U of C ID #

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

1. Find the parametric equations of the straight line perpendicular to the plane $x - 3y + 2z = 7$ and containing the point $(-1, 1, 3)$. $(1, 3, 2)$ is direction vector of line.

$$x(t) = -1 + t$$
$$y(t) = 1 - 3t$$
$$z(t) = 3 + 2t$$

2. Find the equation of the plane containing the point $(-1, 1, 2)$ and the line $\vec{l}(t) = (2, 0, 3) + t(1, 2, 0)$.

$(-1,1,2)$ and $(2,0,3)$ are points in the plane
so vector $(2-(-1), 0-1, 3-2) = (3, -1, 1)$ is
normal of plane.

$(1, 2, 0) \times (3, -1, 1) = (2, -1, -7)$ is normal to plane.

$$2x - y - 7z = 2(2) - 0 - 7 \cdot 3 = -17$$

Eq of plane : $2x - y - 7z = -17$

3. What is the point of interesction (if it exits) of the lines $\vec{l_1}(t) = (5, -1, 2) + t(2, -2, 4)$ and $\vec{l_2}(t) = (4, -1, 1) + t(0, 1, -1)$.

Find t, s with $\vec{l_1}(t) = \vec{l_2}(s)$, so
with
$$5 + 2t = 4$$
$$-1 - 2t = -1 + s$$
$$2 + 4t = 1 - s$$

Solution is $t = -\frac{1}{2}$, $s = 1$,
Point of intersection is $\vec{l_1}(-\frac{1}{2}) = \vec{l_2}(1) = (4, 0, 0)$

4. Find the plane containing the point $(0, 1, 2)$ and not meeting the two lines $l_1(t) = (1, 2, 0) + t(0, -1, 1)$ and $l_2(t) = (1, 1, -2) + t(2, 1, -1)$.

For a line and a plane not to meet the direction vector of the line must be perpendicular to the normal vector of the plane. So, a normal of the plane is $(0, -1, 1) \times (2, 1, -1) = (0, 2, 2)$

and the equation of the plane is

$$y + z = 3.$$

5. Evaluate $\iiint_E (-2x + 2)\, dV$ where E is the solid enclosed by the planes $x = 0, \ y = 0, \ z = 0, \ 2x + 2y + z = 2$.
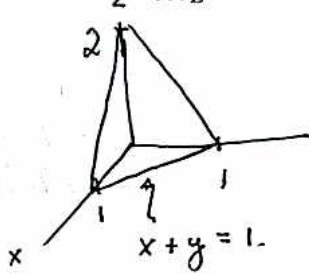
$$\int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} 2(1-x)\, dz \ dy \ dx$$

$$= \int_0^1 \int_0^{1-x} 2(1-x)(2-2x-2y)\, dy\, dx$$

$$= 4 \int_0^1 \left[(1-x)(1-x - \frac{y}{2})y \right]_0^{1-x} dx$$

$$= 4 \int_0^1 \frac{(1-x)^3}{2}\, dx \quad = \quad -\frac{(1-x)^4}{2} \Big|_0^1 \ = \ \frac{1}{2}$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes_____ or No_____