Department of Mathematics and Statistics
AMAT 219 - QUIZ 1 - Thursday, January 26, 2006

U of C ID #

45 Minutes, Open Book, NO Calculators
To obtain credit you need to show your work. Work should be neat and organized.

Determine each of the following integrals : $k \in \mathbb{R}$.

1. $\int (x+1)\ln(x)\,dx$ (parts)

$\frac{(x+1)^2}{2}\ln(x) - \frac{1}{2}\int \frac{(x+1)^2}{x}$

$= \frac{(x+1)^2}{2}\ln(x) - \frac{1}{2}\left[\int\left(x + 2 + \frac{1}{x}\right)dx\right]$

$\frac{(x+1)^2}{2}\ln(x) - \frac{1}{4}x^2 - x - \frac{1}{2}\ln|x| + k$

2. $\int e^{(x+1)}\cos(x)\,dx$ (parts twice)

$e\int e^x\cos(x)\,dx$

$= e\left[e^x\sin(x) - \int e^x\sin(x)\,dx\right]$

$= e\left[e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)\,dx\right]$

$\frac{1}{2}e^{(x+1)}\left[\sin(x) + \cos(x)\right] + k$

3. $\int \frac{x^2}{(1-3x^2)^{3/2}}\,dx$ (inverse trig)

$\left\{ \sqrt{3}\,x = \sin\theta \;,\; \sqrt{3}\,dx = \cos(\theta)\,d\theta \right.$

$\int \frac{\frac{1}{3}\sin^2(\theta)\frac{1}{\sqrt{3}}\cos(\theta)\,d\theta}{(\cos^2(\theta))^{3/2}}$

$= \frac{1}{3\sqrt{3}}\int \tan^2(\theta)\,d\theta = \frac{1}{3\sqrt{3}}\int(\sec^2(\theta)-1)\,d\theta = \frac{1}{3\sqrt{3}}\left[\tan(\theta) - \theta\right]$

$\frac{1}{3}\left[\frac{x}{\sqrt{1-3x^2}} - \frac{1}{\sqrt{3}}\arcsin(\sqrt{3}x)\right] + k$

4. $\int \frac{x+1}{x^3-3x^2+2x}\,dx$ (partial fraction)

$$\frac{x+1}{x^3-3x^2+2x} = \frac{x+1}{x(x-2)(x-1)}$$

$$= \frac{a}{x} + \frac{b}{x-2} + \frac{c}{x-1}$$

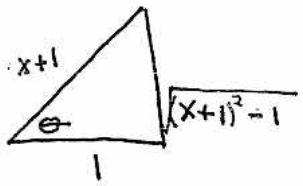
$$x+1 = a(x-2)(x-1) + bx(x-1) + cx(x-2)$$

$a = 1/2$, $b = 3/2$, $c = -2$.

$$\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x-2| - 2\ln|x-1| + k.$$

5. $\int \frac{1}{\sqrt{x^2+2x}}\,dx$ (complete square)

$$= \int \frac{1\,dx}{\sqrt{(x+1)^2-1}}$$



$$\begin{cases} (x+1) = \sec(\theta) \\ dx = \sec(\theta)\tan(\theta)\,d\theta \end{cases}$$

$$= \int \frac{\sec(\theta)\tan(\theta)\,d\theta}{\tan(\theta)} = \int \sec(\theta)\,d\theta$$

$$= \ln|\sec(\theta) + \tan(\theta)|$$

$$\ln\left|(x+1) + \sqrt{(x+1)^2-1}\right| + k.$$

Surname	Given Names	Lab #	Mark (20)

I agree that this paper may be placed at the front of the classroom for pick-up.

Please Initial Yes———— or No————