AMAT 219 PRACTICE SHEET #4

1. In each case find the value of Trapezoidal Rule, Midpoint Rule, and Simpson's Rule estimate for

the given integral and the specified value of n.

$(a)\ \int_0^1 \frac{1}{1+x^2}\,dx\ , \qquad n = 6.$

$(b)\ \int_0^1 \cos(x^2)\,dx\ , \qquad n = 6$

$(c)\ \int_1^7 \frac{1}{x+1}\,dx\ , \qquad n = 6$

2. Refer to problem #1, find the values of T_{12} and S_{12}.

3. Refer to problem #1 part (a), find an estimate for the value of π obtained from each

of the three rules (round your answers to four decimal places).

4. Refer to problem #1 part (c), find an estimate for the value of $\ln(2)$ obtained from each

of the three rules (round your answers to six decimal places).

5. Refer to problem #1part (c).Find an estimate for the absolute value of the **Error** involved in

approximating the integral using:

$(i)\ T_6$ $\qquad (ii)\ M_6$ $\qquad (iii)\ S_6$

6. Find the value of the Simpson's Rule estimate S_n for $\int_0^2 (3x^2 - 4x + 2)$

dx ,

where n is an arbitrary even positive integer.

7. Use the Trapezoidal Rule and the data in the following table to estimate the value of $\int_{14}^{21} y(t)\,dt$.

t	14	15	16	17	18	19	20	21
y	-6	-4	-2	0	2	4	6	8

8. How large should we take n in order to guarantee that the Trapezoidal Rule approximation for

$$\int_1^3 \frac{1}{x}\,dx \text{ is accurate to within } 0.03 \text{ ?}$$

9. How large should we take n in order to guarantee that the Simpson's Rule estimate for

$$\int_1^4 \frac{1}{x}\,dx \text{ is accurate to within } 0.00064 \text{ ?}$$

ANSWERS

1.(a) $T_6 = 0.784240767$, $M_6 = 0.785976857$, $S_6 = 0.785397945$

(b) $T_6 = 0.900628388$, $M_6 = 0.906472209$, $S_6 = 0.904522925$

(c) $T_6 = 1.405357143$, $M_6 = 1.376934177$, $S_6 = 1.387698413$

2.(a) $T_{12} = 0.785108812$, $S_{12} = 0.785398160$

(b) $T_{12} = 0.903550299$, $S_{12} = 0.904524269$

(c) $T_{12} = 1.391145660$, $S_{12} = 1.386408499$

3. Using Trapezoidal Rule T_6, we find $\pi \cong 3.1370$

Using Midpoint Rule M_6, we find $\pi \cong 3.1439$

Using Simpson's Rule S_6, we find $\pi \cong 3.1416$

4. Using Trapezoidal rule T_6, we find $\ln(2) \cong 0.702679$

Using Midpoint Rule M_6, we find $\ln(2) \cong 0.688467$

Using Simpson's Rule S_6, we find $\ln(2) \cong 0.693849$

5. (i) $E_6 \leq 0.125$ (ii) $E_6 \leq 0.0625$ (iii) $E_6 \leq 0.025$

6. $S_n = 4$

7. $T_7 = 7$

8. $n = 7$

9. $n = 16$

Hints

1. (b) Calculator should be in radian mode!

2. Use relations : $T_{2n} = \dfrac{T_n + M_n}{2}$, $S_{2n} = \dfrac{T_n + 2M_n}{3}$ with $n = 6$.

3. Verify that $\displaystyle\int_0^1 \frac{1}{x^2+1}\, dx = \frac{\pi}{4}$ and hence $4T_6$ or $4M_6$ or $4S_6$ are the required estimates of π.

4. Verify that $\displaystyle\int_1^7 \frac{1}{x+1}\, dx = 2\ln(2)$ and hence $\frac{1}{2}T_6$ or $\frac{1}{2}M_6$ or $\frac{1}{2}S_6$ are the required estimates of $\ln(2)$.

5. The function $\dfrac{1}{(x+1)^r}$ is strictly decreasing on $[1,7]$ for $r = 1, 2, 3, ...$ and hence its absolute maximum value say " k " occurs at $x = 1$.

6. Note that the integrand is a polynomial of degree two!