Department of Mathematics and Statistics
AMAT 219 - QUIZ 2 - Tuesday, February 7, 2006

U of C ID #

(log = ln)

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

1.Find $\int \frac{3x+2}{x^3+x^2-2}\,dx$

$x=1$ is a root of x^3+x^2-2, so

$$\frac{3x+2}{x^3+x^2-2} = \frac{3x+2}{(x-1)(x^2+2x+2)} = \frac{a}{x-1} + \frac{bx+c}{x^2+2x+2}$$

$$\boxed{\log|x-1| - \frac{1}{2}\log((x+1)^2+1) + \arctan(x+1) + k}$$

Obtain $a=1$, $b=-1$, $c=0$ and

integral $= \int \frac{1}{x-1}dx - \int \frac{x}{(x+1)^2+1}dx = \int \frac{dx}{x-1} - \int \frac{(x+1)}{(x+1)^2+1}dx + \int \frac{1}{(x+1)^2+1}dx$

2. Find $\int_0^1 \frac{3x}{x^2-1}dx$

$$= \lim_{a \nearrow 1} \int_0^a \frac{3x}{x^2-1} = \frac{3}{2} \lim_{a \nearrow 1} \left[\log|x^2-1|\right]_0^a$$

$$\boxed{\text{diverges.}}$$

$$= \frac{3}{2}\left[\lim_{a \nearrow 1} \log|a^2-1| - 0\right]$$

$$= -\infty$$

3. Find $\int_0^\infty \frac{1}{4+x^2}\,dx$

$$= \lim_{a\to\infty} \frac{1}{4}\int_0^a \frac{1}{\cancel{4}1+\left(\frac{x}{2}\right)^2}$$

$$\boxed{\text{converges to } \pi/4}$$

$$= \lim_{a\to\infty} \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big|_0^a$$

$$= \lim_{a\to\infty} \frac{1}{2}\left[\arctan\left(\frac{a}{2}\right) - \arctan(0)\right]$$

$$= \frac{1}{2}\left[\pi/2 - 0\right] = \pi/4$$

4. Find $\int_1^\infty xe^{-x}\,dx$

$$= \lim_{a\to\infty} \left[-xe^{-x} - e^{-x}\right]_1^a$$

$$\boxed{\text{converges to } 2/e.}$$

$$= \lim_{a\to\infty} \left[-ae^{-a} - e^{-a} + 2e^{-1}\right]$$

$$= \lim_{a\to\infty} \left[\frac{-a-1}{e^a}\right] + 2/e \quad \text{(use l'hopital's rule)}$$

$$= \left[\lim_{a\to\infty} -\frac{1}{e^a}\right] + 2/e = 0 + 2/e$$

5. What is the least value of n that guarantees T_n is within 10^{-2} of $ln(2) = \int_1^2 \frac{1}{x}\,dx$?

$$\boxed{n = 5}$$

- Find n so that error estimate for T_n is less than or equal to 10^{-2}.
- need n so that $\dfrac{K\,|2-1|^3}{12\,n^2} \le 10^{-2}$ where K is max value of $\left|\left(\frac{1}{x}\right)''\right|$ on $[1,2]$. Since $\left(\frac{1}{x}\right)'' = +2x^{-3}$, $K = 2$.
- Find n with $\dfrac{2}{12\,n^2} \le 10^{-2}$; $\dfrac{10^2}{6} \le n^2$; $16.\overline{6} \le n^2$; so $n = 5$ is needed.

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes———— or No————