AMAT 219 PRACTICE SHEET #10

1. Find the cartesian equation of the tangent line to the plane curve given parametrically by $x(t) = t^2 + 1$, $y(t) = t^4 + 2t^2 + 1$, at the point $(1, 1)$.

2. Find the cartesian equations of the tangent and normal lines to the **cycloid** $x(t) = t - \sin(t),\ y(t) = 1 - \cos(t)$ at the point corresponding to $t = \frac{\pi}{2}$.

3. Find the arc length of the plane parametric curve $\overrightarrow{r(t)} = (\cos^3(t), \sin^3(t))$, $\quad 0 \le t \le \frac{\pi}{2}$.

4. Find the cartesian equation of the plane curve given parametrically by $x(t) = 2\cosh(t),\ y(t) = 4\sinh^2(t)$, $\quad t \in \mathbb{R}$.

5. Find the cartesian equation of the tangent line to the plane curve $\overrightarrow{r(s)} = (s^2 + 2s - 6, 7 - s^3)$, at the point $(2, -1)$.

6. Determine the arc length of the plane curve given by the vector function $\overrightarrow{r(t)} = (t, \frac{2}{3}(t + 3)^{3/2})$, $\quad 0 \le t \le 1$.

7. Determine the point of intersection of the two parametric curves $(x(t), y(t)) = (2t\ ,\ 3t^2 + 4)$ and $(x(s), y(s)) = (s, s^2)$.

8. The position vector of a particle moving in space is given by $\overrightarrow{r(t)} = (t^2 + 2t - 8)\overrightarrow{i} + (\frac{1}{2}t^2 - 1)\overrightarrow{j} - \sqrt{2}t^{3/2}\overrightarrow{k}$.Find the velocity, acceleration, and the speed of the particle at the point $(0, 1, -4)$.

9. The position of a particle at time t (in seconds) is given by $(x(t)\ ,\ y(t)\ ,\ z(t)) = (\frac{1}{3}t^3 - 3t\ ,\ \frac{1}{2}t^2,\ 2t + 7)$ where x, y, and z are measured in meters. When will the speed of the particle be $3\ m\ /\ s$?

10. Find the cartesian equation of the plane curve given parametrically by $(x(t), y(t)) = (\frac{1 - t^2}{1 + t^2}, \frac{2t}{1 + t^2})$, $t \in [-1, 1]$. Identify the curve and sketch indicating the orientation.

11. Find the arc length of the space curve given parametrically by $x(t) = 1$, $y(t) = 2t - \sin(2t)$, $z(t) = 1 - \cos(2t)$, $\quad 0 \le t \le \pi$

12. A conic section is given parametrically by $x(t) = -1 + 4\sec(t)$, $\quad y(t) = 6 - 3\tan(t)$, $\quad t \in [0, \frac{\pi}{2})$. Name the curve and sketch.

13. Name the conic section defined by the vector function $\overrightarrow{r(t)} = (1 + 2\sin(t), -6 + 2\cos(t))$, $\ t \in [0, 2\pi]$.

14. Find the arc length of the space curve given by $\overrightarrow{r(t)} = (\frac{1}{3}t^3, t^2, 2t)$, $0 \le t \le 3$.

15 The position vector of a particle is given by $\overrightarrow{r(t)} = 4\sqrt{t}\,\overrightarrow{i} + 9\,\overrightarrow{j} + t^2\,\overrightarrow{k}$. Determine the speed of particle at $t = 2$.

16. Find the cartesian equation of the tangent line to the parametric curve given by $\overrightarrow{r(t)} = (e^t\cos(t), e^t\sin(t))$ at the point corresponding to $t = \frac{\pi}{2}$.

17. Find the arc length of the space curve given by $x(t) = e^{-4t}\cos(2t)$, $y(t) = e^{-4t}\sin(2t)$, $z(t) = e^{-4t}$, $0 \le t \le \frac{\ln 2}{4}$.

18. The position vector of a moving particle is given by $\overrightarrow{r(t)} = (t^2 - 1)\,\overrightarrow{i} + \sqrt{3}t\,\overrightarrow{j} + (t^2 + t)\,\overrightarrow{k}$. At what times is the speed of particle equal to 4 ?

19. Find the cartesian equation of the conic section given parametrically by $x(t) = 3 + 2\cos(t)$, $y(t) = \frac{1}{5}\sin(t)$, $t \in [0, 2\pi]$.

20. Find the area of the surface generated by revolving the arc of the parametric curve $x(t) = 2\ t^3$, $y(t) = 3t^2$, $0 \le t \le 1$ about the $x - axis$.

ANSWERS

1. $y = 2x - 1$ 2. Tangent line: $y = x - \frac{\pi}{2} + 2$, Normal line : $y = -x + \frac{\pi}{2}$
3. $\frac{3}{2}$

4. $y = x^2 - 4$, $x \ge 2$. 5. $y = -2x + 3$ 6. $\frac{2}{3}(5\sqrt{5} - 8)$ 7. ($\pm\,4$, 16)

8. Velocity $\overrightarrow{v} = (6, 2, -3)$, Acceleration $\overrightarrow{a} = (2, 1, -\frac{3}{4})$, Speed = 7

9. $t = \pm\,1$, $\pm\,2$. 10. $x^2 + y^2 = 1$, $0 \le x \le 1$ (Right semi circle) , 11. 8

12. $\frac{(x+1)^2}{16} - \frac{(y-6)^2}{9} = 1$, the part of the hyperbola centred at $(-1\ ,6)$, such that $x \in [3, \infty)$, $y \in (-\infty\ ,\ 6]$.

13. The full circle centred at the point ($1, -6$) and has radius 2 units.

14. 15 15. $3\sqrt{2}$ 16. $y = -x + e^{\pi/2}$ 17. $\frac{3}{4}$

18. $t = -\frac{3}{2}$ or 1 19 $\frac{(x-3)^2}{2^2} + \frac{y^2}{(1/5)^2} = 1$ (An ellipse)

20. $\frac{12}{5}(\sqrt{2} + 1)$. Hint : Use the substitution $u^2 = t^2 + 1$.