By popular demand, some quick solution to the 2005 AMAT 307 exams. These have NOT been checked for accuracy. M. Lamoureux.

1a) Use an integrating factor, soln $y = t^2/5 + C/t^3$.

b) This is an exact equation, soln $e^{xy} + x^2 y + y^2/2 = C$

c) We didn't cover equilibria in this class. But anyhow, the solutions are $= 1$ is unstable, $y = 5$ is stable, $y = 6$ is unstable.

d) Linear constant coefficient, soln $y = Ae^{-t} + Be^{-2t} - te^{-2t}$

e) The Wronskian is given by $W = C \exp \int 1/(t+1)dt = C/(t+1)$ and the initial condition gives $W = 3/(1+t)$

f) $A\cos(2t) + B\sin(2t) + Ct\cos(2t) + Dt\sin(2t)$.

g) $a_n = 2^n/n!$, radius of convergence is infinity.

h) Radius of convergence is 3, and it does not converge at $x = -2$.

i) $6e^{-2s+2}/(s-1)^4$

j) $2te^{-3t}$

k) $2e^t\cos(t) + 3e^t\sin(t)$

l) $y = Ae^t u_1 + Be^{-2t}u_2 + Ce^{-2t}u_3$

m) $y = 2e^t u_1 + e^{3t}u_2$

n) eigenvectors $u_1 = [1 \quad 0]'$, $u_2 = [1 \quad 1]'$ and solution $y = Ae^{2t}u_1 + Be^{3t}u_2$

o) Yuck, this involves some complex eigenvalues. Eigenvalues are $2 \pm i$. Eigenvector $[1 \quad i]$ corresponds to the eigenvalue $2 + i$. Soln is $y = Ae^{2t}[\cos(t) \quad -\sin(t)] + Be^{2t}[\sin(t) \quad \cos(t)]$.

2. Exponential growth is given by $y = y_o 2^{t/7}$ to get doubling every day, or $y = y_o e^{t\ln 2/7}$. So the predator free equation is $y' = (\ln(2)/7)y$. With the predators, it changes to $y' = (\ln(2)/7)y - 20,000$. This is a linear 1st order equation which is easily solved. Use the initial condition $y_o = 200,000$.

3. This is a Bernouli equation. Do I have to explain Bernouli again?

4. The homogenous equation $y'' - 4y' + 5y = 0$ has characteristic roots $2 \pm i$, so the general homogeneous solution is $Ae^{2t}\cos t + Be^{2t}\sin t$. To find the particular solution, first look for a particular solution $y_1 = C\cos(2t) + D\sin(2t)$ to get the $\sin(2t)$ part (solving for numerical values for C,D). Then look for $y_2 = Et^2 + Ft + G$ to get the t^2 part, again solving for numerical values of E, F, G. The final answer is

$$y = Ae^{2t}\cos t + Be^{2t}\sin t + (8/65)\cos(2t) + (1/65)\sin(2t) + (8/5)t^2 + (...)t + (...)$$

where you can figure out those last two constants.

5. Well, we didn't do Euler's equations in this class, but we can solve this anyway. To solve the homogeneous equation, $x^2y'' - 2y = 0$, look for solutions of the form $y = x^r$. Easy to see that $r = 2, -1$ work, so the general homogeneous solution is $y = Ax^2 + Bx^{-1}$. We then do variation of parameters (yikes!) and get the general solution

$$y = Ax^2 + Bx^{-1} - (1/3)x^{-1}\ln x.$$

6. Just write $y = 2 + x + Ax^2 + Bx^3 + \dots$ and plug into the DE and solve for A, B. Soln:

$$y = 2 + x - x^2 + (1/3)x^3 + \dots$$

7. $a_{n+2} = [(n^2 - n + 2)/(n^2 + 3n + 2)]a_n$

8. Soln is $y = Ae^{-t} + Bte^{-t} + 2t^2e^{-t}$.

9. Yuck. The eigenvalue 3 is degenerate, we only get one eigenvector $u_1 = [1 \quad -1]$. The corresponding generalized eigenvector is $u_2 = [-0.5 \quad -0.5]$. The general solutions is

$$y = Ae^{3t}u_1 + Be^{3t}(u_2 + tu_1).$$