Department of Mathematics and Statistics AMAT 219 - QUIZ 5 - Tuesday, April 4, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

1. Find $\frac{\partial^2 z}{\partial x \partial y}$ for $z = f(2x - 3y)$ given that $f''(t) = \sin(t)$.

Let $t = 2x - 3y$.

$$\frac{\partial z}{\partial x} = \frac{df}{dt} \cdot \frac{\partial t}{\partial x} = 2 \cdot \frac{df}{dt}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 2 \cdot \frac{d^2 f}{dt^2} \cdot \frac{\partial t}{\partial y} = -3 \cdot 2 \cdot \frac{d^2 f}{dt^2} = -6 \sin(2x - 3y).$$

2. Find the equation of the plane tangent to the surface $z = 3 - x^2 + y^3$ at the point $x = 1$, $y = -1$.

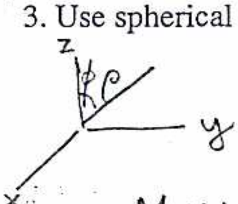
$\boxed{2x - 3y + z = 6}$

(1,-1) tangent plane has normal vector $= \left(\frac{\partial f}{\partial x}(1,-1), \frac{\partial f}{\partial y}(1,-1), -1\right)$ where

$f(x,y) = 3 - x^2 + y^3$. So $\frac{\partial f}{\partial x} = -2x$, $\frac{\partial f}{\partial y} = 3y^2$ and normal vector for plane is $(-2, 3, -1)$. A point on the plane is $(1, -1, f(1,-1)) = (1, -1, 1)$.

Equation is $-2x + 3y - z = -6$.

3. Use spherical coordinates to find the mass of the hemisphere $x^2 + y^2 + z^2 \le 4$, $z \ge 0$, with density $\delta(x, y, z) = z$



$z = \rho \cos\phi$; $dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

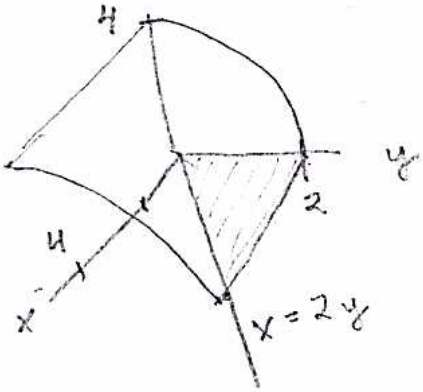
$\boxed{4\pi}$

$$\text{Mass} = \iiint_H \delta(x,y,z)\, dV \quad \text{where}$$

H is $\begin{cases} 0 \le \rho \le 2 \\ 0 \le \theta \le 2\pi \\ 0 \le \phi \le \pi/2 \end{cases}$ so $\text{Mass} = \int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta$

$$= \int_0^{2\pi} \int_0^{\pi/2} \frac{\rho^4}{4}\Big|_0^2 \sin\phi \cos\phi \, d\phi \, d\theta = 4 \cdot 2\pi \left[\frac{\sin^2\phi}{2}\right]_0^{\pi/2} = 4\pi$$

4. Use triple integrals to find the volume of the solid enclosed by the region, $0 \le y$, $0 \le x \le 2y$, and $0 \le z \le 4 - y^2$.



$$Vol = \int_0^2 \int_0^{2y} \int_0^{4-y^2} dz\, dx\, dy$$

$$= \int_0^2 (4-y^2)(2y)\, dy$$

$$= \left[4y^2 - \frac{y^4}{2}\right]_0^2 = 16 - 8 = 8.$$

5. Find the arclength of the planar parametric curve $c(t) = (\cos(t^3), \sin(t^3))$, $0 \le t \le 2$

$$\int_0^2 \sqrt{(x')^2 + (y')^2}\, dt =$$

$$\int_0^2 \sqrt{9t^4 \sin^2(t^3) + 9t^4 \cos^2(t^3)}\, dt =$$

$$\int_0^2 3t^2 \sqrt{1}\, dt = t^3 \Big|_0^2 = 8$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please initial: Yes__________ or No__________