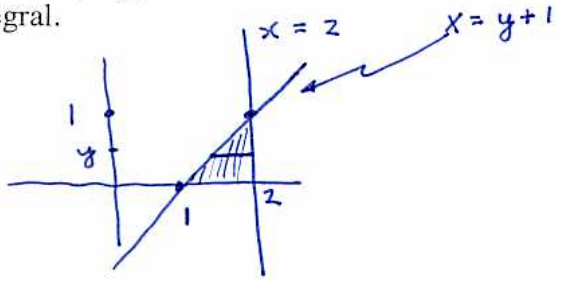
Department of Mathematics and Statistics AMAT 219 - QUIZ 3 - Tuesday, February 28, 2006

U of C ID #

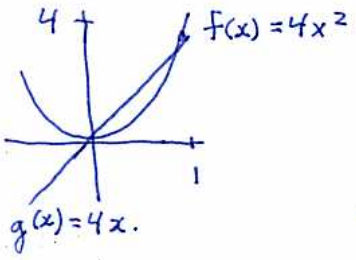
45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

1. Write $\int_0^1 \left(\int_{y+1}^2 \cos(x^2 + 1)\, dx \right) dy$ as an iterated integral with the order of integration reversed. Do not evaluate the integral.



$$\int_1^2 \int_0^{x-1} \cos(x^3+1)\, dy\, dx$$

2. Find $\iint_R x\, dA$, where R is the region in the xy – plane bounded by $f(x) = 4x^2$ and $g(x) = 4x$.



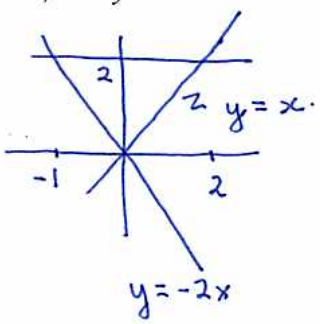
$$\iint_R x\, dA = \int_0^1 \int_{4x^2}^{4x} x\, dy\, dx$$

$$= \int_0^1 \left(xy \Big|_{y=4x^2}^{y=4x} \right) dx = \int_0^1 (4x^2 - 4x^3)\, dx$$

$$= \frac{4}{3}x^3 - x^4 \Big|_0^1 = \frac{1}{3}$$

1/3

3. Find $\iint_R y\, dA$ by viewing R as an x –simple region, where R is the region in the xy – plane bounded by $y = -2x$, $y = x$, and $y = 2$.



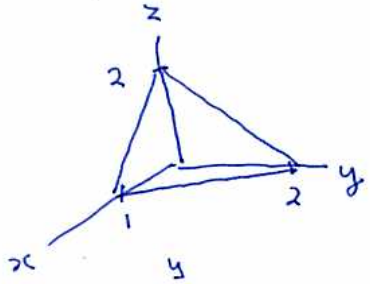
$$\iint_R y\, dA = \int_0^2 \int_{-y/2}^{y} y\, dx\, dy$$

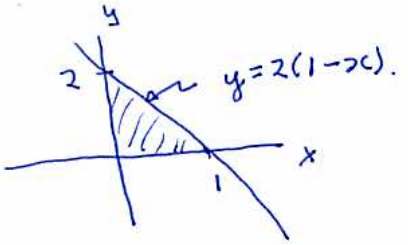
$$= \int_0^2 yx \Big|_{-y/2}^{y} dy = \int_0^2 \left(y^2 + y^2/2 \right) dy$$

$$= 3/2 \int_0^2 y^2\, dy = 3/2 \cdot y^3/3 \Big|_0^2 = 4$$

4

4. Use double integrals to find the volume of the region in the first octant $(x, y, z \geq 0)$ below the plane $2x + y + z = 2$.





R

$z = f(x,y) = 2(1-x) - y.$

$$\text{Volume} = \iint_R f(x,y)\,dA$$

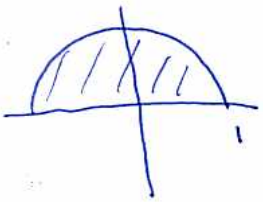
$$= \int_0^1 \int_0^{2(1-x)} [2(1-x) - y]\,dy\,dx$$

$$= \int_0^1 [2(1-x)]^2 - \frac{[2(1-x)]^2}{2}\,dx$$

$$= \int_0^1 2(1-x)^2\,dx = -\frac{2}{3}(1-x)^3\Big|_0^1 = 2/3$$

2/3

5. Use polar coordinates to find $\iint_R y\,dA$ where R is the region bounded by $x^2 + y^2 = 1$ with $y \geq 0$.



$y = r\sin\theta$

$dA = r\,dr\,d\theta$

$R: \begin{cases} 0 \leq r \leq 1 \\ 0 \leq \theta \leq \pi \end{cases}$

$$\iint_R y\,dA$$

$$= \int_0^\pi \int_0^1 r^2 \sin\theta\,dr\,d\theta$$

$$= \int_0^\pi \frac{r^3}{3}\sin\theta\Big|_0^1\,d\theta = -\frac{1}{3}\cos\theta\Big|_0^\pi = \frac{1}{3} + \frac{1}{3}$$

2/3

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes_____ or No_____