AMAT 219 PRACTICE SHEET #6

1. Let R be the planar region enclosed by the semi-circle $y = \sqrt{1 - x^2}$ and $y = 0$.

Use polar coordinates to find $\iint_R y \, dA$.

2. Compute $\iint_D (x^2 + y^2)^{\frac{5}{2}} dA$, where D is the closed disk in the plane bounded by the circle of radius b centred at the origin .

3. Suppose R is the quarter circle bounded by $x^2 + y^2 = 3$, with $x \geq 0$, $y \geq 0$.

Use polar coordinates to find $\iint_R \frac{1}{\sqrt{1 + x^2 + y^2}} \, dA$.

4. Let R be the semi-circle centred at $(0, 0)$ and has radius 2 units with $y \geq 0$.

Use polar coordinates to find $\iint_R y^2 \, dA$.

5. Find the volume enclosed by the surfaces $z = 5 - (x^2 + y^2)$, and $z = 4$.

6. Use double integrals to find the volume enclosed by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 6 - x^2 - y^2$.

7. Use double integrals to find the volume of the solid enclosed by the two surfaces $z = (x^2 + y^2)^{\frac{3}{2}}$, and $z = 16 - (x^2 + y^2)^{\frac{3}{2}}$.

8. Find the volume of the region described by $x^2 + y^2 \leq z \leq 8 - (x^2 + y^2)$.

9. Use double integrals to find the volume enclosed by the two surfaces $z = x^2 + y^2$ and $z = 2 - \sqrt{x^2 + y^2}$.

10. Determine $\iint_R e^{-(x^2+y^2)} \, dA$, where R is the quarter circle $x^2 + y^2 = \alpha^2$, $\alpha > 0$,with $x \geq 0$, $y \geq 0$.

ANSWERS

1. $\frac{2}{3}$ 2. $\frac{2\pi}{7} b^7$ 3. $\frac{\pi}{2}$ 4. 2π 5. $\frac{\pi}{2}$

6. $\frac{32\pi}{3}$ 7. $\frac{192}{5}\pi$ 8. 16π 9. $\frac{5\pi}{6}$ 10. $\frac{\pi}{4}(1 - e^{-\alpha^2})$